

# A SEMI-EMPIRICAL MODEL FOR CHARACTERISATION OF FLOW COEFFICIENT FOR PNEUMATIC SOLENOID VALVES

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## Abstract

A semi-empirical model has been elaborated for analyzing and predicting the flow characteristics of small electro-pneumatic (EP) valves within a wide range of pressure ratio. As a basis for characterization of flow coefficient, an analytical model has been established for a simplified geometry. This model has been corrected corresponding to more complex valve geometries, utilizing the results of axisymmetric quasi-3D (Q3D) computations using the Computational Fluid Dynamics (CFD) code FLUENT. By this means, a semi-empirical modelling methodology has been elaborated for characterization of through-flow behavior of pneumatic valves of various geometries.

*Keywords:* pneumatic valve, flow coefficient, computational fluid dynamics.

## 1. Introduction

Electro-pneumatic (EP) valves are widely applied in several areas of industry. The knowledge on the dynamic flow characteristics of such valves is especially important in the cases when they are integrated in fast-response, controlled fluid power systems as control devices. A typical application of this kind is the realization of control functions in intelligent EP braking systems of motor vehicles [1], [2]. The dynamic flow characteristics of EP valves influence the successful operation of the entire fluid power equipment.

As illustrated in SZENTE et al. [3], a simplified 1D simulation tool can be effectively used in design, research and development regarding controlled fluid power systems. The EP valve models integrated in such systems must represent reliably the transmission characteristics of the EP valve, without time-consuming and practically unnecessary resolution of 3D flow details. A key factor in such 1D models is the flow coefficient  $C_q$ , representing the contraction of the isentropic or sonic gas jet in the orifice cross-section.

In the last decades, several attempts have been made to describe the pressure ratio and valve geometry dependent flow coefficient of pneumatic orifices. The classic measurements by PERRY [4] provide truly empirical data – summarized in

the ‘Perry polynomial’ – for sharp-edged circular orifices over the entire pressure ratio range. Since the through-flow geometry is considerably different in the case of pneumatic solenoid valves, the applicability of Perry data is doubtful in this area. Contrarily, the lack of knowledge compels the researcher even recently to use the Perry polynomial in pneumatic simulation in certain cases [5]. BUSEMANN [6] presents an analytical discharge coefficient model for a two-dimensional planar slit using the tangent-gas approximation, whereas OSWATITSCH outlines a more comprehensive model [7] for a Borda-type orifice. Both models, however, analyze the subcritical regime only. BROWER et al. [8] suggest a new analytical model based on Busemann over the entire pressure ratio range, but for axisymmetric sharp-edged circular orifices. There are several other sources with measurement data, e.g. GRACE and LAPPLE [9], JOBSON [10], or TSAI and CASSIDY [11]. They usually suggest some constant  $C_q$  values, and are mainly concerned about sharp-edged orifices or poppet valves, both of which are quite different from the geometry used in EP valves. The above suggest the necessity to establish a widely applicable model for prediction of flow coefficient for characteristic EP valve geometries.

This paper presents a semi-empirical model, supplying reliable information on the flow transmission characteristics of the EP valve. It is based on a simplified analytical model using the law of linear fluid momentum. The data of the analytical model have been corrected with use of CFD results supplied by the finite volume CFD code FLUENT and thus, a semi-empirical model has been established. The application of the model is demonstrated in a case study EP valve. The semi-empirical model has been extended to a number of different EP geometries, serving as a knowledgebase for future developments.

## 2. The EP Valve of Case Study

The valve under investigation is applied in fast-response pneumatic systems as control valve providing e.g. pressure signal for relay valves. Such miniature valves must provide rapid, pulsed fluid transmission between enclosures of relative pressures in the orders of magnitude of 10 bar and 0 bar within a time period in the order of magnitude of 0.01 s. It is of critical importance to elaborate a reliable fluid dynamical model for the valve to be applied in design of the fluid power hardware and its control.

