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NON-STATIONARY TEMPERATURE FIELD OF INFINITE CYLINDER AT CO-CURRENT CONTACT WITH LIQUID MEDIUM

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Abstract

In this paper, an analytical solution of the Fourier–Kirchoff equation for heat conduction is presented for an infinite cylinder assuming co-current flow contact with liquid. The solutions are obtained in non-closed form as an expansion of series and is rearranged into a non-dimensional form.

Keywords: extrusion, non-stationary temperature field.

1. Introduction

At the formulation of the task the following simplifications were considered. At the origin of the coordinate system the processed extruded bar enters into the calculation with an ideal cylindrical shape of radius R and isotropic material properties. (*Fig.1*) The bar material at the beginning is uniformly heated and it has an initial temperature T_{s0}. Around the bar there is a cylindrical space created by a perfectly isolated bigger size pipe, where the co-current cooling (heating) medium enters with an initial temperature T_{f0} and it is in direct contact with the extruded bar. The motion of the bar is steady and according to the moving piston effect it predetermines the liquid flow in the cylinder. Inside the solid phase we do not consider heat sources. During the solution we assume the thermo-mechanical material properties of the bar material and the liquid (c_f, c_s, λ_f, λ_s) to be constant, that is independent of the temperature. The coefficient of heat transfer between the bar wall and the liquid remains constant too. The heat due to radiation is included in the coefficient of heat transfer α. The mass flow of the liquid M_f and the bar material M_s does not vary with time.

2. Mathematical Formulation of the Problem

Considering the aforementioned suppositions the Fourier–Kirchoff equation of heat conduction can be transformed into the following term

$$\frac{\partial T}{\partial t} = a\left(\frac{\partial^2 T}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial T}{\partial \rho}\right). \tag{1}$$

The initial and boundary conditions are the following ones. The temperature of the bar and the gas at the entry is constant.

$$t = 0, \qquad T_s = T_{s0}, \qquad T_f = T_{f0}. \tag{2}$$

The heat exchange on the border between the phases is given by the equation

$$\alpha\left[T_f - (T_s)_{r=R}\right] = -\lambda_s\left[\frac{\partial T_s}{\partial r}\right]_{r=R}. \tag{3}$$

The gas temperature can be calculated according to the heat balance law

$$M_s c_s (T_{sc} - T_{s0}) = M_f c_f (T_{f0} - T_f), \tag{4}$$

where

$$T_s\Big|_{r=R} = T_{sp}.$$

The symmetry conditions imply the temperature gradient to be zero on the surface of symmetry.

$$\left[\frac{\partial T_s}{\partial r}\right]_{r=0} = 0. \tag{5}$$

Let's introduce the following dimensionless variables

$$\mathrm{Bi} = \frac{\alpha R}{\lambda_s} \qquad \text{Biot number,}$$

$$\mathrm{Fo} = \frac{at}{R^2} \qquad \text{Fourier number,}$$

$$m = \frac{M_s c_s}{M_f c_f} \qquad \text{thermal capacitance ratio of the contact phases}$$

$$\rho = \frac{r}{R} \qquad \text{dimensionless coordinate,} \tag{6}$$

$$\Theta_s = \frac{T_s - T_{s0}}{T_{f0} - T_{s0}} \qquad \text{relative temperature difference of the solid phase,}$$

$\Theta_{sc} = \dfrac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$ average calorimetric relative temperature difference,

$\Theta_{sp} = \dfrac{T_{sc} - T_{s0}}{T_{f0} - T_{s0}}$ surface relative temperature difference,

$\Theta_{f} = \dfrac{T_{f} - T_{s0}}{T_{f0} - T_{s0}}$ relative temperature difference of the gas phase.

The heat conduction equation can be combined as follows.

$$\frac{\partial \Theta_s}{\partial Fo} = \frac{\partial^2 \Theta_s}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial \Theta_s}{\partial \rho} \tag{7}$$

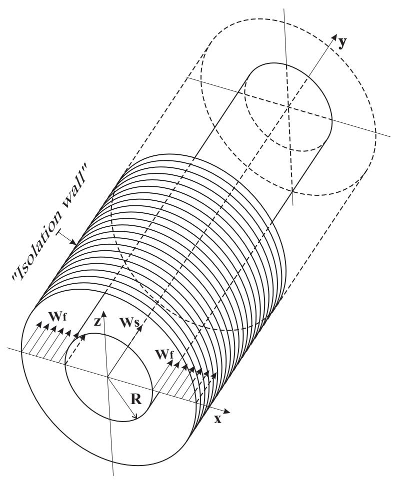

Fig. 1. Solid and liquid phase flux direction.

