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OPTIMIZATION OF LOOPED WATER SUPPLY NETWORKS

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Abstract

The paper approaches the optimization of looped networks supplied by direct pumping from one or more node sources, according to demand variation. Traditionally, in pipe optimization, the objective function is always focused on the cost criteria of network components. In this study an improved nonlinear model is developed, which has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, operating expenses etc. Discharge continuity at nodes, energy conservation in loops and energy conservation along some paths between the pump stations and the adequate 'critical nodes' are considered as constraints. This problem of nonlinear programming with equality constraints finally turns into a system of nonlinear equations to be solved by the 'gradient method'. The nonlinear optimization model considers head losses or discharges through pipes as variables to be optimized in order to establish the optimal diameters of pipes and is coupled with a hydraulic analysis. The paper compares a nonlinear optimization model to some others, such as the classic model of average economical velocities and Moshnin optimization model. This shows the good performance of the new model. For different analyzed networks, the saving of electrical energy, due to diminishing pressure losses and operation costs when applying the new model, represents about 10 . . . 30%.

Keywords: water supply, distribution, looped networks, optimal design, nonlinear model.

1. Introduction

Distribution networks are an essential part of all water supply systems. The reliability of supply is much greater in the case of looped networks. Distribution system costs within any water supply scheme may be equal to or greater than 60% of the entire cost of the project. Also, the energy consumed in a distribution network supplied by pumping may exceed 60% of the total energy consumption of the system [16].

Attempts should be made to reduce the cost and energy consumption of the distribution system through optimization in analysis and design.

A water distribution network that includes pumps mounted in the pipes, pressure reducing valves, and check-valves can be analyzed by several common methods such as Hardy–Cross, linear theory, and Newton–Raphson.

Traditionally, pipe diameters are chosen according to the average economical velocities (Hardy–Cross method) [6]. This procedure is cumbersome, uneconomical, and requires trials, seldom leading to an economical and technical optimum.

Linear programming is one of the common methods that has been used to design water distribution systems, specially, in branched pipes systems. SÂRBU and BORZA [15] approach distribution networks with concentrated outflows or uniform outflow along the length of each pipe. In this method, pipe lengths are considered variables to be optimized.

For optimizing the design of pipe network with closed loops that is a nonlinear problem, the bulk transport function can be used as an objective function. Strictly, this will not be the optimum for nonlinear flow rate – cost relationships, since economy of scale is not introduced [17].

DIXIT and RAO [7] have used a method in which only the cost of pipes is minimized. There are other analytical and numerical models which make use of optimization of cost criteria [1], [3], [5], [12]. Some of these methods either require more feasible variants, or do not include the case of looped networks supplied by more sources and having pumps mounted in the pipes. On the other hand, all of these optimization models consider quadratic turbulence regime of water flow.

In this study it is thought that water pumping is to be performed directly in the network according to demand variation, by means of a complex automation of pump stations, and that the distribution network does not have any impounding reservoir (*Fig. 1*). This water pumping system is used especially for large distribution networks, situated on flat ground. The solution, even if it creates some difficulty in operation, is flexible; if, after a while, the distribution system needs to be further developed (new higher buildings are built), the pressure or water flow in the network can be increased by changing the pumps and, subsequently, modifying the network.

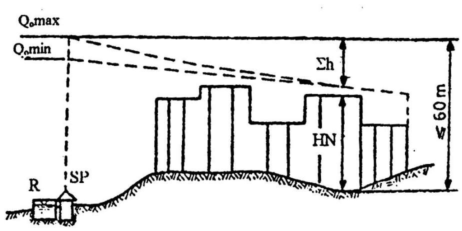

Fig. 1. Hydraulic scheme for supply of distribution network

The present paper develops a nonlinear model for optimal design of looped networks supplied by direct pumping from one or more node sources, which has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, operating expenses etc. Also, this new model considers the transitory or quadratic turbulence regime of water flow. The discharge continuity at nodes, energy conservation in loops, and energy conservation along some paths between the pump stations and the adequate 'critical nodes' are considered as constraints. The nonlinear optimization model considers head losses or discharges through pipes as variables to be optimized in order to establish the optimal diameters of pipes and is coupled with a hydraulic analysis. This model can serve as guidelines to supplement existing procedures of network design.

2. Networks Design Optimization Criteria

Optimization of distribution network diameters considers a mono- or multicriterial objective function. Cost or energy criteria may be used, simple or complex, which considers the network cost, pumping energy cost, operating expenses, included energy, consumed energy etc.

Newtork cost C_c is obtained by adding the costs of each compound pipe by the relation:

$$C_c = \sum_{ij=1}^{T} (a + bD_{ij}^{\alpha}) L_{ij}, \tag{1}$$

where: T is the number of pipes in a network; a, b, α – cost parameters depending on pipe material [16]; D_{ij}, L_{ij} – diameter and the length of pipe ij.