*Figs. 1a and 1b* show the simplified scheme of the valve (SZENTE and VAD [12]). The valve body is equipped with flexible seal and contact surfaces. In absence of solenoid excitation, the valve body is kept at its closed end-position by the return spring. The solenoid is energized by DC voltage. The frame and the jacket assist in development of a magnetic circuit. The resultant magnetic force displaces the valve body against the return spring. As a consequence, a flow cross-section develops through the orifice. The original valve has a valve seat with angle of  $\alpha = 8^\circ$  (see the explanation for  $\alpha$  in Chapter 4).

As a first step of modelling the flow transmission characteristics of the EP

valve, an analytical model has been elaborated.

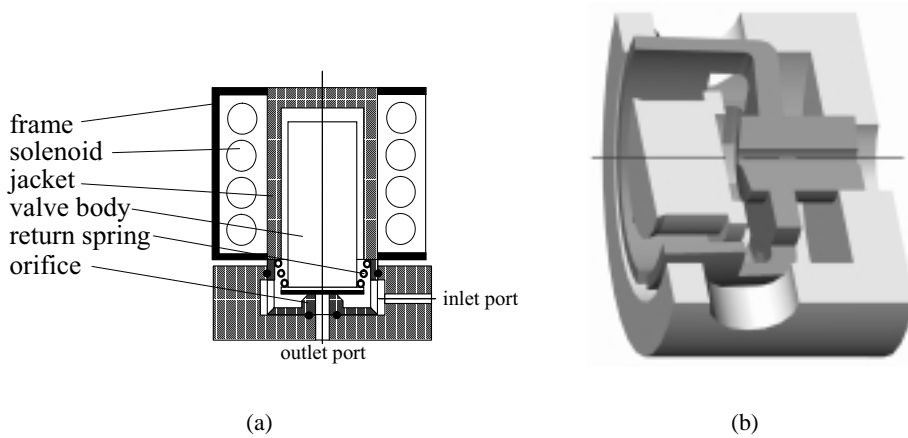


Fig. 1. (a) Scheme of the EP valve, (b) Detailed view of the valve

### 3. Analytical Model

The analytical model of the valve uses the fluid momentum law applied onto a Borda-type orifice. The scheme of the orifice is shown in Fig. 2. The Borda-type orifice is a circular, sharp-edged, straight, short pipe section immersed in the enclosure serving as the source of incoming flow. For incompressible fluids, the analytical model for a Borda-type orifice represents a flow coefficient of 0.5 [13], justified by measurements. The novelty of the present model compared to the one proposed by Oswatitsch is that it provides flow coefficient data over the entire pressure ratio range, thus taking compressibility effects into account as appropriate.

The upstream surfaces of control volume presented in Fig. 2 extend sufficiently far from the orifice to guarantee no through-flow and undisturbed static pressure  $p_{up}$ . The control volume excludes the Borda-type orifice. Downstream of the orifice, the control volume ends at the vena contracta (i.e. the limit of applicability of the isentropic law).

The following assumptions have been taken for the present model:

- The flow is stationary through the orifice,
- The effects of force fields are negligible,
- The flow is isentropic (inviscid flow with no heat transfer) upstream of and inside of the vena contracta (narrowest cross-section of pneumatic jet). This means that even for sonic flow (throttled expansion), the shock losses appear only downstream of the vena contracta.

