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A NONLINEAR OBSERVER FOR FLEXIBLE JOINT ROBOTS[1]

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Abstract

This paper presents the design of a kind of semiglobal nonlinear observers for flexible joint robots which needs the measurements of the positions of each motor rotor and link. For the proposed observer, the error equation is locally exponentially stable about the origin, with a region of convergence which may be arbitrarily enlarged by a suitable choice of the observer gain. The results are illustrated by simulation examples.

Keywords: flexible joints; nonlinear systems; nonlinear observers; exponential stability.

1. Introduction and Preliminaries

The observer design problem for flexible joint manipulators has attracted considerable attention in the last decade (see e.g. [11], [7], [3], and the references therein). The need for a state observer arises from the fact that the control of flexible joint robots by state feedback requires the knowledge of four state variables for each of the joints, but the measurement of all variables is too expensive if not impossible. Therefore, observers that reconstruct the whole state vector by using a reduced set of measurements are needed for controller design (see e.g. [4] – [6]).

In the literature, design of state observers is reported under the assumption of different measurements: in [11] an observer was proposed assuming link position and speed available from measurement, which has globally asymptotically stable error dynamics, while in [7] only the link positions are assumed to be measurable; in this case, the error dynamics is semiglobally asymptotically stable. In this paper, a parametrized family of semiglobally exponentially stable observers is

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proposed under the assumption that both link and motor positions are available for measurement. Since most robots are equipped with sensors on the motor shaft, the measurement of motor positions seems to be a reasonable assumption. On the other hand, the proposed parametrization gives more flexibility in adjusting the behaviour of the observer.

We use standard notation in the paper. In particular, $\|x\|$ denotes the Euclidean norm of vector x, $\|A\|$ denotes the induced matrix norm of matrix A. A^T denotes the transpose of matrix A, $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimum and maximum eigenvalues of symmetric matrix A, respectively.

Detailed description of dynamic models of elastic joint robots is reported in [2], [9] and [10] (see also [11]). Assuming that the motion of the actuator rotors may be considered as pure rotations with respect to an inertial frame, the model of an elastic joint robot is given by

$$\begin{aligned} B_1(q_1)\ddot{q}_1 + C_1(q_1, \dot{q}_1)\dot{q}_1 + K(q_1 - q_2) + h(q_1) &= 0, \\ B_3\ddot{q}_2 - K(q_1 - q_2) &= u, \end{aligned} \tag{1}$$

where q_1 and q_2 are the $n \times 1$ vectors of the link and rotor relative displacements, respectively,

$$C_1(q_1, \dot{q}_1)\dot{q}_1 = \dot{B}_1(q_1)\dot{q}_1 - \frac{1}{2}\frac{\partial \dot{q}_1^T B_1(q_1)\dot{q}_1}{\partial q_1} \tag{2}$$

and $\dot{B}_1 - 2C_1$ is a skew-symmetric matrix, for a suitable definition of C_1 (see [2] and [7]). For rotational joints there exist positive constants such that

$$B_{1m} \le \lambda_m(B_1(q_1)) \le \|B_1(q_1)\| \le \lambda_M(B_1(q_1)) \le B_{1M}, \qquad \forall q_1 \in R^n, \tag{3}$$

$$\|C_1(q_1, \dot{q}_1)\| \le C_{1M}\,\|\dot{q}_1\|, \qquad \forall q_1, \dot{q}_1 \in R^n. \tag{4}$$

Matrix C_1 has the property (see [7])

$$C_1(q_1, y)z = C_1(q_1, z)y, \tag{5}$$

where y and z are arbitrary $n \times 1$ vectors.

We shall assume that the positions of all links and motor rotors are measured so that the $n \times 1$ output vectors y_1 and y_2 are given by

$$y_1 = q_1, \qquad y_2 = q_2.$$

2. A Design of Nonlinear Observers

In this section we show how to design a locally exponentially stable nonlinear observer which requires the measurement of the link and motor positions and for which the region of convergence may be arbitrarily enlarged by increasing some observer gains. Such a nonlinear observer may be used in a dynamic output feedback controller to perform asymptotic tracking of a desired trajectory. We shall assume that a sufficiently smooth reference trajectory $q_d(.)$ for the link position is given and we construct the observer directly in terms of tracking error variables. We define the error vectors