Pumping station cost C_p, proportional to the installed power, is given by:

$$C_p = \frac{9.81}{\eta} f \sigma Q_p \left(\sum h_{ij} + H_o \right), \tag{2}$$

where: η is the efficiency of pumping station; f – installation cost of unit power; σ – a factor greater than one which takes into account the installed reserve power; Q_p – pumped discharge; $\sum h_{ij}$ – sum of head losses along a path between the pump station and the critical node; H_o – geodesic and utilization component of the pumping total dynamic head.

Pumping energy cost C_e is defined by:

$$C_e = W_e e = \frac{9.81}{\eta} 730 e \tau \sum_{1}^{12} \Phi_k Q_p \left(\sum h_{ij} + H_o \right), \tag{3}$$

where: W_e is the pumping energy; e – cost of electrical energy; $\tau = T_p/8760$ – pumping coefficient, which takes into account the effective number T_p of pumping hours per year; Φ_κ – ratio between the average monthly discharge and the pumped discharge [15].

Annual operating expenses C_{ex} are given by:

$$C_{ex} = p_1 C_c + p_2 C_p + C_e, \tag{4}$$

where p_1 and p_2 are represented by repair, maintenance and periodic testing part for network pipes and pump stations respectively.

Annual total expenses C_{an} are defined by the multicriterial function:

$$C_{an} = \beta_o (C_c + C_p) + C_{ex}, \tag{5}$$

where $\beta_o = 1/T_r$ is the amortization part for the operation period T_r.

Total updated expenses C_{ac} are given by the multicriterial function:

$$C_{ac} = C_c + C_p + \frac{(1+\beta_o)^t - 1}{\beta_o(1+\beta_o)^t} C_{ex} \tag{6}$$

and is considered during the whole operation period ($t = T_r$).

Network included energy W_c is defined by the binomial objective function of the form (1), where a, b, α parameters have statistically corresponding determined values [16].

Energetic consumption W_t represents the energy included in the pipes of the network and the energy consumed in network exploitation during one year and is expressed by:

$$W_t = (\beta_o + p_1)W_c + W_e, \tag{7}$$

where W_e is the pumping energy, having the expression determined from (3).

Taking into account relations (1)...(7) and denoting:

$$r_a = \frac{(1+\beta_o)^t - 1}{\beta_o(1+\beta_o)^t}, \tag{8}$$

$$\xi_1 = r_a p_1 + \frac{t}{T_r}; \qquad \xi_2 = r_a p_2 + \frac{t}{T_r}, \tag{9}$$

$$\psi = \frac{9.81}{\eta}\left(f\sigma\xi_2 + 730 r_a e\tau \sum_{1}^{12} \Phi_k\right), \tag{10}$$

a complex objective multicriterial function is determined, with the general form:

$$F_c = \xi_1 \sum_{ij=1}^{T} (\alpha + bD_{ij}^{\alpha}) L_{ij} + \psi \sum_{j=1}^{NP} Q_{p,j} \left(\sum h_{ij} + H_o\right)_j, \tag{11}$$

where: t is the period for which the optimization criterion expressed by the objective function is applied, having the value 1 or T_r; NP – number of pump stations.

The general function (11) enables us to obtain a particular objective function by particularization of the time parameter t and of the other economic and energetic parameters, characteristic of the distribution system. For example, from $t = 1$, $r_a = 1$, $e = 1$, $f = 0$ the minimum energetic consumption criterion is obtained.

For networks supplied by pumping, the literature [1], [5], [12], [17] suggests the use of *minimum annual total expenses criterion (CAN)*, but choosing the optimal diameters obtained in this way, the networks become uneconomical at some time after construction, due to inflation.

Therefore, it is recommended the fore-mentioned criterion to be subject to dynamization by using the *criterion of total updated minimum expenses (CTA)*, the former being in fact a specific case of the latter when the investment is realised within a year; the operating expenses are the same from one year to another and the expected life-time of the distribution system is high. In particular, the use of energetical criteria different from cost criteria is recommendable.

Thus, another way to approach the problem which has a better validity in time and the homogenization of the objective function is network dimensioning according to *minimum energetic consumption (WT)*.

3. Nonlinear Model of Optimization in Designing Distribution Networks

3.1. Functional Relationship Head Loss – Discharge for Pipes

The head loss is given by the Darcy–Weisbach functional relation:

$$h_{ij} = \frac{8}{\pi^2 g} \lambda_{ij} \frac{L_{ij}}{D_{ij}^r} Q_{ij}^2, \tag{12}$$

where: r is an exponent having the value 5.0; g – acceleration of gravity; λ_{ij} – friction factor of pipe ij which can be calculated using the Colebrook–White formula; Q_{ij} – discharge of the pipe ij.

