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NON-LINEAR FINITE ELEMENT ANALYSIS OF A POLYMER-MADE MACHINE PART

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Abstract

Polymers are more and more popular as structural materials nowadays. However, their material behaviour is much more complicated than that of metals. In the present paper, a polypropylene-made automobile part is studied. The point of interest is its response to periodic excitations. Both geometric and material non-linearities are present in the problem. To solve it, a finite element model, composed of thick shell elements is prepared and presented. It uses a three-parameter Maxwell material model that is built by the authors based on measurement data from the literature. To verify the finite element results, measurements are also done and evaluated. Eigenfrequencies (of the non-linear system) are quite well approximated. However, as shown, other effects such as modal dissipation depending on frequency can not be modeled with this material law.

Keywords: polypropylene, finite element, viscoelastic, eigenfrequency.

1. Introduction

In the industry, the need for a quick, design-time analysis of machine parts is almost always present. In mechanical engineering finite element analysis is a common choice, both for metals and non-metals. For polymers, however, there seems to be no generally used material behaviour approximation. Material models found in the literature are either based on some theoretical considerations (e.g. [4], [5]) or of an empirical form (e.g. [6]), fitted on actual measurement data by parameters. These are not so well suited to finite element application. On the other hand, current finite element studies use mostly non-linear elastic laws (e.g. [1], [2], [3]) that do not take dissipation into account, and are therefore only adequate for static loads.

The final goal of our work is to estimate the expected lifetime of a polypropylene-made bracket supporting a box in a car. In this case load is clearly cyclic (the vibration of the chassis). Thus, phenomena to be taken into account include mechanical dissipation, temperature dependency and heat probably as well, generated by dissipation. Furthermore, since the box is relatively heavy, large strains are also expected. Consequently, both the finite element model and the material law should be prepared according to this.

The main objective of the present paper is first to develop and test a finite element model (composed of thick shell elements), and second to show a method to find 'eigenfrequencies' of a non-linear system. In order to do so, measurements have been performed and will be presented in the next section. Then, the finite element model will be described and its results shown. Finally, results by measurement and by computer simulation are compared, and conclusions are made.

2. Measurement

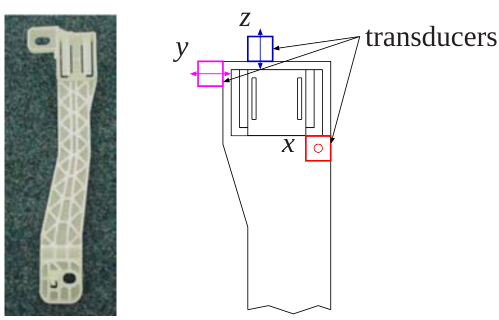

Fig. 1. Junction box bracket. The bottom part is fixed to the car body while the top part holds the actual box. In our measurements, the bottom part is fixed to the jarring table while accelerations are measured at the indicated positions.

The sample is of polypropylene, and is an actual product (see *Fig. 1*). It is about 20 cm long. In its preparation we used nothing special – it is just one piece from the production line.

For vibration measurements a Bruel&Kjær system has been used. A function generator with a jarring table, two piezo-electric accelerometers and a signal processing software have been used. (The mass of the accelerometers is considerable compared to that of the piece. However, it has also been taken into account in the finite element model so in this case, it is not a significant drawback.) A 10-100 Hz excitation frequency was applied to the bracket at the part that is fixed to the car body in operating condition. In a steady state, the time-acceleration function of this part and of the other end of the piece have been recorded (at the positions and directions marked in *Fig. 1*, but only with one accelerometer at a time). Results have then been Fourier-transformed, and a frequency (square of acceleration amplification) function has been calculated. Curves relevant to the first (both excitation and measured acceleration are in x-direction, i.e. perpendicular to the paper's plane) and second (excitation and measured acceleration are in y-direction) modes are shown in *Fig. 2*.