$$x_1 = q_1 - q_d, \qquad x_2 = \dot{q}_1 - \dot{q}_d, \tag{6}$$

and introduce the variables

$$x_3 = q_2, \qquad x_4 = \dot{q}_2. \tag{7}$$

In the new coordinates model (1) becomes

$$\begin{aligned}
&\dot{x}_1 = x_2,\\
&B_1(x_1+q_d)\dot{x}_2 + C_1(x_1+q_d, \dot{x}_2)\dot{x}_2 +\\
&\qquad + h_1(x_1+q_d) + K(x_1+q_d-x_3) = -B_1(x_1+q_d)\ddot{q}_d,\\
&\dot{x}_3 = x_4,\\
&\dot{x}_4 = B_3^{-1}K(x_1+q_d-x_3) + B_3^{-1}u,\\
&y_1 = x_1 + q_d,\\
&y_2 = x_3.
\end{aligned} \tag{8}$$

Let the state variables of the observer be denoted by ξ_i, $i = 1, \ldots, 4$, and let us estimate x_i in (8) by $\widehat{x_i}$, where

$$\begin{aligned}
\widehat{x_1} &= \xi_1, & \widehat{x_2} &= \xi_2 + k_1(y_1 - q_d - \xi_1),\\
\widehat{x_3} &= \xi_3, & \widehat{x_4} &= \xi_4 + k_2(y_2 - \xi_3),
\end{aligned} \tag{9}$$

and k_1, k_2 denote design parameters. As usual, the input variables of the observer are the input and output variables of the original model (8), i.e. u, y_1 and y_2. The observer is looked for in the following form:

$$\begin{aligned}
1)\quad & \dot{\xi}_1 = \xi_2 + k_1(y_1 - q_d - \xi_1) + H_{11}(y_1 - q_d - \xi_1) + H_{12}(y_2 - \xi_3),\\
2)\quad & B_1(y_1)\dot{\xi}_2 + C_1(y_1, \xi_2 + k_1(y_1 - q_d - \xi_1) + \dot{q}_d)\times\\
&\quad \times (\xi_2 + k_1(y_1 - q_d - \xi_1) + \dot{q}_d) =\\
&\quad = -K(y_1 - \xi_3) - h_1(y_1) - B_1(y_1)\ddot{q}_d +\\
&\quad + \widetilde{H}_{21}(y_1 - q_d - \xi_1) + \widetilde{H}_{22}(y_2 - \xi_3),
\end{aligned} \tag{10}$$

$$
\begin{aligned}
&3) \quad \dot{\xi}_3 = \xi_4 + k_2(y_2 - \xi_3) + H_{31}(y_1 - q_d - \xi_1) + H_{32}(y_2 - \xi_3),\\
&4) \quad \dot{\xi}_4 = B_3^{-1}K(y_1 - \xi_3) + B_3^{-1}u + \widetilde{H}_{41}(y_1 - q_d - \xi_1) + \widetilde{H}_{42}(y_2 - \xi_3),
\end{aligned}
$$

where H_{ij} and $\widetilde{H}_{lj}$, (i=1,3, j=1,2, l=2,4,) are further design parameters to be fixed later. To investigate the estimation error $\widetilde{x}_i = x_i - \widehat{x}_i$, i=1,...,4, first we rewrite the second and the fourth equations of (10) in terms of variables $\widehat{x}_i$. We have

$$
\begin{aligned}
2) \quad & B_1(y_1)\,\dot{\widehat{x}}_2 + C_1(y_1, \widehat{x}_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d) + K(y_1 - \widehat{x}_3) + h_1(y_1) =\\
&\quad = k_1 B_1(y_1)(\dot{x}_1 - \dot{\widehat{x}}_1) - B_1(y_1)\ddot{q}_d + \widetilde{H}_{21}(y_1 - q_d - \xi_1) +\\
&\quad + \widetilde{H}_{22}(y_2 - \widehat{x}_3) =\\
&\quad = k_1 B_1(y_1)(x_2 - \widehat{x}_2) - B_1(y_1)\ddot{q}_d + (\widetilde{H}_{21} - k_1 B_1(y_1)H_{11}) \times\\
&\quad \times (y_1 - q_d - \xi_1) + (\widetilde{H}_{22} - k_1 B_1(y_1)H_{12})(y_2 - \widehat{x}_3), \qquad (11)\\
4) \quad & \dot{\widehat{x}}_4 = k_2(\dot{x}_3 - \dot{\widehat{x}}_3) + B_3^{-1}K(y_1 - \widehat{x}_3) + B_3^{-1}u + \widetilde{H}_{41}(y_1 - q_d - \xi_1) +\\
&\quad + \widetilde{H}_{42}(y_2 - \widehat{x}_3) =\\
&\quad = k_2(x_4 - \widehat{x}_4) + B_3^{-1}K(y_1 - \widehat{x}_3) + B_3^{-1}u +\\
&\quad + (\widetilde{H}_{41} - k_2 H_{31})(y_1 - q_d - \xi_1) + (\widetilde{H}_{42} - k_2 H_{32})(y_2 - \widehat{x}_3).
\end{aligned}
$$