In the case of the transitory turbulence regime of water flow (Moody criterion Re $\sqrt{\lambda}\Delta/D$ is between 14 and 200), the friction factor λ is calculated with the following explicit formula [2]:

$$\sqrt{\lambda} = \frac{C + \sqrt{C^2 + 16\sqrt{\lambda_p}\mathrm{Re}\,\Delta/D}}{2\mathrm{Re}\,\Delta/D}, \tag{13}$$

in which:

$$\frac{1}{\sqrt{\lambda_p}} = -2\lg\frac{\Delta}{D} + 1.138, \tag{14}$$

$$C = \mathrm{Re}\,\frac{\Delta}{D}\sqrt{\lambda_p} + 8\sqrt{\lambda_p} - 4, \tag{15}$$

where: Re is Reynolds number; D – pipe diameter; Δ – absolute roughness of the pipe wall; λ_p – friction factor for quadratic turbulence regime of water flow.

Eq. (12) is difficult to use in the case of pipe networks and therefore it is convenient to write it similar to the Chézy–Manning formula:

$$h_{ij} = R_{ij} Q_{ij}^{\beta}, \tag{16}$$

where: $R_{ij} = KL_{ij}/D_{ij}^r$ is the hydraulic resistance of pipe ij; β – exponent which has values between 1.85 and 2 [16].

Specific consumption of energy for distribution of water w_{sd}, in kWh/m^3, is obtained by referring the hydraulic power dissipated in pipes to the sum of node discharges:

$$w_{sd} = 0.00272 \frac{\sum_{ij=1}^{T} R_{ij} |Q_{ij}|^{\beta+1}}{\sum_{\substack{j=1 \\ q<0}} |q'_j|}, \tag{17}$$

where q'_j is the outflow at the node j.

3.2. Formulation of the Mathematical Model

The nonlinear optimization model (MON) allows the optimal designing of looped networks by using one of the CAN, CTA or WT optimization criteria, expressed by the objective function (11).

If the diameter D_{ij} is expressed in relation (17) through the discharge and head losses:

$$D_{ij} = K^{\frac{1}{r}} Q_{ij}^{\frac{\beta}{r}} h_{ij}^{-\frac{1}{r}} L_{ij}^{\frac{1}{r}}, \tag{18}$$

and in the objective function (11) the resulting expression is replaced, we have:

$$F_c = \xi_1 \sum_{ij=1}^{T} \left(a + bK^{\frac{\alpha}{r}} Q_{ij}^{\frac{\beta\alpha}{r}} h_{ij}^{-\frac{\alpha}{r}} L_{ij}^{\frac{\alpha}{r}} \right) L_{ij} + \psi \sum_{j=1}^{NP} Q_{p,j} \left(\sum h_{ij} + H_o \right)_j \tag{19}$$

which is limited by the following constraints:

- discharge continuity at nodes:

$$\sum_{\substack{i=1 \\ i \neq j}}^{N} Q_{ij} + q_j = 0 \qquad (j = 1, \ldots, N - NP) \tag{20}$$

- energy conservation in loops:

$$\sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m = 0 \qquad (m = 1, \ldots, M) \tag{21}$$

- energy conservation along some paths between the pump stations and adequate ‘critical nodes’:

$$Z_{SP,j} - \sum_{ij=1}^{NT_j} \varepsilon_{ij} (h_{ij} - H_{p,ij}) - Z_{o,j} = 0 \qquad (j = 1, \ldots, NP) \tag{22}$$

in which: Q_{ij} is the discharge through pipe ij, with the sign (+) when entering node j and (−) when leaving it; q_j – concentrated discharge at node j with the sign (+) for node inflow and (−) for node outflow; h_{ij} – head loss of the pipe ij; ε_{ij} – orientation of flow through the pipe, having the values (+1) or (−1) as the water flow sense is the same as or opposite to the path sense of the loop m and (0) value if $ij \notin m$; f_m – pressure head introduced by the potential elements of the loop m [15]; M – number of independent loops (closed-loops and pseudo-loops); $Z_{SP,j}$ – available piezometric head at the pump station SP_j; $H_{p,ij}$ – pumping head of the pump mounted in the pipe ij, for the discharge Q_{ij}, approximated by parabolic interpolation on the pump curve given by points [16]; $Z_{o,j}$ – piezometric head at the critical node O_j; NT_j – pipe number of a path $SP_j - O_j$.

