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BASIC EQUATIONS AND DIFFERENT APPLICATIONS (BEDA) OF CONTINUUM MECHANICS[1]

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Abstract

The equations of continuum mechanics can be split into four groups, one of them contains the constitutive equations. The revision of the system shows that all groups can be completed, but most of the latest researches aim to the group of constitutive equations. In the present paper we display the results of this revision.

Keywords: continuum mechanics, thermo-hygro, composite, second sound, similarity.

1. Introduction

The basic equations of continuum mechanics [1] can be split into four groups: laws of nature, kinematical, constitutive and complementary equations. The revision of the system shows that more or less all groups can be completed, but most of the latest researches aim to the group of constitutive equations [2].

The following results were obtained through systematic review of the basic equations of continuum mechanics and by their application in engineering, teaching and generalization. In this paper we are only going to mention them, mainly referring to the original papers.

2. Revision of Basic Equation of Thermo-Hygro-Mechanics (THM)

The revision of the basic equations of thermo-elasticity (TE) shows that the generalized equation of heat conduction is needed to be completed with a new member, according to the large changes of temperature. See *Eq.* (8) in [3] and the comment following it.

[1]This paper is based on the presentation given in the frame of Beda Symposium at the FUDoM'01 International Conference of Mechanics. The Symposium was devoted to honouring Professor Beda on the occasion of his 70th birthday.

According to the second sound phenomenon the TE needs to be rewritten by the modified law of heat conduction. Details of the modification method and the application to TE can be seen in [4, 5, 6].

To make the equation system of TE complete, a further equation, the so-called *thermokinematical* one is needed, with the new notion of *thermokinematical potential* [7]. By these in $1D$ the thermokinematical equation reads

$$h_t = q_x, \tag{1}$$

where $h(x, t)$ is the heat amount, $q(x, t)$ is the heat flux and by the $\beta(x, t)$ thermokinematical potential they can be obtained through the expressions

$$h = \beta_x, \qquad q = \beta_t. \tag{2}$$

Applying *Eq.* (2) in (1), the names, their origins and the analogy to kinematic notions (u, ε, v) are obvious. By the way with the thermokinematical potential, $\beta(x, t)$ also the system of TE potentials can be completed [7].

The biggest item on the list is the analogy between heat and moisture. Most of the consideration are based on the analogies between FOURIER's & FICK's laws and on SORET & DUFOUR effects [8, 9]. The investigations are extended for the coupling of diffusion and convection [9, 10], for the application of theory of similitude [11], resulting new nondimensional numbers. The reciprocal relations generally and especially the ONSAGER's one are also included into the investigations [12, 13].

As a special case of THE, also the HE is dealt. In this case, beside of the analogy, further theoretical investigations were needed [14].

3. Different Applications

The application of the previously mentioned theoretical results is very widespread. First of all we have to mention the engineering applications e.g. [15], but also the teaching and the generalization are important fields of applications.

The most important engineering application is the THM tailoring of composites. There are several papers in this field, as typical examples let us recall, e.g. [8], [16]–[24].

Other, very practical engineering applications are the THM composites in automotive industry [25, 26] and in civil engineering & architecture [14].

Generally, in case of any application the problem of material properties, i.e. the numerical values come up. To solve this problem, experimental possibilities and analogies were worked out [16], [27]–[35].

As we mentioned before, a wide range application was possible in our teaching courses, e.g. [7].

A further possibility was the generalization of the problem on the basis of parabolic and hyperbolic partial differential equations, see e.g. [36]–[38].

References

[1] BÉDA, GY. – KOZAK, I. – VERHAS, J., *Continuum Mechanics*, Akadémiai Kiadó, Budapest, 1995.

[2] SZEKERES, A. – BÉDA, GY., The Role of Acceleration Wave in the Research of Basic Equations of Solid Mechanics, *ZAMM*, **63** (1983) 4/5.

[3] SZEKERES, A., Some Remarks on the Equations of Thermo-elasticity, *ZAMM*, **57** (1978) pp. 160–161.

[4] SZEKERES, A., The Equation System of Thermoelasticity Using the Modified Law of Thermal Conductivity, *Periodica Polytechnica*, **24** No. 3 (1980), pp. 253–261.