The mass flow rate  $q_m$  through the orifice is a function of upstream absolute pressure  $p_{\text{up}}$ , upstream temperature  $T_{\text{up}}$ , orifice cross-section  $A$ , flow coefficient  $C_q$  and mass flow parameter  $C_m$  ([5], [14]):

$$q_m = A \cdot C_q \cdot C_m \cdot \frac{p_{\text{up}}}{\sqrt{T_{\text{up}}}}, \quad (1)$$

where

$$C_m = \sqrt{\frac{2 \cdot \kappa}{R \cdot (\kappa - 1)} \cdot \left( \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)^{\frac{2}{\kappa}} - \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)^{\frac{\kappa+1}{\kappa}} \right)}$$

if

$$\frac{p_{\text{down}}}{p_{\text{up}}} > \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}} \quad (\text{subsonic flow}), \quad (2a)$$

$$C_m = \sqrt{\frac{2 \cdot \kappa}{R \cdot (\kappa - 1)} \cdot \left( \frac{2}{\kappa + 1} \right)^{\frac{1}{\kappa-1}}}$$

if

$$\frac{p_{\text{down}}}{p_{\text{up}}} \leq \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}} \quad (\text{transonic flow}), \quad (2b)$$

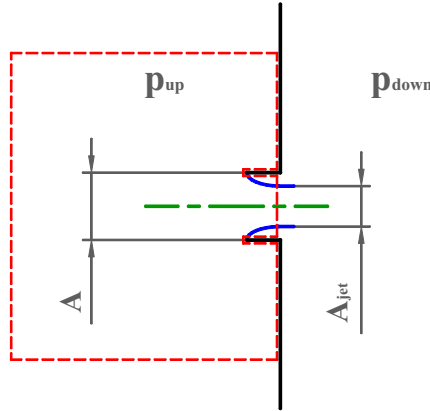


Fig. 2. Scheme of the Borda-type orifice

and the critical pressure ratio is

$$\left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}} = \left( \frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa-1}} = 0.528 \quad \text{if} \quad \kappa = 1.4. \quad (3)$$

According to [5] and [14], all the parameters except the flow coefficient  $C_q$  can be assessed using explicit functions or measurements, therefore, to build an analytical or semi-empirical model, the function of  $C_q$  has to be determined.

By applying the fluid momentum law to the control volume, the following equation can be obtained, assuming that the process is isentropic in the control volume, and, according to the energy equation, the total enthalpy remains constant in the control volume:

$$-\rho \cdot v_{\text{jet}}^2 \cdot A_{\text{jet}} = p_{\text{jet}} \cdot A_{\text{jet}} - p_{\text{up}} \cdot A + p_{\text{down}} \cdot (A - A_{\text{jet}}). \quad (4)$$

In this equation, it has been assumed that the static pressure valid in the downstream flow field influences the development of the jet on the annular cross-section  $(A - A_{\text{jet}})$ . Therefore, it is supposed that even in case of throttled expansion, the orifice is suitably short to avoid its blockage by shocked flow from the downstream flow field.

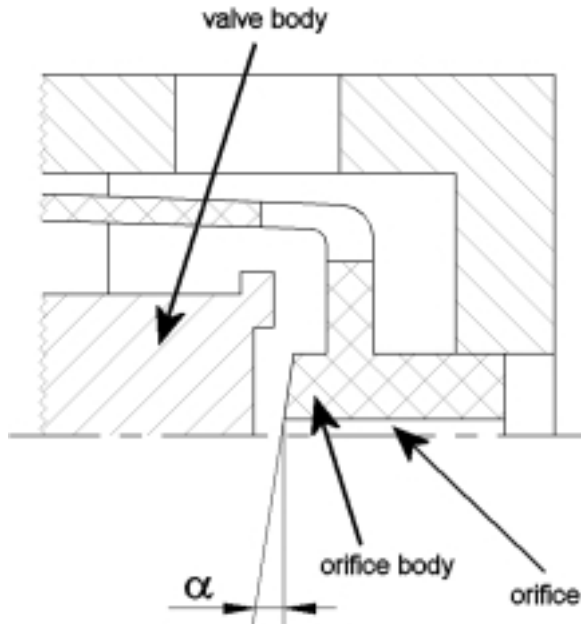


Fig. 3. Q3D scheme of the valve

Define  $C_q$  as the ratio of the flow- and the orifice cross-section:

$$C_q = \frac{A_{\text{jet}}}{A} = \frac{p_{\text{up}} - p_{\text{down}}}{p_{\text{jet}} - p_{\text{down}} + \rho \cdot v_{\text{jet}}^2}. \quad (5)$$

