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THE SECOND SOUND PHENOMENON: PRO AND CONTRA

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Abstract

The paper deals with the problem of modification of Fourier Law. After a short summary on the basic problem of the second sound phenomenon the different opinions are listed and commented. Also the effect of the second sound phenomenon on the thermoelasticity is analyzed.

Keywords: heat conduction, second sound phenomenon, thermoelasticity.

1. Introduction

In 1867 Maxwell turned the attention to the theoretical contradiction hidden in the Fourier's law. As a result, also the practical application is limited by this contradiction. It has been a surprise because this law (and its relatives, as Fick's, Darcy's, etc.) is one of the most effective and widely used tools in physics. Maybe this Janus face is the reason of the discussion on the topic. The problem divides almost all the experts having any kind of relation to the topic: thermodynamicians, mechanicians, thermotechnicians, etc.

The core of the problem is very simple. The heat propagation task described by the Fourier's law leads to the parabolic differential equation and as a consequence to unlimited speed of propagation. As a resolution of this kind of contradiction, dozens of modified laws [1, 2] and modification methods, e.g. [3], have been worked out. The common feature of these efforts is that all lead to hyperbolic differential equation and the speed of propagation becomes limited. This is the origin of the name 'second sound' that became the emblem of the whole phenomenon (SSP).

While the core of the problem is as simple as described above, the practical aspect is very complicated. First of all, because the modification needs at least one new material property, the so called 'relaxation time'. And the reliable experimental determination of this property has not been solved yet [4]. It's clear that without this, the whole theory is unstable. There are several other problems, too, e.g. the solution applicable in heat conduction is not satisfactory in thermoelasticity [5]. It is an other practical problem that the question has importance only in the dynamical tasks, that are very seldom. Similarly, in case of metals, the problem has less importance than in case of more up to date engineering materials as e.g. composites.

Based on all of these, it is understandable that the opinion of the experts on the SSP is really widely differing. The different opinions have been collected. Beside this, the paper deals with a basic summary on the SSP and analyzes the relation of SSP and thermoelasticity.

2. Short Summary on the Second Sound Phenomenon

The heat conduction problem by the energy equation, equation of state and Fourier's law, neglecting the mechanical interactions, leads to the parabolic differential eq. on the temperature T,

$$\mathrm{d}\, T_{xx} + T_t = 0, \tag{1}$$

where $\mathrm{d} = k/\rho c$ is the diffusivity, t and x are the time and space coordinates, the subscript denotes partial derivation. It results in unlimited speed of temperature wave propagation, $v_T \to \infty$.

To resolve this contradiction several attempts were made [1, 2]. One of them, maybe the first one was the Cattaneo-Vernotte modified law $(C - V)$ of heat conduction, as

$$\tau h_t + h = -kT_x, \tag{2}$$

where h is the heat flux, k is the conductivity and τ is the so called relaxation time, the new material property. Replacing the Fourier's law by the $C - V$, the result is the

$$\mathrm{d}T_{xx} + T_t + \tau T_{tt} = 0, \tag{3}$$

hyperbolic equation with the propagation speed

$$v_T = \sqrt{\mathrm{d}/\tau}. \tag{4}$$

An other possible resolution is to change the equation of state to

$$e = cT + c_1 T_t, \tag{5}$$

where e is the internal energy, c is the heat capacity, $c_1 = \tau c$, while keeping the law of heat conduction (Fourier) unaltered. The result is the same as above [3].

The result has been generalized in two directions: several new modified laws of heat conduction were published [1, 2], and also modification methods were worked out e.g. [3]. In spite of all these efforts, the problem of the new material properties has remained [4, 6].

There are further aspects of the SSP. One of them is the other phenomena of physics described by similar laws: moisture diffusion (Fick's law), groundwater flow (Darcy's law), dislocation diffusion, solution process, etc [7]. Also the problem of material arises. While the relaxation time in case of metals has a very small value, this material property is bigger by order of magnitudes in case of polymers (composites), human tissues, etc.

3. Pro and Contra

The variety of the experts' opinions is very wide. The mechanicians, first of all those dealing with thermomechanics, support the idea of applying the SSP in mechanics.The reason seems to be very simple. The theoretical aspects of the SSP are very convincing and this group has minor problem with the application. Of course, there are exceptions, see later.

The thermotechnicians are mainly against it. The reason is clear and practical. The whole theory of heat-exchangers is based on the Fourier's law. Knowing the importance of this field of engineering, nobody wants to risk the value of this huge knowledge accumulated in the last 60-80 years. Of course, there are exceptions also in this case.

One of my basic experiences concerning the rejection of SSP is coupled to a first class thermotechnician who was the referee of my first paper on the topic. Looking at the τh_t member of the $C - V$, he asked me about the physical meaning compared to the Fourier's law.