The optimization model (19)...(22) represents a nonlinear programming problem, which results in a system of non-linear equations by applying the Lagrange non-determined coefficients method. The Lagrange function Γ can be written as:

$$\Gamma = F_c + \sum_{n=1}^{N-NP} \Lambda_n \left(\sum_{\substack{i \neq j \\ i=1}}^{N} Q_{ij} + q_j \right) + \sum_{m=1}^{M} \Lambda_m \left(\sum_{\substack{ij \in m \\ ij=1}}^{T} \varepsilon_{ij} h_{ij} - f_m \right) + \sum_{j=1}^{NP} \Lambda_j \left[Z_{SP,j} - \sum_{ij=1}^{NT_j} \varepsilon_{ij} (h_{ij} - H_{p,ij}) - Z_{o,j} \right] \tag{23}$$

in which Λ_n, Λ_m, Λ_j are Lagrange multipliers.

The optimal solution of the model described by the relations (19)...(22) requires that first order derivatives of function Γ by the variables $y_l \in \{Q_{ij}, h_{ij}\}$ and multipliers Λ_n, Λ_m, Λ_j be equal to zero. By eliminating the multipliers Λ_n, Λ_m, Λ_j, this system will come to a $2T + NP$ equations system with unknown variables Q_{ij}, h_{ij}, $Z_{SP,j}$, formed by:

a) $N - NP$ nodal equations (20);
b) M loop equations (21);
c) NP functional equations (22);
d) $N - NP$ energy-economy equations for nodes:

$$\sum_{\substack{i \neq j \\ i=1}}^{N} Q^*_{ij} = \begin{cases} -\dfrac{\psi}{A} Q_{p,j}; & \text{for pumping nodes } (j = 1, \ldots, NP) \\ 0; & \text{for other nodes } (j = NP+1, \ldots, N-NP) \end{cases} \tag{24}$$

in which:

$$Q^*_{ij} = Q_{ij}^{\frac{\beta\alpha}{r}} L_{ij}^{\frac{\alpha+r}{r}} h_{ij}^{-\frac{\alpha+r}{r}}, \tag{25}$$

$$A = \frac{\alpha}{r} \xi_1 b K^{\frac{\alpha}{r}}; \tag{26}$$

e) M energy-economy equations for loops:

$$\sum_{\substack{ij\in m\\ ij=1}}^{T} H_{ij}^{*} = 0 \qquad (m = 1, \ldots, M) \tag{27}$$

in which:

$$H_{ij}^{*} = h_{ij}^{-\frac{\alpha}{r}} L_{ij}^{\frac{\alpha+r}{r}} Q_{ij}^{\frac{\beta\alpha-r}{r}}. \tag{28}$$

Eqs. (24) can be expressed similarly with the discharge continuity equations, by giving Q_{ij}^{*} the same sign as Q_{ij}. *Eqs.* (27) are similar to the energy conservation equations in the loops, by giving H_{ij}^{*} the same sign as h_{ij}.

Generally, the (20), (21), (22), (24) and (27) equation system allows the determination of variables Q_{ij} and h_{ij}, but we must also consider the existence of the objective function's extreme.

Second order derivatives of function F_c by h_{ij} and Q_{ij} are:

$$\frac{\partial^2 F_c}{\partial h_{ij}^2} = \frac{\alpha + r}{r} A Q_{ij}^{\frac{\beta\alpha}{r}} h_{ij}^{-\frac{\alpha+2r}{r}} L_{ij}^{\frac{\alpha+r}{r}}, \tag{29}$$

$$\frac{\partial^2 F_c}{\partial Q_{ij}^2} = \beta A \frac{\beta\alpha - r}{r} Q_{ij}^{\frac{\beta\alpha-2r}{r}} h_{ij}^{-\frac{\alpha}{r}} L_{ij}^{\frac{\alpha+r}{r}}. \tag{30}$$

As $Q_{ij} \geq 0$ and $h_{ij} \geq 0$, and considering that for usual values of α [16], $(\alpha+r)/r > 0$, it results that $\partial^2 F_c/\partial h_{ij}^2 > 0$. For practical values of α and β, $(\beta\alpha - r)/r < 0$, so it results that $\partial^2 F_c/\partial Q_{ij}^2 < 0$.

Consequently, in all cases the objective function F_c has a convex-concave form for its definition range, and, therefore, has no extreme. In order to establish an extreme, we should specify a set of variables (Q_{ij} or h_{ij}). Thus, if the flow discharges in pipes are known, the values h_{ij} are to be determined by minimizing the objective function F_c. If only the head losses are the given values, the variables Q_{ij} are to be determined by maximizing the objective function F_c.

Considering variables h_{ij} to be unknown, pipes discharges could be calculated in a variety of ways for *Eqs.* (20) to be satisfied; this, however, affects the reliability and technical and economic-energetical conditions of the system. That is why optimization of the flow discharges in pipes must be performed according to the minimum bulk transport criterion [13], which takes into account the network reliability.

In this case, computation of the optimal design of looped networks must be performed in the following stages:

- Establishment of optimal distribution for discharges through pipes, Q_{ij} [15].
- Determination of head losses through pipes (h_{ij}) and piezometric heads at the supply nodes ($Z_{SP,j}$), by solving the nonlinear equation system (21), (22), (24) using the gradients method [11].