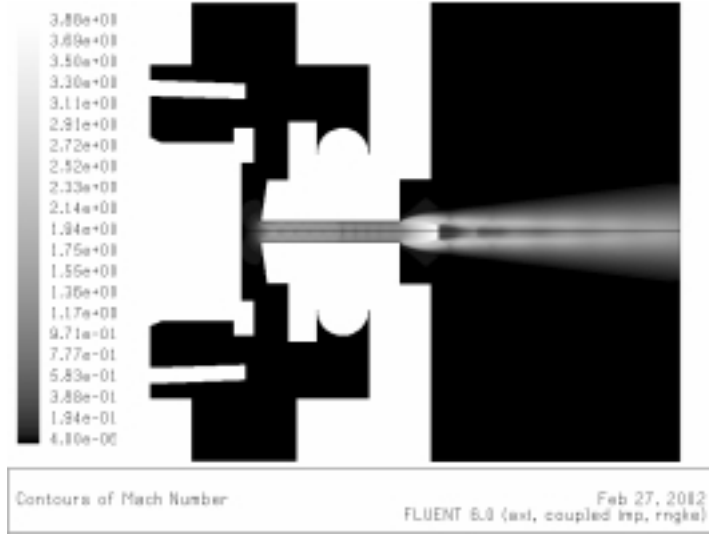


Fig. 4. Mach contour plot at  $p_{\text{down}}/p_{\text{up}} = 1 : 10$

The exit velocity of the Borda-type orifice for subsonic (6a) and transonic (6b) flows:

$$v_{\text{jet}}^2 = \frac{2 \cdot \kappa}{\kappa - 1} \cdot R \cdot T_{\text{up}} \cdot \left( 1 - \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)^{\frac{\kappa-1}{\kappa}} \right) \quad \text{if} \quad \frac{p_{\text{down}}}{p_{\text{up}}} > \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}, \quad (6a)$$

$$v_{\text{jet}}^2 = \frac{2 \cdot \kappa}{\kappa - 1} \cdot R \cdot T_{\text{up}} \cdot \left( 1 - \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)^{\frac{\kappa-1}{\kappa}} \right)_{\text{crit}} \quad \text{if} \quad \frac{p_{\text{down}}}{p_{\text{up}}} \leq \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}. \quad (6b)$$

By assuming the following circumstances in the vena contracta

$$\frac{p_{\text{jet}}}{p_{\text{up}}} = \frac{p_{\text{down}}}{p_{\text{up}}} \quad \text{if} \quad \frac{p_{\text{down}}}{p_{\text{up}}} > \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}, \quad (7a)$$

$$\frac{p_{\text{jet}}}{p_{\text{up}}} = \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}} \quad \text{if} \quad \frac{p_{\text{down}}}{p_{\text{up}}} \leq \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}, \quad (7b)$$

and substituting Eq. (6a) and Eq. (6b) into Eq. (5), the result is the analytical function

of  $C_q$ :

$$C_q = \frac{1 - \frac{P_{\text{down}}}{P_{\text{up}}}}{\frac{2 \cdot \kappa}{\kappa - 1} \cdot \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)^{\frac{1}{\kappa}} \cdot \left( 1 - \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)^{\frac{\kappa - 1}{\kappa}} \right)}$$

if

$$\frac{P_{\text{down}}}{P_{\text{up}}} > \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)_{\text{crit}}, \quad (8a)$$