To simplify these equations, we introduce the notations

$$
\begin{aligned}
H_{21} &= \widetilde{H}_{21} - k_1 B_1(y_1)H_{11}, & H_{22} &= \widetilde{H}_{22} - k_1 B_1(y_1)H_{12},\\
H_{41} &= \widetilde{H}_{41} - k_2 H_{31}, & H_{42} &= \widetilde{H}_{42} - k_2 H_{32}.
\end{aligned}
$$

Now, by taking into consideration (9) and relations $\widetilde{x}_1 = y_1 - q_d - \widehat{x}_1$, $\widetilde{x}_3 = y_2 - \widehat{x}_3$, subtraction of the corresponding equations of (10) and (11) from the equations of (8) gives the error equation as follows:

$$
\begin{aligned}
\dot{\widetilde{x}}_1 &= \widetilde{x}_2 - H_{11}\widetilde{x}_1 - H_{12}\widetilde{x}_3,\\
B_1(y_1)\,\dot{\widetilde{x}}_2 &= -k_1 B_1(y_1)\widetilde{x}_2 + (K - H_{22})\widetilde{x}_3 - H_{21}\widetilde{x}_1 -\\
&\quad - (C_1(y_1, x_2 + \dot{q}_d)(x_2 + \dot{q}_d) - C_1(y_1, \widehat{x}_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d)), \qquad (12)\\
\dot{\widetilde{x}}_3 &= \widetilde{x}_4 - H_{31}\widetilde{x}_1 - H_{32}\widetilde{x}_3,\\
\dot{\widetilde{x}}_4 &= -k_2\widetilde{x}_4 - (B_3^{-1}K + H_{42})\widetilde{x}_3 - H_{41}\widetilde{x}_1.
\end{aligned}
$$

We want to find conditions for the unknowns H_{ij} and k_i under which all solutions $\widetilde{x}_i$, $i = 1, \ldots, 4$ of (12) starting from a certain neighbourhood of the origin tend to the origin. Let us introduce the notation $\widetilde{w}^T = (\widetilde{x}_1^T, \widetilde{x}_2^T, \widetilde{x}_3^T, \widetilde{x}_4^T)$ and consider the candidate Lyapunov function

$$
V_0(t, \widetilde{w}) = \frac{1}{2}\left[\widetilde{x}_1^T P_1 \widetilde{x}_1 + \widetilde{x}_2^T B_1(q_1(t))\widetilde{x}_2 + \widetilde{x}_3^T P_3 \widetilde{x}_3 + \widetilde{x}_4^T P_4 \widetilde{x}_4\right] \qquad (13)
$$

with arbitrary positive definite matrices P_1, P_3, P_4. The time derivative of V_0 along the solution of (12) is the following

$$\dot{V}_0 = \frac{1}{2}\widetilde{x}_2^T \dot{B}_1(q_1)\widetilde{x}_2 + \widetilde{x}_1^T P_1 \dot{\widetilde{x}}_1 + \widetilde{x}_2^T B_1(q_1) \dot{\widetilde{x}}_2 + \widetilde{x}_3^T P_3 \dot{\widetilde{x}}_3 + \widetilde{x}_4^T P_4 \dot{\widetilde{x}}_4 \,. \tag{14}$$