Of course, it is impossible to give a satisfactory answer. It is more a philosophical problem whether we are able to explain the modification on the basis of the law we want to modify.

Another example of the rejection is from a mechanician [8]. His argument was more covincing, but not satisfactory. Let us start with the $C - V$, see *Eq.* (2), but supposing uniform temperature distribution, i.e. $T_x = 0$ and as a consequence

$$\tau h_t + h = 0. \tag{6}$$

The solution of this differential *Eq.* is

$$h = A \exp\left(-t/\tau\right). \tag{7}$$

It means, the heat flux exists even though $T_x = 0$, i.e. inspite of missing the driving power, and that is impossible. The argument is false, because Cattaneo–Vernotte's main statement is that not only the temperature gradient could be the reason of the heat flux.

The argument of the very practical thermotechnician is understandable from that point of view, too, that the experience doesn't help too much. The problem comes up only in dynamical cases, mostly with nonmetallic materials. By the way, this two parallel features of the problem (process and material) are shown very characteristically by the theory of similitude that results the dimensionless number $\tau\pi/t_0$, where τ is a material while t_0 is a process property.

The argument of the mechanician is not understandble, because in the continuummechanics the SSP is an everyday problem. Also the modern life of the question has been started in the early 1950s in the thermoelasticity. And it is not by chance. The thermoelasticity is the most often used coupled field of the engineering and within it the dynamical tasks became very frequent, which brought the need of modification, i.e. SSP. Danilovskaya [9], having solved the thermal shock task of elastic material, experienced disturbances before the wave front and she found the reason in the not satisfactory law of heat conduction. Even though, the symbiosis of the SSP and the thermoelasticity is not bilateral. As we will see in the next chapter, the solution which is satisfactory in heat conduction doesn't give a solution in the thermoelasticity.

The complicated feature of SSP can be illustrated by personal experience on the R&D field. A few years ago I was invited to the ANSYS headquarters to discuss the problem of SSP. I tried to convince them to extend their code into this direction. I wasn't able, and the final argument was: no need for SSP from the customers. I tried to make it clear that it is a 'vicious circle': no need to buy it because there is no possibility to buy it. Now, I am in a similar relation to the ESI Group, I hope to have more success.

I have to mention one more aspect of the SSP. The tradition causes very hard problem. Of course, in some sense it is advantageous, but if it becomes too hard it creates more problems than advantage. I guess, this is the case with the modification of one of the most celebrated laws of the physics, i.e. the Fourier's law.

4. Second Sound Phenomenon and Thermoelasticity

As we saw previously, dealing with the pure heat conduction problem the modification in both cases according to the *Eq.* (2) (modified law of heat conduction), or *Eq.* (5) (modified state equation) lead to the same hyperbolic equation, and the problem of unlimited wave propagation is solved.

Dealing with the coupled problem of the thermoelasticity, let us derive the generalized equations of motion and the heat conduction based on the basic equations of the system:

$$u_{tt} - \frac{E}{\rho} u_{xx} + \frac{E\alpha}{\rho} T_x = 0, \tag{8}$$

$$\mathrm{d}T_{xx} + \tau T_{tt} + T_t - \frac{\tau E}{\rho c} (u_x - \alpha T)\, u_{xtt} - \frac{E}{\rho c} (u_x - \alpha T)\, u_{xt} = 0, \tag{9}$$

where u is the displacement, E is the Young modules and α is the coefficient of thermal expansion.

The following comments apply to the equation system (8) and (9).

a. Neglecting the interactions between temperature and displacement, the system splits into the independent equations of

$$u_{tt} - \frac{E}{\rho} u_{xx} = 0, \tag{10}$$

$$\mathrm{d}T_{xx} + \tau T_{tt} + T_t = 0. \tag{11}$$

Eq. (11) shows the different cases of the heat conduction:

- by the modified laws for both cases $C - V$ and state equation,

- by the Fourier taking $\tau = 0$.

b. For the cross-coupled case the generalized equation of motion, *Eq.* (8), is not affected by the modification.

c. While the generalized equation of heat conduction, *Eq.* (9), is different from Fourier ($\tau = 0$), for $C - V$ (all members exist) and for modified state equation (4$^{\text{th}}$ member is equal to zero).

It verifies the basic statement that the solution of the SSP satisfactory in heat conduction does not give perfect solution in thermoelasticity. Generalized, it shows the basic role of mechanics in thermodynamics.

5. Summary

The old problem of the SSP hasn't been solved yet. The very complicated feature of the question has several signs. One of them is the diversity of opinions of the different experts. Another one is connected to the thermoelasticity, i.e. while the contradiction seems to be solved in the heat conduction, the coupled temperature and displacement causes unsolved problems.

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