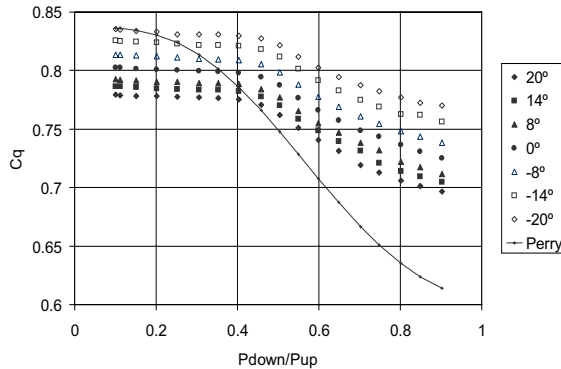
$$C_q = \frac{1 - \frac{P_{\text{down}}}{P_{\text{up}}}}{\frac{2 \cdot \kappa}{\kappa - 1} \cdot \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)_{\text{crit}}^{\frac{1}{\kappa}} \cdot \left( 1 - \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)_{\text{crit}}^{\frac{\kappa - 1}{\kappa}} \right) + \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)_{\text{crit}} - \frac{P_{\text{down}}}{P_{\text{up}}}}$$

if

$$\frac{P_{\text{down}}}{P_{\text{up}}} \leq \left( \frac{P_{\text{down}}}{P_{\text{up}}} \right)_{\text{crit}}. \quad (8b)$$

The analytical model is presented in *Figs. 7* and *8* (' $C_q$ -analytical' curves). For the incompressible case represented by the pressure ratio of unity, the model represents the flow coefficient value of 0.5 formerly deduced for incompressible cases. It is conspicuous in the figures that the model represents the trends of the Perry model qualitatively; serving as a kind of explanation of the underlying physics.

The next chapter reports the CFD campaign carried out on the case study EP valve, having geometrical cross-sections different from a Borda-type orifice.



*Fig. 5.*  $C_q$  values for different seat angles ( $\alpha$ )

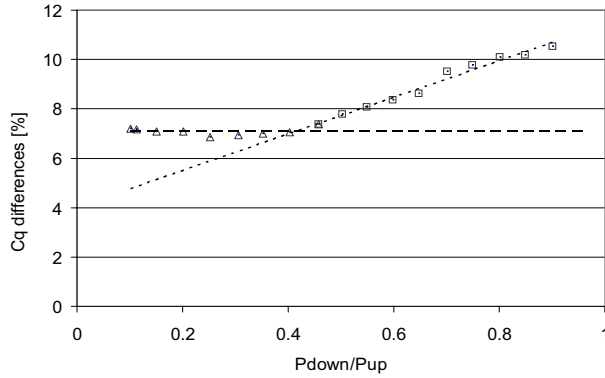


Fig. 6. Differences between min. and max.  $C_q$  values

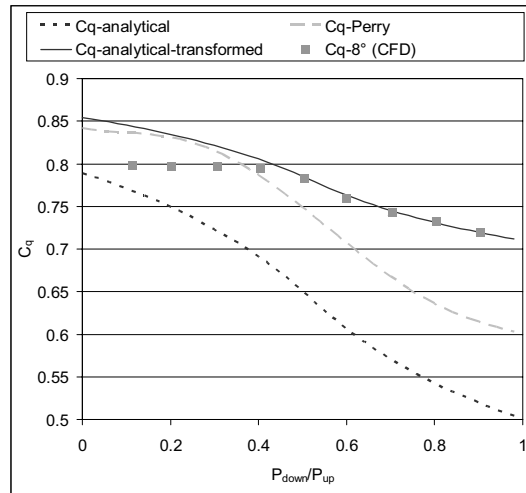


Fig. 7. Transformation for the subsonic region

#### 4. CFD Studies

In order to build up a data base from which the corrections of the analytical model can be deduced, a number of different 3D models were prepared, based on a previously validated Computational Fluid Dynamics (CFD) code FLUENT [15], [16]. Because of axial symmetry, the 3D valve model has been transformed to Q3D axisymmetric domain. Fig. 3 shows the 2D scheme that has been used in CFD simulation. Because of present limitations in the simulation software available, the movement of the valve body has not been incorporated into the model. In order to analyze the influence of the geometry on the flow parameters, a number of different Q3D models were