Substituting the derivatives from the equations (12) into (14), we get

$$\begin{aligned}\dot{V}_0 =& \widetilde{x}_1^T P_1\widetilde{x}_2 - \widetilde{x}_1^T P_1 H_{11}\widetilde{x}_1 - \widetilde{x}_1^T P_1 H_{12}\widetilde{x}_3 + \frac{1}{2}\widetilde{x}_2^T \dot{B}_1(q_1)\widetilde{x}_2 - \\ & - k_1\widetilde{x}_2^T B_1(q_1)\widetilde{x}_2 - \widetilde{x}_2^T (C_1(x_1, x_2 + \dot{q}_d)(x_2 + \dot{q}_d) - \\ & - C_1(y_1, \widehat{x}_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d)) + \widetilde{x}_2^T (K - H_{22})\widetilde{x}_3 - \widetilde{x}_2^T H_{21}\widetilde{x}_1 + \\ & + \widetilde{x}_3^T P_3\widetilde{x}_4 - \widetilde{x}_3^T P_3 H_{31}\widetilde{x}_1 - \widetilde{x}_3^T P_3 H_{32}\widetilde{x}_3 - \\ & - k_2\widetilde{x}_4^T P_4\widetilde{x}_4 - \widetilde{x}_4^T P_4(B_3^{-1}K + H_{42})\widetilde{x}_3 - \widetilde{x}_4^T P_4 H_{41}\widetilde{x}_1.\end{aligned} \tag{15}$$

By taking into account property (5) and adding and subtracting $C_1(y_1, x_2 + \dot{q}_d)(x_2 - \widehat{x}_2)$, the expression in the second line of (15) can be transformed in the following way:

$$\begin{aligned}& C_1(y_1, x_2 + \dot{q}_d)(x_2 + \dot{q}_d) - C_1(y_1, \widehat{x}_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d)\pm \\ & \quad \pm C_1(y_1, x_2 + \dot{q}_d)(x_2 - \widehat{x}_2) = \\ & \quad = C_1(y_1, x_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d) + C_1(y_1, x_2 + \dot{q}_d)(x_2 - \widehat{x}_2) - \\ & \quad - C_1(y_1, \widehat{x}_2 + \dot{q}_d)(\widehat{x}_2 + \dot{q}_d) = \\ & \quad = C_1(y_1, \widehat{x}_2 + \dot{q}_d)\widetilde{x}_2 + C_1(y_1, x_2 + \dot{q}_d)\widetilde{x}_2.\end{aligned}$$

Because of the skew-symmetry of $\frac{1}{2}\dot{B}_1(q_1) - C_1(q_1, \dot{q}_1)$, the pure quadratic terms of $\widetilde{x}_2$ in (15) are reduced to

$$- \widetilde{x}_2^T (C_1(q_1, \widehat{x}_2 + \dot{q}_d) + k_1 B_1(q_1))\widetilde{x}_2. \tag{16}$$

If the design parameters are chosen to satisfy conditions

$$\begin{aligned}P_1 = H_{21}, \qquad H_{12} = -P_1^{-1}H_{31}P_3, \qquad H_{22} = K, \\ H_{41} = 0, \qquad H_{42} = P_3P_4^{-1} - KB_3^{-1},\end{aligned} \tag{17}$$

then we have

$$\begin{aligned}\dot{V}_0 = -\widetilde{x}_1^T P_1 H_{11}\widetilde{x}_1 - \widetilde{x}_2^T (k_1 B_1(q_1) + \\ + C_1(q_1, \widehat{x}_2 + \dot{q}_d))\widetilde{x}_2 - \widetilde{x}_3^T P_3 H_{32}\widetilde{x}_3 - k_2\widetilde{x}_4^T P_4\widetilde{x}_4.\end{aligned} \tag{18}$$

Since

$$\|C_1(q_1, \widehat{x}_2 + \dot{q}_d)\| \leq C_{1M}\,\|\widehat{x}_2 + \dot{q}_d\| \leq C_{1M}\,(\|\dot{q}_1\| + \|\widetilde{x}_2\|)\,, \tag{19}$$

and $\|B_1(q_1)\| \le B_{1m}$, (16) can be estimated in the domain

$$\|\dot{q}_1\| \le \alpha_1, \quad \|\widetilde{x}_2\| \le \alpha_2 \tag{20}$$

as follows

$$\widetilde{x}_2^T(C_1(q_1, \widehat{x}_2 + \dot{q}_d) + k_1 B_1(q_1))\widetilde{x}_2 \ge [k_1 B_{1m} - C_{1M}(\alpha_1 + \alpha_2)]\widetilde{x}_2^T\widetilde{x}_2 \ge \beta_0 \widetilde{x}_2^T\widetilde{x}_2,$$