- Computation of optimal pipes diameters D_{ij} using expression (18) and their approximation to the closest commercial values.
- A new computation of the head losses using relation (12) or (16) and the hydraulic equilibrium for pipes network using Hardy–Cross method.

If the head losses are the given values, the unknown variables Q_{ij} are to be determined by solving the equation system (20), (22), (27), and used to calculate the optimal diameters in relation (18).

The piezometric heads Z_n can be determined starting from a node of known piezometric head. The residual pressure head H_n at the node n is calculated from the relation:

$$H_n = Z_n - ZT_n, \tag{31}$$

where ZT_n is the elevation head at the node n.

For an optimal design, the piezometric line of a path of NT_j pipes, situated in the same pressure zone, must represent a polygonal line which resembles as closely as possible the optimal form expressed by the equation:

$$Z_n = Z_{SP,j} - \left[1 - \left(1 - \frac{d}{\sum\limits_{ij=1}^{NT_j} L_{ij}} \right)^{\frac{\beta\alpha}{\alpha+r}+1} \right] \sum_{ij=1}^{NT_j} h_{ij}, \tag{32}$$

in which: Z_n is piezometric head at the node n; d – distance between the node n and the pump station SP_j.

The computer program OPNELIRA was designed [16] based on the nonlinear optimization model. It was realized in the FORTRAN 5.1 programming language, for IBM-PC compatible computers.

4. Numerical Application

The looped distribution network with the topology from *Fig.* 2 is considered. It is made of cast iron and is supplied with a discharge of 1.22 m^3/s, provided from two pump stations ($Q_{p,1} = 0.806$ m^3/s, $Q_{p,2} = 0.404$ m^3/s). The following data are known: pipe length L_{ij}, in m, elevation head ZT_j, in m, and necessary pressure $HN_j = 24$ m H_2O.

Table 1. Hydraulic characteristics of the pipes

Pipe $i-j$	L_{ij} [m]	Classic model (MVE)				Moshnin model (MOM)				Nonlinear model (MON)			
		Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]	Q_{ij} [m^3/s]	D_{ij} [mm]	h_{ij} [m]	V_{ij} [m/s]
0	1	2	3	4	5	6	7	8	9	10	11	12	13
2–1	480	0.01372	150	2.370	0.78	0.01488	300	0.097	0.21	0.01715	200	0.838	0.55
3–2	560	0.04476	250	1.992	0.91	0.04467	400	0.221	0.36	0.04557	250	2.062	0.93
4–3	450	0.07719	300	1.812	1.09	0.07195	400	0.460	0.57	0.07005	350	0.682	0.73
5–4	410	0.10431	350	1.345	1.08	0.08931	350	1.316	0.93	0.08692	400	0.477	0.69
6–5	470	0.12616	350	2.235	1.31	0.07299	350	1.008	0.76	0.08514	350	1.039	0.89
12–6	380	0.15178	400	1.306	1.21	0.09861	350	1.487	1.03	0.11076	400	0.707	0.88
8–7	470	0.02729	200	2.010	0.87	0.01984	300	0.169	0.28	0.01934	200	1.034	0.62
9–8	540	0.06848	300	1.722	0.97	0.04405	350	0.422	0.46	0.03863	250	1.444	0.79
10–9	460	0.11621	350	1.864	1.21	0.06718	350	0.836	0.70	0.07668	350	0.830	0.80
11–10	515	0.16874	400	2.177	1.34	0.08398	350	1.462	0.87	0.10410	400	0.850	0.83
12–11	450	0.23271	500	1.133	1.19	0.13028	400	1.509	1.04	0.13969	450	0.720	0.88
32–12	350	0.42005	600	1.097	1.49	0.26445	500	1.472	1.35	0.28601	700	0.236	0.74
14–13	485	0.04911	250	2.066	1.00	0.02912	300	0.377	0.41	0.02019	200	1.159	0.64
15–14	545	0.11293	350	2.088	1.17	0.07325	400	0.578	0.58	0.07628	350	0.974	0.79
16–15	470	0.14540	450	0.812	0.91	0.07548	350	1.078	0.78	0.06911	350	0.694	0.72
17–16	180	0.19425	500	0.319	0.99	0.13865	400	0.684	1.10	0.14401	450	0.305	0.91
18–17	220	0.22695	500	0.527	1.16	0.15660	400	1.066	1.25	0.16457	450	0.484	1.04
32–18	500	0.27757	500	1.777	1.41	0.25322	500	1.928	1.29	0.26553	600	0.640	0.94
20–19	475	0.03088	200	2.582	0.98	0.05114	300	1.137	0.72	0.06085	350	0.549	0.63
21–20	530	0.08192	350	1.087	0.85	0.15016	450	1.260	0.94	0.14410	450	0.900	0.91
21–22	430	0.10602	350	1.456	1.10	0.03066	300	0.370	0.43	0.02680	300	0.226	0.38
22–23	550	0.05587	300	1.183	0.79	−0.04011	300	−0.810	0.57	−0.01985	250	−0.410	0.40
23–24	420	0.01337	150	1.973	0.76	−0.07448	350	−0.938	0.77	−0.05484	350	−0.397	0.57
31–24	300	0.00833	125	1.440	0.68	0.09618	400	0.548	0.77	0.07654	400	0.273	0.61