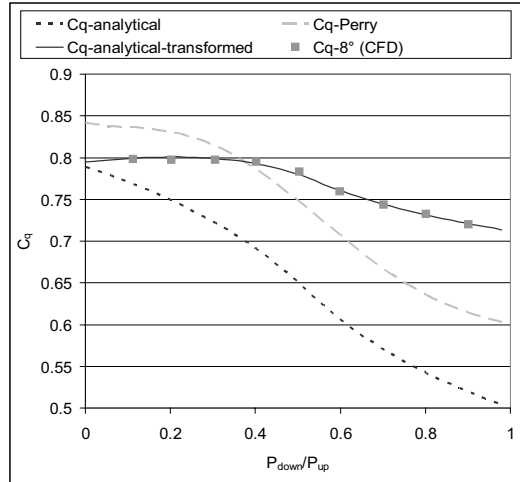


Fig. 8. Transformation for both regions

prepared. The current investigations were concentrated on the influence of the angle of the valve seat  $\alpha$  at the inlet of the orifice, shown in Fig. 3, on the flow coefficient. The seat angle  $\alpha$  is positive if a meridional line of the valve seat cone forms an acute angle with the section of orifice axis at the output port. Therefore, the Borda-type orifice in Chapter 3 is considered as a valve seat with  $\alpha = 90^\circ$ .

The simulation software computed the mass flow rate for the stationary state.  $p_{up}$ ,  $p_{down}$  and  $T_{up}$  have been specified as boundary conditions. A computed Mach number contour distribution can be seen in Fig. 4 as example. Such images can be used in the future for a detailed analysis of flow field within the valve. The value of  $C_q$  has been deduced from the CFD simulation using Eq. (1).

In Fig. 5 the results for the different geometries are plotted against the pressure ratio. The values of the Perry model are also shown for comparison purposes. The main trends of the computed curves are similar to those for the Perry model as well as for the analytical model of Chapter 3. Fig. 5 suggests the tendency that the increase of the angle decreases the flow coefficient, as the separation bubble developing at the inlet edge of the orifice reduces the flow cross-section. The angle variation has more influence at higher pressure ratios, as it can be seen in Fig. 6. Furthermore, it is apparent that the domain can be separated into two parts: at lower pressure ratios up to 0.5 the relative difference between the  $C_q$  values for the largest and smallest valve seat angles almost remains constant, and then starts increasing linearly from there (see the two jagged lines in Fig. 6, the dashed horizontal line shows the region where the difference is constant, while the dotted line shows the region where the difference increases linearly). This partitioning is the same as it is with the flow coefficient characteristics:  $C_q$  remains constant at lower pressure ratios, and starts to decrease at about the critical pressure ratio. It suggests that

the analytical model should be corrected differently in the subsonic and transonic regions.

## 5. Fitting the Analytical Model to CFD Data

On the basis of the perception that the analytical model follows trends similar to those apparent in the computational results, it has been supposed that the curve representing the analytical model can be transformed to the computed ones with use of simple transformation functions.

The first step of the model correction was to select one case for which the transformation functions can be tested and verified. This case was the one where the value of  $\alpha$  was  $8^\circ$  as this was the angle of the original valve seat. Numerous attempts have been made to find the simplest solution for transformation. It has been found that the regions of subcritical and supercritical pressure ratio indeed have to be treated separately. *Fig. 7* shows that the analytical model shares the same tendency as the Perry model, both of which are quite different from the validated CFD values. It also shows, however, that the correction of the subsonic range can be quite simple, as the  $C_q$ -analytical-transformed curve fits almost perfectly to the CFD data by using a simple linear transformation, represented later by *Eq. (9a)*. This suggested that the correction for the transonic range should be a similar transformation, and should use the parameters from the subsonic range correction function as well.