where the last inequality is fulfilled for any positive β_0 if parameter k_1 is chosen according to

$$k_1 \ge \frac{1}{B_{1m}}(\beta_0 + C_{1M}(\alpha_1 + \alpha_2)). \tag{21}$$

Let P_1, P_3, H_{11} and H_{32} be chosen to satisfy equations

$$H_{11}^T P_1 + P_1 H_{11} = Q_1 \qquad \text{and} \qquad H_{32} P_3 + P_3 H_{32} = Q_2 \tag{22}$$

with symmetric and positive matrices Q_1, Q_2.

Theorem 1 If (20) is satisfied, β_0, k_2 are arbitrarily fixed positive numbers, k_1 is chosen according to (21) and the further design parameters of the observer satisfy relations (17) and (22), the origin is locally exponentially stable for (12) in the region defined by (20).

Proof. The function $V_0(t, \widetilde{w})$ satisfies the following inequality

$$\gamma_1 \|\widetilde{w}\|^2 \le V_0(t, \widetilde{w}) \le \gamma_2 \|\widetilde{w}\|^2, \tag{23}$$

where

$$\gamma_1 = \frac{1}{2}\min\left[\lambda_m(P_1), B_{1m}, \lambda_m(P_3), \lambda_m(P_4)\right],$$
$$\gamma_2 = \frac{1}{2}\max\left[\lambda_M(P_1), B_{1M}, \lambda_M(P_3), \lambda_M(P_4)\right].$$

Under the conditions of the theorem, we have

$$\begin{aligned} \dot{V}_0 \le -\left[\frac{1}{2}\lambda_m(Q_1)\widetilde{x}_1^T\widetilde{x}_1 + \beta_0\widetilde{x}_2^T\widetilde{x}_2 + \frac{1}{2}\lambda_m(Q_3)\widetilde{x}_3^T\widetilde{x}_3 + k_2\lambda_m(P_4)\widetilde{x}_4^T\widetilde{x}_4\right] \le \\ \le -\gamma_3 \|\widetilde{w}\|^2, \end{aligned} \tag{24}$$

where $\gamma_3 = \min\left[\frac{1}{2}\lambda_m(Q_1), \beta_0, \frac{1}{2}\lambda_m(Q_3), k_2\lambda_m(P_4)\right]$. The local exponential stability of (12) follows from $Theorem$ 4.2 [8].

Remark 1 Theorem 1 states local exponential stability of (12). The exponential convergence takes place in the region where estimations (23) and (24) are valid, i.e. when (20) and (21) hold true. Therefore the region of attraction can be made arbitrarily large by choosing a sufficiently large k_1. In this sense, we can speak about semiglobal convergence.

A possible choice of the parameters can be given as follows:

$$\begin{aligned}
&H_{11} = H_{21} = H_{32} = I, \quad H_{12} = H_{31} = H_{41} = 0, \quad H_{22} = K, \quad H_{42} = I - KB_3^{-1},\\
&\widetilde{H}_{41} = 0, \qquad \widetilde{H}_{22} = K, \qquad \widetilde{H}_{21} = I + k_1 B_1(y_1),\\
&\widetilde{H}_{42} = H_{42} + k_2\, H_{32} = I - KB_3^{-1} + k_2 I = (1 + k_2) I - KB_3^{-1},\\
&P_1 = P_3 = P_4 = I.
\end{aligned}$$

Therefore the observer will be

$$\begin{aligned}
\dot{\xi}_1 &= -(k_1 + 1)\xi_1 + \xi_2 + (1 + k_1)(y_1 - q_d),\\
B_1(y_1)\dot{\xi}_2 &+ C_1(y_1, \xi_2 + k_1(y_1 - q_d - \xi_1) + \dot{q}_d)(\xi_2 + k_1(y_1 - q_d - \xi_1) + \dot{q}_d) =\\
&= -B_1(y_1)\ddot{q}_d - h_1(y_1) + K(y_2 - y_1) + (I + k_1 B_1(y_1))(y_1 - q_d - \xi_1),\\
\dot{\xi}_3 &= -(k_2 + 1)\xi_3 + \xi_4 + (k_2 + 1) y_2,\\
\dot{\xi}_4 &= -(k_2 + 1)\xi_3 + B_3^{-1} K y_1 + ((1 + k_2) I - KB_3^{-1}) y_2 + B_3^{-1} u.
\end{aligned} \tag{25}$$

3. Simulations

The performance of the proposed observer has been tested by simulations with respect to two robots, the models and the system parameter values of which are given in the literature ([7] and [1]).