Table 1. (continued)

0	1	2	3	4	5	6	7	8	9	10	11	12	13
26–25	490	0.03270	200	2.976	1.04	0.03873	300	0.673	0.55	0.03619	250	1.155	0.74
27–26	510	0.07723	300	2.056	1.09	0.07319	350	1.099	0.76	0.08074	400	0.515	0.64
27–28	440	0.03105	200	2.417	0.99	−0.02257	250	−0.542	0.46	−0.01942	250	−0.315	0.40
28–29	190	0.00791	125	0.825	0.64	−0.05624	300	−0.550	0.80	−0.05534	300	−0.401	0.78
30–29	250	0.00399	100	0.914	0.51	0.08290	350	0.691	0.86	0.07939	400	0.244	0.63
31–30	400	0.03376	250	0.824	0.69	0.13064	400	1.349	1.04	0.11834	450	0.464	0.74
32–31	340	0.07343	250	3.176	1.50	0.25816	600	0.516	0.91	0.22622	600	0.319	0.80
7–1	310	0.01009	125	2.154	0.82	0.00893	200	0.196	0.28	0.00666	125	0.969	0.54
8–2	300	0.00934	125	1.795	0.76	0.01059	200	0.268	0.34	0.01196	150	1.138	0.68
9–3	335	0.00811	125	1.526	0.66	0.01325	200	0.468	0.42	0.01605	200	0.516	0.51
10–4	320	0.00845	125	1.577	0.69	0.01821	200	0.843	0.58	0.01869	200	0.659	0.60
11–5	370	0.01582	150	2.408	0.90	0.05399	300	0.987	0.76	0.03946	250	1.031	0.80
13–7	340	0.01655	150	2.415	0.94	0.02285	300	0.162	0.32	0.02108	200	0.883	0.67
14–8	320	0.01728	150	2.470	0.98	0.03550	300	0.369	0.50	0.04179	250	0.996	0.85
15–9	345	0.01101	125	2.836	0.90	0.04076	300	0.525	0.58	0.02864	250	0.518	0.58
16–10	330	0.00489	100	1.784	0.62	0.05038	300	0.767	0.71	0.04025	300	0.377	0.57
18–11	340	0.00234	100	0.453	0.30	0.05818	300	1.053	0.82	0.05435	350	0.316	0.57
25–13	360	0.00315	100	0.842	0.40	0.02944	300	0.286	0.42	0.03661	250	0.867	0.75
26–14	350	0.00470	100	1.753	0.60	0.04261	300	0.582	0.60	0.03694	250	0.858	0.75
27–15	340	0.02977	200	1.721	0.95	0.08977	350	1.103	0.93	0.08705	400	0.397	0.69
16–28	330	0.00447	100	1.500	0.57	−0.02669	250	−0.568	0.54	−0.00482	250	−0.018	0.10
17–29	320	0.01101	125	2.632	0.90	−0.00375	125	−0.438	0.31	−0.00114	100	−0.112	0.15
18–30	370	0.00518	100	2.231	0.66	−0.00465	200	−0.064	0.15	0.00352	150	0.138	0.20
19–25	340	0.00632	100	3.009	0.81	0.02657	250	0.581	0.54	0.03628	250	0.805	0.74
20–26	330	0.01080	125	2.615	0.88	0.05878	300	1.044	0.83	0.04302	350	0.196	0.45
21–27	320	0.18657	400	1.646	1.49	0.18891	450	1.204	1.19	0.19690	500	0.582	1.00
22–28	320	0.01097	125	2.612	0.89	0.03159	300	0.292	0.45	0.00747	250	0.038	0.15
23–30	290	0.00453	100	1.353	0.58	−0.00360	150	−0.139	0.20	−0.00299	125	−0.199	0.24