[5] SZEKERES, A., Experiments on Thermal Shock of Long Rods (With M. Szalontay), *Periodica Polytechnica*, **24** No. 3 (1980), pp. 243–252.

[6] SZEKERES, A., Application of the Modified Law of Heat Conduction and State Equation to Dynamical Problems of Thermoelasticity. (With dr. I. Farkas) *Periodica Polytechnica*, **28** No. 2–3 (1984), pp. 163–170.

[7] SZEKERES, A., Course on Thermomechanics, Virginia Tech. 1994, TU Budapest, 1998–2003.

[8] SZEKERES, A. – ENGELBRECHT, J., Coupled Thermal and Moisture Fields with Application to Composites, *Periodica Polytechnica,* **41** No. 2. (1997), pp. 151–161.

[9] SZEKERES, A., Analogy Between Heat and Moisture. Thermo-Hygro-Mechanical Tailoring of Composites by Taking into Account the Second Sound Phenomenon, *Journal of Computers & Structures* **76** (2000), pp. 145–152.

[10] SZEKERES, A. – ENGELBRECHT, J., Coupling of Generealized Heat and Moisture, *Periodica Polytechnica* **44** No. 1 (2000), pp. 161–170.

[11] SZEKERES, A., Possible Extension of Theory of Similitude for Moisture Convection, *Proc. SolMec, Zakopane*, Poland, Sept 1–5, 1998.

[12] SZEKERES, A. – PRAMILA, A. – HANNILA, R., Cross-Coupled Heat and Moisture Diffusion – Application of Onsager's Relation, Pres. held at the 11th Int. *Conf. on Mechanics to Composite Materials*, Riga, Latvia, June 11–15, 2000.

[13] SZEKERES, A., *Application of Reciprocal Relations in THM*, Seminar held in the Institute of Cybernetics, Tallinn, Estonia, De 2. 1999.

[14] SZEKERES, A. – PRAMILA, A., Fiber Reinforced Composite Structures with Hygroscopic Element in Civil Engineering, *Proc. of the Centenary Conference on Innovation in Civil and Structural Engineering*, Cambridge, England, August 19–21. 1997.

[15] SZEKERES, A., *Application of Thermo-Hygro-Elastic (THE) Fiber Reinforced Composites (FRCs) in Engineering Constructions*, Seminar held in the Aerospace Eng. Dept. University of Michigan, Ann Arbor, October 2, 1997.

[16] SZEKERES, A., *Thermo-Hygro-Viscoelastic (THVE) Tailoring of Fiber Reinforced Composites (FRCs): Theoretical, Numerical, Experimental and Practical Problems*, Seminar held in the Dept. of Engg. Science and Mechanics, Virginia Tech. November 6, 1996.

[17] SZEKERES, A., Possible Extension of Tailoring of Composites through Inhomogeneity (Co-author R. Heller) Pres. held at Fifth Pan American Congress of Applied Mechanics, San Juan, Puerto Rico, January 2–4, 1997.

[18] SZEKERES, A., Theory and Practice of Thermo-Hygro-Mechanical Tailoring of Fiber Reinforced Composites, Some Results on Diffusivities. (Co-authors A. Pramila and R. Heller), *Proc. of Fourth International Conference on Composite Engineering*, ICCE 4, 6-12 July, 1997, Big Island of Hawaii.

[19] SZEKERES, A., Thermodynamics of Complex Systems: Special Problems of Coupled Thermal and Moisture Fields and Application to Tailoring of Composites (Co-author R. Heller), *Periodica Polytechnica, Ser. Chem Eng.* **42** No. 1 (1998), pp. 55–64.

[20] SZEKERES, A. – ENGELBRECHT, J. – HELLER, R. – KISS, P. – PRAMILA, A., Thermo-Hygro-Mechanical (THM) Tailoring of Fiber Reinforced Composites (FRC) for Mechanical Engineering Application, *Proc. of 1st Conference on Mechanical Engineering*, TU Budapest, May 28–29, 1998.

[21] SZEKERES, A., Thermo-Hygro-Mechanical (THM) Tailoring of Composites by Taking into Account the Second Sound Phenomenon, *Proc. of NATO Advanced Study Institute on Mechanics of Composite Materials and Structures*, Troia, Portugal, 12–24 July, 1998.