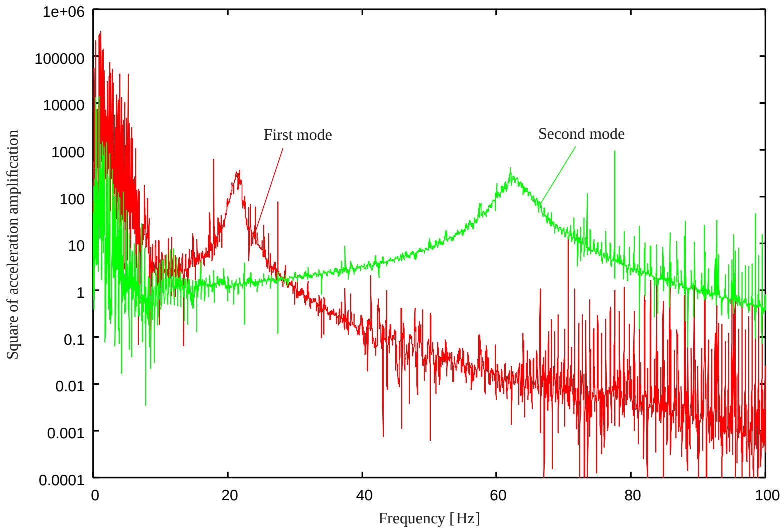

Fig. 2. Measurement results. The first mode is a bending vibration (excitation and accelerometer along x), while the second one is torsional (excitation and accelerometer along y).

3. Computer Simulation

3.1. Physical Model and Calculation Methods

The physical problem is non-linear, as mentioned in section 1. More precisely, the sources of non-linearity are:

- large displacements expected
- possibly large strains
- material behaviour

The short-term goal was to develop a method to determine the 'eigenfrequencies' of the system. In order to do so, two procedures have been applied:

- Linear eigenfrequency calculation is a fast and efficient way to determine the first few eigenfrequencies that are interesting from an engineering point of view. Furthermore, it also provides information about vibration modes (eigenvectors). However, it cannot take into account non-linear effects such as large displacements and non-linear material behaviour.
- Time integration is a very general way to find the time history of a certain physical quantity - for example the acceleration in some interesting points. These data can then be processed similarly to measurement data. The main drawback of this method is the huge computational effort that it requires.

To perform the calculations in both ways, the `MSC.Marc`[1] general-purpose finite element software suite has been used. The discrete Fourier transform has been made by `Octave`[2] (both for calculated and measured time-acceleration functions).

3.2. *Material Model*

Our material model has been built on a tensile test done by Duffo and his colleagues in [6]. Their measurements have been done at constant strain rates. In that paper they published a constitutive equation of the form

$$\sigma\left(\varepsilon,\dot{\varepsilon}\right)=k\left(1-\mathrm{e}^{-w\varepsilon}\right)\mathrm{e}^{h\varepsilon^{2}}\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0}}\right)^{m},\tag{1}$$

with $\dot{\varepsilon}_0$ being $1\frac{1}{\mathrm{s}}$ and the other parameters chosen according to measured results. At 20 °C their values are:

$$k=63.6\,\mathrm{MPa},\quad h=0.52,\quad m=0.082,\quad w=31.$$

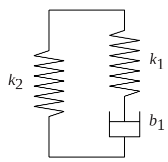

Fig. 3. Three-parameter Maxwell model

For the finite element study it was more appropriate to replace the above material law with a viscoelastic one. Thus, a three-parameter Maxwell model can be used (*Fig. 3*). In the Laplace-operator space it gives

$$G\left(s\right)=\frac{\Sigma\left(s\right)}{E\left(s\right)}=k_{2}+\frac{s}{\frac{s}{k_{1}}+\frac{1}{b_{1}}}=\cdots=\frac{\left(k_{1}+k_{2}\right)s+k_{2}a_{1}}{s+a_{1}},\tag{2}$$

where $a_1=\frac{k_1}{b_1}$, $\Sigma\left(s\right)$ and $E\left(s\right)$ are the Laplace-transformed forms of $\sigma\left(t\right)$ and $\varepsilon\left(t\right)$ respectively, while $G\left(s\right)$ is the transfer function between the two. From this, the response for a constant strain rate load ($E\left(s\right)=\frac{\dot{\varepsilon}_0}{s^2}$) by inverse Laplace transformation is[3]:

$$\sigma\left(t\right)=\dot{\varepsilon}_{0}\left(\frac{k_{1}}{a_{1}}+k_{2}t-\frac{k_{1}}{a_{1}}\mathrm{e}^{-a_{1}t}\right)=\cdots=\dot{\varepsilon}_{0}\left[b_{1}\left(1-\mathrm{e}^{-\frac{k_{1}}{b_{1}}t}\right)+k_{2}t\right],\tag{3}$$

[1] homepage: www.marc.com

[2] a free `MatLAB` clone for linux, homepage: www.octave.org

[3] The idea comes from [7].