As it has been mentioned previously, the  $C_q$ -analytical-transformed curve is based on a simple linear transformation. The curve has been rotated around the  $C_q$  value of the critical pressure ratio, then shifted along the  $Y$  axis by a constant value. This transformation produced the curve which can be seen in *Fig. 7*. This follows the CFD data quite well in the subsonic range, but breaks away in the supersonic range. To correct this departure, a pressure-dependent transformation has been applied. This transformation uses one more constant ( $K_3$ ) in addition of the two ( $K_1$ ,  $K_2$ ) used in the subsonic correction. The final curve, using both transformations for the appropriate region, can be seen in *Fig. 8*, and the two transformations in *Eqs. (9a)–(9b)*.

$$C_{q \text{ corr}} = (C_q - C_{q \text{ crit}}) \cdot K_1 + C_{q \text{ crit}} + K_2$$

if

$$\frac{p_{\text{down}}}{p_{\text{up}}} > \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}, \quad (9a)$$

$$C_{q \text{ corr}} = (C_q - C_{q \text{ crit}}) \cdot \left( K_3 + \frac{p_{\text{down}}}{p_{\text{up}}} \cdot \frac{K_1 - K_3}{C_{q \text{ crit}}} \right) + C_{q \text{ crit}} + K_2$$

if

$$\frac{p_{\text{down}}}{p_{\text{up}}} \leq \left( \frac{p_{\text{down}}}{p_{\text{up}}} \right)_{\text{crit}}. \quad (9b)$$

After specifying the correction functions, the values of the constants  $K_i$  have been determined for the  $\alpha$  values used in the CFD calculations by using the least-squares method. After specifying these constants for each  $\alpha$  value, it became apparent that the variations of the constants can be defined with simple linear functions of  $\alpha$  as it can be seen below in Eqs. (10a)–(10c). The new  $K_i$  values provided by these functions are capable to keep the corrected  $C_q$  values within a  $\pm 1\%$  maximum relative difference compared to the CFD data.

$$K_1 = 0.0028 \cdot \alpha + 0.4307, \quad (10a)$$

$$K_2 = -0.0015 \cdot \alpha + 0.1433, \quad (10b)$$

$$K_3 = 0.0003 \cdot \alpha + 0.1482. \quad (10c)$$

## 6. Conclusions

An analytical model has been elaborated for description of flow coefficient for a Borda-type orifice over the entire pressure ratio range. This new model has been compared to literature data. It formed the basis for a semi-empirical model describing the flow coefficient of EP valves with various valve seat angles. The semi-empirical model has been obtained by simple transformations from the analytical model. The model parameters have been established by fitting the model to Q3D CFD data obtained by means of code FLUENT. The semi-empirical model is capable of predicting the flow coefficient within a  $\pm 1\%$  maximum relative difference compared to the CFD data. The model is to be generalized and experimentally verified by future studies.

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## Nomenclature

|       |   |
|-------|---|
| $p$   | absolute pressure [bar]                                     |
| $q_m$ | mass flow rate [kg/s]                                       |
| $v$   | flow velocity [m/s]   |
| $A$   | orifice cross-section [m <sup>2</sup> ]                     |
| $C_m$ | orifice mass flow parameter [ $\sqrt{\alpha K} / (m / s)$ ] |
| $C_q$ | orifice flow coefficient [–]                                |
| $K$   | constant used in the correction functions [–]               |
| $R$   | perfect gas constant [J/kg <sup>o</sup> K]                  |
| $T$   | temperature [°K]  |

## Greek letters

|          |                                    |
|----------|------------------------------------|
| $\alpha$ | angle of valve seat [°]            |
| $\kappa$ | specific heat ratio [–]            |
| $\rho$   | fluid density [kg/m <sup>3</sup> ] |

## Subscripts

|      |   |
|------|---|
| up   | upstream values                               |
| down | downstream values                             |
| jet  | values in the fluid jet at the vena contracta |
| crit | values at critical pressure ratio             |
| corr | transformed values                            |

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