[22] SZEKERES, A., Special Possibilities for Tailoring of Composites on the Basis of Coupled Fields, *Proc. of Sixth International Conference on Composite Engineering. ICCE 6*, June 27 – July 3, 1999. Orlando, Florida.

[23] SZEKERES, A., *Thermo-Hygro-Mechanical (THM) Tailoring of Fiber Reinforced Composite (FRC) Material and Structures Theory and Application in Engineering*, (Co-authors A. Pramila, R. Hannila), Seminar held in the Engineering Mechanics Laboratory, University of Oulu, June 5, 2000, Oulu, Finland.

[24] SZEKERES, A., *Thermo-Hygro-Mechanical (THM) Tailoring of Fiber Reinforced Composites (FRC)*, (Coauthor A. Pramila). Seminar in the Dept. Applied Mechanics, TU Budapest, Oct. 25, 2001.

[25] SZEKERES, A., *THM Composites in Engineering, Application in Automotive Industry*, Pres. held at the Int. Mech. Eng. 2000 Congress & Exposition, Orlando, Florida, Nov 5–10, 2000.

[26] SZEKERES, A., *Ideas on Hygro-Thermal Composites for Automotive Industry*, Pres. held at 6^{th} Technical Conference and Reviews of Center for Composite Materials and Structures, Virginia Tech, VA, April 19–21, 1998.

[27] SZEKERES, A., Similarities and Differences Between Heat Conduction and Moisture Diffusion (Co-authors R. Helles and S. Thangjitham), *Proc. of Thermal Stresses'97*, 8–11 June, 1997, RIT, Rochester, NY.

[28] SZEKERES, A., *Feasibility of Electrical Analogy in Compled Fields of Mechanics*, Pres. held at McNU'97 29 June – 2 July 1997, Northwestern University, Evanstone, IL.

[29] SZEKERES, A., *Coupled Fields and Electrical Analogies of Mechanics*, Pres. held at 25^{th} Midwestern Mechanics Conf. Sept. 21-24, 1997, South Dakota School of Mining and Technology, Rapid City, SD.

[30] SZEKERES, A., *Possible Analytical Solution of Crosscoupled Heat and Moisture Transfer by Electrical Analogy*, Seminar held in the Dept. of Mechanical Engg. Saginaw Valley State University, October 3, 1997.

[31] SZEKERES, A. – KOIVUROVA, H., Theoretical and Experimental Investigation on the Diffusivities of Composites, *Periodica Polytechnica*, **41** No. 2 (1997), pp. 143–150.

[32] SZEKERES, A., Possible Extension of Theory of Similitude for Moisture Convection, *Proc. of 32^{nd} Polish Solid Mechanics Conference*, Zakopane, Poland, Sept. 1–5, 1998.

[33] SZEKERES, A., Possible Method for Obtaining the Material Properties of Thermo-Hygro-Elastic Materials (In Slovakian, with P. Élesztős), *Proc. of Conference on Mechanical Engineering*, Faculty of Mechanical Engineering, Slovak Technical University, Bratislava, Sept 17, 1998.

[34] SZEKERES, A., Modelling of Composites by Electrical Analogies, *Proc. of Annual Technical Meeting of Society of Engineering Science*, Washington State University, Pullman, WA, Sept. 27–30, 1998.

[35] SZEKERES, A., Symmetry of Cross-Coupled Heat and Moisture Diffusion with Second Sound Phenomenon Application of Onsager's Reciprocal Relation and of Electrical Analogy, *Proc. of 4th Int. Workshop on Similarity Methods*, University of Stuttgart, Nov. 5/6. 2001. pp. 79–89.

[36] SZEKERES, A., Special Problems of Bio-Thermo-Hygro-Mechanics, *Proc. of CANCAM'99, McMaster Univ.*, Hamilton, Ontario, May 30 – June 3, 1999.

[37] SZEKERES, A., *Possible Generalization of Thermo-Hygro-Mechanics: Application in Agriculture*, (Co-author I. Adam). Pres. held at the South African International Conference on Applied Mechanics, Jan 11–13, 2000, Durban South Africa.

[38] SZEKERES, A., *Feather and Down Reinforced Composites and Structural Thermomechanics*, (Co-author I. Adam Pres. held at the 3nd International Conference on Composite Science and Technology, Jan 11-13, 2000, Durban, South Africa.