Example 1 [7] The robot consists of one elastic joint, rotating in a vertical plane. Frictional forces have not been considered. Its dynamic model is represented by

$$\begin{aligned}
J_L \ddot{q}_1 + k(q_1 - q_2) + \frac{1}{2} mgl \sin q_1 &= 0,\\
J_R \ddot{q}_2 - k(q_1 - q_2) &= u,
\end{aligned}$$

where J_L and J_R are, respectively, the inertias of the link and of the motor rotor, m is the link mass, g is the gravity constant, l is the link length, and k is the elastic constant of the joint. The robot parameters are (all values are in SI units)

$$m = 1, \quad l = 1, \quad k = 100, \quad J_R = 0.02 \quad J_L = 0.4.$$

The desired reference trajectory for the link position is given by

$$q_d(t) = 1 - \frac{2}{1 + e^t}.$$

The initial conditions for the robot are

$$q_1(0) = 0, \quad q_2(0) = 0, \quad \dot{q}_1(0) = 0, \quad \dot{q}_2(0) = 0$$

and the initial conditions for the observer are

$$\widehat{x}_1(0) = 0, \quad \widehat{x}_2(0) = 0.01, \quad \widehat{x}_3(0) = 0, \quad \widehat{x}_4(0) = 0.01\,.$$

The design parameters are chosen to be

$$k_1 = 100, \quad k_2 = 10,$$

$$P_3 = P_4 = H_{11} = H_{32} = 1, \quad H_{12} = H_{31} = H_{41} = 0,$$
$$H_{22} = k.$$

Fig. 1 shows the obtained observer error for $P_1 = H_{21} = 1; 10; 30.$

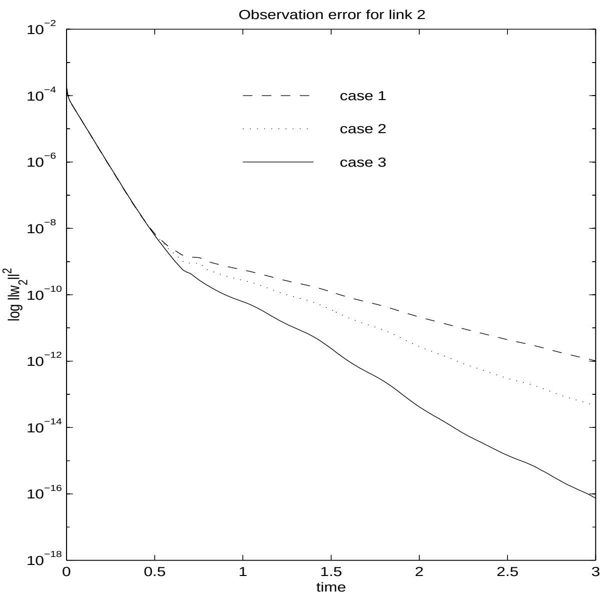

Fig. 1.

Example 2 The robot $PUMA$ 560. The explicit dynamic model and inertial parameters have been published in [1]. The desired reference trajectory for the link position is given by

$$q_d(t) = [0, q_{d_2}(t), q_{d_3}(t), 0, 0, 0]^T,$$

where

$$q_{d_2}(t) = q_{d_3}(t) = 1 - \frac{2}{1+e^t}.$$

The initial conditions for the robot are

$$q_1(0) = [0,0,0,0,0,0]^T, \qquad q_2(0) = [0,0,0,0,0,0]^T,$$
$$\dot{q}_1(0) = [0,0,0,0,0,0]^T, \qquad \dot{q}_2(0) = [0,0,0,0,0,0]^T$$

and the initial conditions for the observer are

$$\widehat{x}_1(0) = [0,0,0,0,0,0]^T, \qquad \widehat{x}_2(0) = [0,0.01,0.01,0,0,0]^T,$$
$$\widehat{x}_3(0) = [0,0,0,0,0,0]^T, \qquad \widehat{x}_4(0) = [0,0.01,0.01,0,0,0]^T.$$