The heat exchange can be expressed as follows:

$$1 - m\Theta_{sc} - \Theta_{sp} = -\frac{1}{\mathrm{Bi}}\left[\frac{\partial \Theta_s}{\partial \rho}\right]_{\rho=1}. \tag{8}$$

Initial and boundary conditions are the following ones:

$$\mathrm{Fo} = 0; \qquad \Theta_{sp} = 0, \qquad \Theta_{sc} = 0, \tag{9}$$

Balance equation

$$\Theta_f = 1 - m\Theta_{sc}, \tag{10}$$

where the average calorimetric temperature of the solid phase is defined as

$$\Theta_{sc} = 2\int_0^1 \rho\Theta_s \,\mathrm{d}\rho. \tag{11}$$

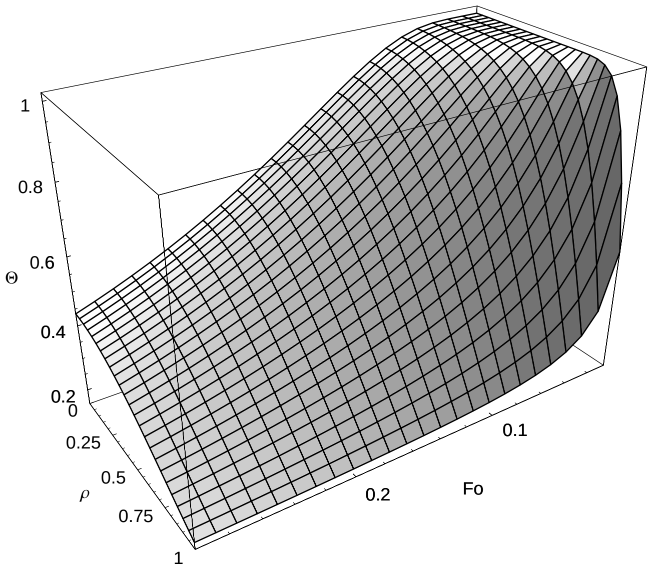

Fig. 2. Temperature distribution over the cylinder for Bi $= 10$ and $m = 0.2$

Utilizing the Fourier method after substitution, the temperature field of the infinite cylinder can be found in terms of an infinite series as a function of dimensionless time Fo, coordinate ρ, temperature capacitance ratio m and Biot number Bi.

$$\Theta_s = \frac{1}{1+m} + \sum_{i=1}^{\infty} \frac{-2k_i J_1(k_i)}{4m J_1^2(k_i) + k_i^2 \left[J_0^2(k_i) + J_1^2(k_i) \right]} e^{-k_i^2 \mathrm{Fo}} J_0(k_i \rho). \tag{12}$$

Average calorimetric temperature of the bar also depends on the time Fo, the temperature capacitance ratio m and Biot number Bi.

$$\Theta_{sc} = \frac{1}{1+m} - \sum_{i=1}^{\infty} \frac{2 J_1^2(k_i)}{4m J_1^2(k_i) + k_i^2 \left[J_0^2(k_i) + J_1^2(k_i) \right]} e^{-k_i^2 \mathrm{Fo}}. \tag{13}$$

The liquid phase temperature depends on the time Fo, the temperature capacitance ratio m and Biot number Bi as well.

$$\Theta_f = \frac{1}{1+m} + m \sum_{i=1}^{\infty} \frac{4 J_1^2(k_i)}{4m J_1^2(k_i) + k_i^2 \left[J_0^2(k_i) + J_1^2(k_i) \right]} e^{-k_i^2 \mathrm{Fo}}. \tag{14}$$

Where the constants $k_1, k_2, \ldots k_I$ (which are dependant on the Biot number and the thermal capacitance ratio of contact phases) can be determined according to the following transcendental equation

$$\frac{k_i}{B_i} = \frac{2m}{k_i} + \frac{J_0(k_i)}{J_1(k_i)}. \tag{15}$$

Fig. 2 shows the dimensionless temperature distribution as a function of the dimensionless radius and the dimensionless time, included into the Fourier number, for parameters Bi $= 10$ and $m = 0.2$.

3. Closure

The presented paper solves the non-stationary temperature field of the infinite cylinder at the co-current contact with liquid medium. The derived analytical solution shows us the dimensionless temperature dependence of the cylinder on the dimensionless coordinate ρ, Fourier number Fo, Biot number Bi and temperature capacitance ratio of both phases m.

4. Used Symbols

a	coefficient of temperature conductivity	$[m^2s^{-1}]$
M	mass flow	$[kgs^{-1}]$
m	thermal capacitance ratio of contact phases	[–]
R	outer radius of cylinder	[m]
t	time	[s]
T	temperature	[K]
c	specific heat	$[Jkg^{-1}K^{-1}]$
α	coefficient of heat transfer	$[Wm^{-2}K^{-1}]$
ρ	radial coordinate	[m]
λ	heat conductivity	$[Wm^{-1}K^{-1}]$
Θ	dimensionless temperature	[–]

Subscripts

f	fluid phase
s	solid phase
0	initial value
p	variable value on the surface
c	calorimetric
0	0-th order
1	1-th order

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