Fitting this function to the one resulting from equation (1) up to $\varepsilon = 0.5$ leads to

$$k_1 = 1000\,\mathrm{MPa}, \quad b_1 = 35764\,\mathrm{MPa}\cdot\mathrm{s}, \quad k_2 = 10.685\,\mathrm{MPa}.$$

(See also *Fig. 4*.)

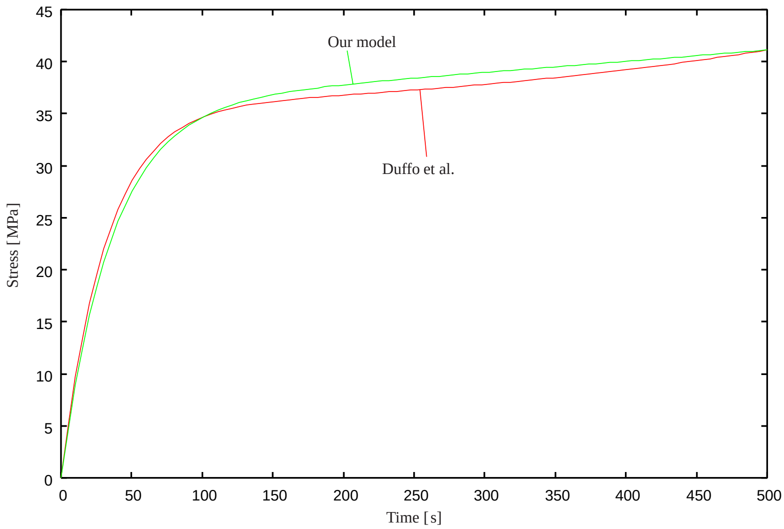

Fig. 4. Material law fitting (at $\dot{\varepsilon} = 10^{-3}\,\frac{1}{\mathrm{s}}$)

3.3. Finite Element Model

To produce the numerical results, a finite element model has been built. It is made of 4-node, bilinear, thick shell elements, and has 900 unconstrained degrees of freedom. The only boundary condition is the excitation of the jarring table. It is modeled by a prescribed displacement function (entered via a user subroutine) of the part that has been fixed to the table. For the eigenvalue-eigenvector extraction these displacements have been fixed to zero.

In the case of the time integration method, the integration time step is an important question.[4] Using 10% of the minimal time period of interest (i.e. the one belonging to 100 Hz here), a good result is given by the method. Cutting back this time step more and more, there are changes in the acceleration vs. time function but the spectrum itself does not vary considerably.

[4] So is the time integration scheme. Currently, we can only use the single-step Houbolt method of `Marc 2003`.

4. Results and Discussion

4.1. Comparison of Results

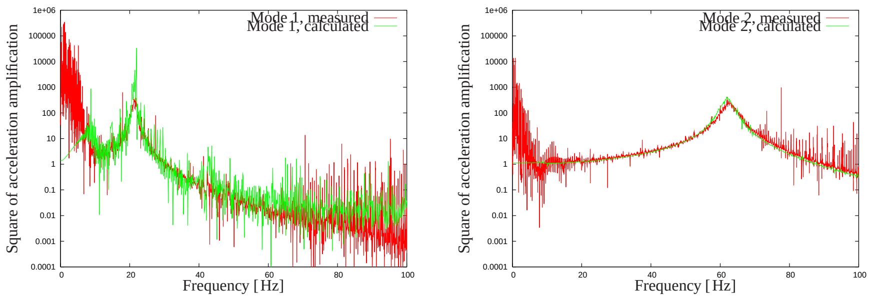

Fig. 5. Measured and calculated transfer functions for modes 1 and 2.