The design parameters are chosen to be

$$k_1 = 100, \qquad k_2 = 10.$$

$$P_3 = P_4 = H_{11} = I, \qquad H_{12} = H_{31} = H_{41} = 0,$$
$$H_{22} = K, \qquad H_{42} = I - KB_3^{-1},$$

$$P_1 = H_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad H_{32} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & p_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Simulations have been performed for the following values of the parameters p:

$$\begin{array}{llll} case\ 1: & p_1 = 170, & p_2 = 50, & p_3 = 30, & p_4 = 25, \\ case\ 2: & p_1 = 350, & p_2 = 100, & p_3 = 60, & p_4 = 50, \\ case\ 3: & p_1 = 700, & p_2 = 200, & p_3 = 120, & p_4 = 100. \end{array}$$

The obtained observer errors are shown for the 2^d link in *Fig.* 2 and for the 3^d link in *Fig.* 3. In these figures the notations $\|w_k\|^2 = \sum_{j=1}^4 \widetilde{x}_{jk}^2$, $k = 2, 3$ have been used.

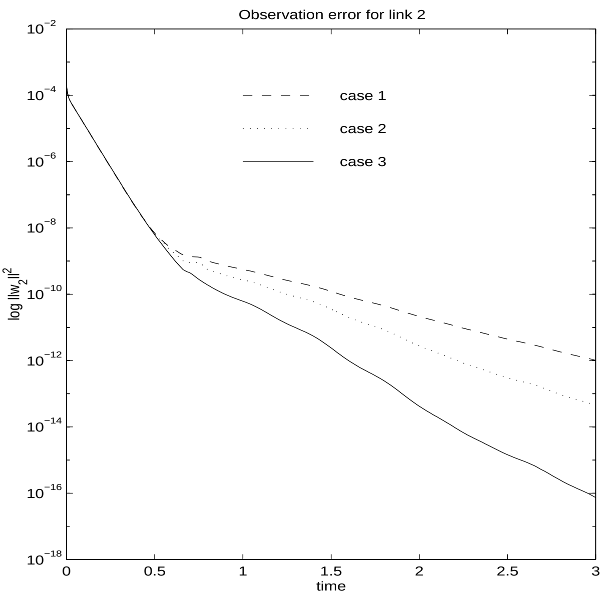
Observation error for link 2
case 1
case 2
case 3
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
log $||w_2||^2$
0
0.5
1
1.5
2
2.5
3
time

Fig. 2.

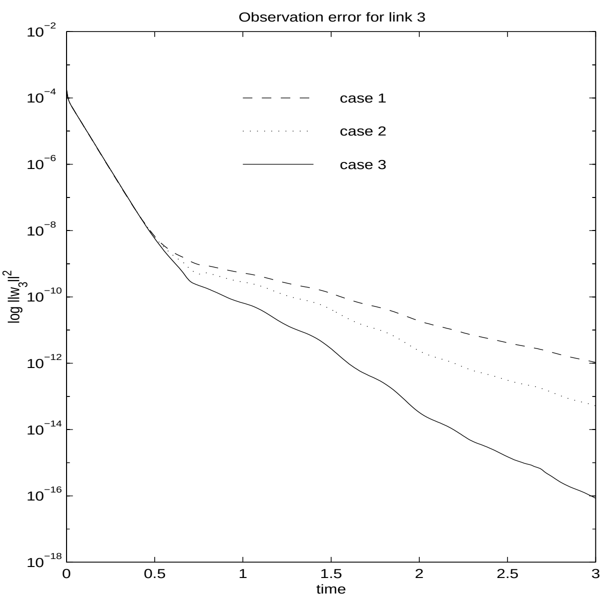
Observation error for link 3
case 1
case 2
case 3
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
10^{-18}
log $||w_3||^2$
0
0.5
1
1.5
2
2.5
3
time

Fig. 3.

4. Conclusion

In this paper we have provided a nonlinear observer for robots with elastic joints. The proposed observer requires the measurements of both the link and motor positions. It estimates the velocity of each link and motor rotor and is locally exponentially stable. The region of convergence may be arbitrarily enlarged by increasing some gains.

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