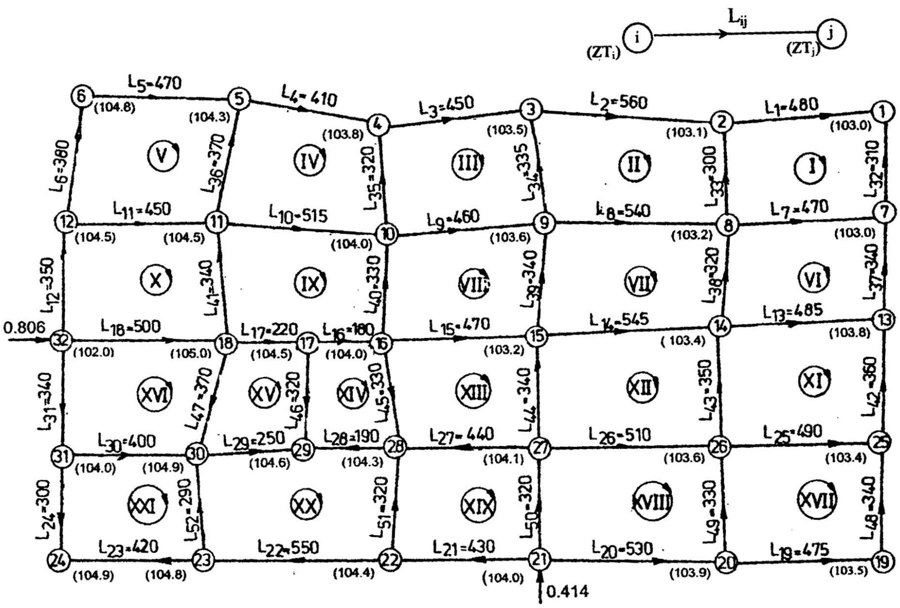

Fig. 2. Scheme of the designed distribution network

A comparative study of network dimensioning is performed using the classic model of average economical velocities (MVE), Moshnin optimization model (MOM) [1] and the nonlinear optimization model (MON) developed above.

Calculus was performed considering a transitory turbulence regime of water flow and the optimization criterion used was that of minimum total updated expenses.

Results of the numerical solution performed by means of an IBM-PC PENTIUM III computer, referring to the hydraulic characteristics of the pipes (discharge, diameter, head loss, velocity) are presented in *Table1*. The significance of the (−) sign of discharges and head losses in *Table 1* is the change of flow sense in the respective pipes with respect to the initial sense considered in *Fig.2*.

The piezometric heads at the node sources 32 and 21 computed using MON are of 133.107 m and 131.941 m respectively. The piezometric heads at the adequate critical nodes 1 and 13 have the values of 127.059 m and 127.853 m, respectively. The residual pressure heads at all the nodes of the network are over 24 m.

In *Fig. 3* there is a graphic representation, starting from the node source 32 to the critical node 1, on the path 32–18–17–16–15–14–13–7–1, the piezometric lines being obtained by using the three mentioned models of computation, and highlighting their deviation from the optimal theoretical form. *Fig.3* also includes the corresponding values of the objective function F_c, pumping energy W_e, as well as specific energy consumption for water distribution w_{sd}.

According to the performed study it was established that:

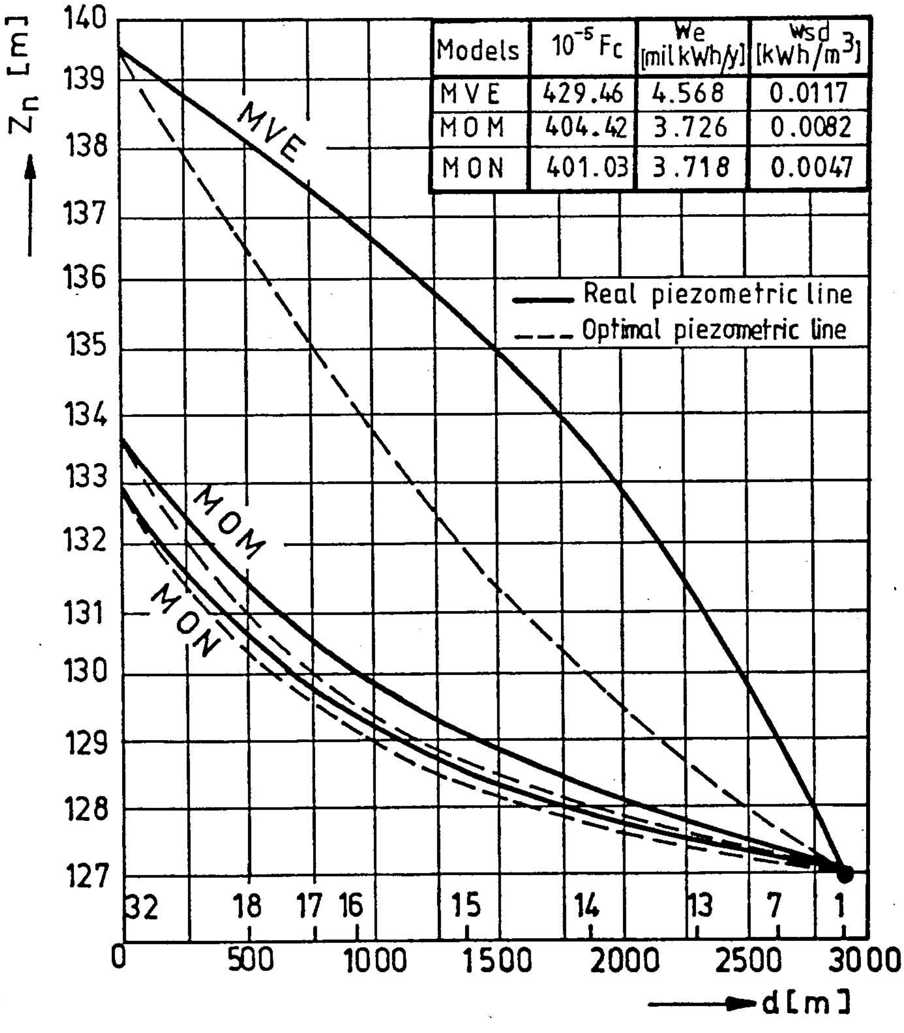

Fig. 3. Representation of piezometric lines along the path 32–18–17–16–15–14–13–7–1

- all the pipes of the network are functioning in a transitory turbulence regime of water flow;
- there is a general increase of pipe diameters obtained by optimization models (MOM, MON) with respect to MVE, because the classical model does not take into account the minimum consumption of energy and the diversity of economical parameters;
- in comparison with the results obtained by MVE, those obtained by optimization models are more economical, a substantial reduction of specific energy consumption for water distribution is achieved (MOM - 29.9%, MON

- 59.8%), as well as a reduction of pumping energy (MOM - 15.4%, MON - 18.8%); at the same time the objective function has also smaller values (MOM - 5.8%, MON - 6.6%);

- the optimal results obtained using MON are superior energetically to those offered by MOM, leading to pumping energy savings of 3.7%;
- also, the application of MON has led to the minimum deviation from the optimal form of the piezometric line, especially to a more uniform distribution of the pumping energy, by elimination of a high level of available pressure at some nodes. The smallest value of the specific energetic consumption, namely that of 0.0047 kWh/m^3, also supports this assertion;
- reduction of the pressure in the distribution network achieved in this way, is of major practical import, contributing to the diminishing of water losses from the system.

5. Conclusions

The mathematical programming, as a fundamental procedure for optimizing the structures in general, together with the graph theory and the increasing implication of computers in solving mathematical formulations have created conditions for solving efficiently some optimization problems of design of water distribution networks. The different types of programming which exist (linear, nonlinear, whole, geometric etc.) provide multiple possibilities for solving specific problems.

The computer program developed in this study, a very general and practical one, offers the possibility of optimal design of water supply networks using multiple criteria of optimization and considers the transitory or quadratic turbulence regime of water flow. It has the advantage of using not only cost criteria, but also energy consumption, consumption of scarce resources, and other criteria can be expressed by simple options in the objective function (11). The optimization approach used in this study does not require calculation of derivatives. This makes the method more efficient and consequently helps the designer to get the best design of water distribution systems with fewer efforts.

The nonlinear optimization model could be applied to any looped network, when piezometric heads at pump stations must be determined. A more uniform distribution of pumping energy is achieved so that head losses and parameters of pump stations can be determined more precisely.

For different analyzed networks, the saving of electrical energy due to diminishing pressure losses and operation costs when applying this new optimization model, represents about 10 . . . 30%, which is of great importance, considering the general energy issues.

Notations

a, b, α	–	cost parameters of the pipes
D_{ij}	–	diameter of pipe ij
d	–	distance between node n and the pump station SP_j
F_c	–	objective function
g	–	acceleration of gravity
H_o	–	geodesic and utilization component of the pumping total dynamic head
H_n	–	residual pressure head at the node n
$H_{p,ij}$	–	pumping head of the pump mounted in the pipe ij
h_{ij}	–	head loss of the pipe ij
L_{ij}	–	length of pipe ij
M	–	number of independent loops
N	–	number of nodes in network
NP	–	number of pump stations
NT_j	–	pipe number of a path $SP_j - O_j$
$Q_{p,j}$	–	pumped discharge of pump station j
Q_{ij}	–	discharge through pipe ij
q_j	–	concentrated discharge at node j
R_{ij}	–	hydraulic resistance of pipe ij
Re	–	Reynolds number
T	–	number of pipes in network
Z_n	–	piezometric head at the node n
$Z_{o,j}$	–	piezometric head at the critical node O_j
$Z_{SP,j}$	–	available piezometric head at the pump station SP_j
ZT_n	–	elevation head at the node n
β	–	exponent of discharge, which has values between 1.85 and 2
Δ	–	absolute roughness of the pipe wall
ε_{ij}	–	orientation of flow through pipe ij
ψ	–	economical-energetical factor
λ_{ij}	–	friction factor of pipe ij

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