	Mode 1	Mode 2
Linear calculation	22,1 Hz	66,3 Hz
Non-linear calculation	21,9 Hz	61,9 Hz
Measurement	21,3 Hz	62,0 Hz
		

Table 1. Eigenfrequency values

Results can be seen in *Fig. 5* and are also resumed in *Table 1*. An immediate observation is that eigenfrequencies calculated with a linear (eigenproblem approach) and a non-linear (time integration) material model are almost identical, but the latter is slightly lower. It is, in fact, expectable because non-linear effects mean a softening here.

It can also be seen that the calculated amplification peak for mode 1 is 1-2 orders of magnitude higher than the measured one. From this fact, a trivial conclusion is that real dissipation is higher than the one in our material model. Thus, a next step could be doing measurements related to dissipation (e.g. measuring the decrease of amplitude in free vibration) and to modifying the material model according to the results. In the next section, this will be attempted.

4.2. Comments about the Material Model

As mentioned earlier, as a material law, the three-parameter Maxwell model has been used (see section 3.2). However, obtaining more data on the material and

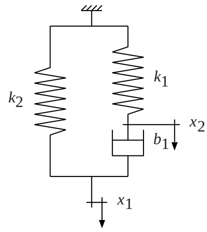

Fig. 6. Three-parameter Maxwell model

trying to adjust our model accordingly, the following problems are encountered:

- With $\dot{\varepsilon} = \text{const.}$ and $t \to \infty$, spring k_1 opens sufficiently for $b_1 (\dot{x}_1 - \dot{x}_2) = k_1 \dot{x}_2$ and remains in that state. Therefore, a constant force is supplied by them and the slope of the $\sigma(t)$ function is entirely given by k_2. (This is logically a small value.)
- In a static problem, i.e. for a given and constant force and $t \to \infty$, the force on k_1 tends to zero, and long-term stiffness is given by k_2. (This is logically not such a small value as it was in the previous paragraph - thus, both needs can not be satisfied simultaneously.)
- Calculations can be made for dissipations with $F(t) = F_0 \mathrm{e}^{\mathrm{i}\omega t}$, F being the force applied to the Maxwell element. Here F vs. x_1 functions can be found (i.e. hysteresis ellipses) – however, such measurements are not too simple to carry out with our facilities.
- Measurements on dissipation that are the simplest for us are free vibration measurements. For the two modes presented in section 4.1, these experiments have been done. Out of them, dissipation values can be found for the two frequencies. Thus, the next question is how to set the model for frequency-dependent dissipation values.

  For free vibrations one has to add a mass m to the Maxwell element. The eigenfrequency of the element depends then on the value of this mass, and then the question is the dissipation of the free vibration in function of m. It is evident to write the system of governing equations

$$m\ddot{x}_1 + k_2 x_1 + b_1 (\dot{x}_1 - \dot{x}_2) = 0, \tag{4}$$

$$k_1 x_2 - b_1 (\dot{x}_1 - \dot{x}_2) = 0. \tag{5}$$

  However, solving this one finds that there are three eigenvalues λ_i, two of which (λ_2 and λ_3) are complex conjugate and one (λ_1) real, with all real parts negative. Moreover, $\Re\lambda_{2,3}$ is practically independent from $\Im\lambda_{2,3}$ – i.e. from the frequency (that depends on m). Thus, it seems that frequency-dependent dissipation can not be entered into this model.

From the above, one can conclude that this material model, although simple and easy to handle, has only a limited usability for our problems and it seems, it needs to be replaced by a more complex one later.

5. Conclusions

- In this paper it has been shown that thick shell elements in `MSC.Marc` are able to give satisfying results for eigenvalue problems of the piece that has been studied. Moreover, by time integration and Fourier transformation, non-linear cases can also be analyzed.
- A finite element calculation with a three-parameter viscoelastic material model has been carried out, and its results have been compared to measurement results. A fair correspondence has been observed between them.
- The three-parameter material model has also been examined. We have seen that it is unable to produce every phenomenon in material behaviour.

Countinuing our work, the next steps are to find a more general material model and then do measurements to fit it to reality.

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