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DEVELOPMENT OF A CONTROL CIRCUIT FOR A RADIAL ACTIVE MAGNETIC BEARING

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Abstract

The electromagnetic bearings are the result of the modern control technique. Utilizing this principle frictionless bearing is reachable, but an appropriate control circuit is needed. In this report a development of an analogue PID controller will be demonstrated.

Keywords: active magnetic bearing, PID controller, electromagnet, displacement sensor.

1. The Operating Principle

The schematic figure of the analyzed construction can be seen in *Fig.1*. The operating principle is based on the force effect in a magnetic field; the magnetic flux lines strive for energy minimum. The rotor (1) is made of a magnetically passive material, for example aluminium. The flux conducting ferromagnetic ring (2) is fixed on the rotor. The actuators are electromagnets, which consist of two parts, the coil (4) and the ferromagnetic body (3).

1.1. Calculating of the Arising Forces

Each of the electromagnets can be independently analyzed, because the distance between them is large enough. The arising force can be calculated from the derivative of the energy of the magnetic field with respect to the displacement:

$$F = \frac{\partial W_m}{\partial x} = A\frac{\partial \delta}{\partial x}\int B\mathrm{d}H = \frac{\Phi^2}{2\mu_0 A} = \mu_0 A H^2. \quad (1)$$

where A is the surface, δ is the air gap, μ_0 is the permeability of the free air, B is the magnetic induction and H is the strength of the magnetic field.

The strength of the magnetic field can be determined from the field excitation law. Ignoring the magnetoresistance of the iron parts the strength of the magnetic field becomes:

$$H = \frac{NI}{\delta}, \tag{2}$$

where N is the turn of the coil and I is the current. According to the above the acting force by the two electromagnets to the rotor can be written:

$$F = \mu_0 A \left(\frac{(NI_1)^2}{(\delta + x)^2} - \frac{(NI_2)^2}{(\delta - x)^2} \right). \tag{3}$$

It is obvious that the force is proportional to the second power of the current and the reciprocal value of the change of the air gap.

2. Dynamical Analysis of the System

According to *Eq.* (3) it is obvious that the system is highly non-linear. To design an analogue control circuit it is needed to linearize the system near the operating point. For the linearization the following equation can be used as a starting point:

$$\Delta F = \frac{\partial F}{\partial i}\Delta i + \frac{\partial F}{\partial x}\Delta x. \tag{4}$$

There are two ways for the linearization of the system. The first is when permanent magnets are applied to produce a constant equilibrium magnetic field, like in *Fig*2

The resultant magnetic flux is the sum of the flux from the electric current (Φ_{el}) and the flux of the permanent magnet (Φ_m). The magnetic field, which is

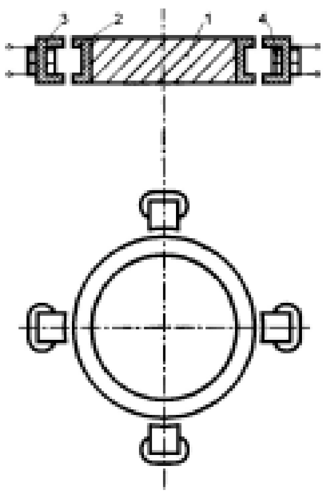

Fig. 1. Schematic figure

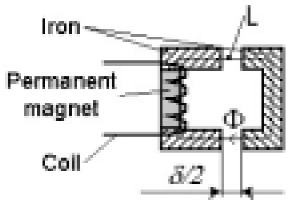

Fig. 2. Utilizing a permanent magnet

excited by the coils, is assumed to small compared to the field of the permanent magnet. For the acting force the following equation can be written [5]:

$$F = 2\frac{\Phi_0 L}{\mu_0 AN}(i_1 - i_2) + 2\frac{B_0^2 A}{\mu_0 \delta_0}(x_1 - x_2) = h\Delta i_c + k\Delta x, \tag{5}$$

where L is the inductance of the coil, i_1 is the current of the first and i_2 is the current of the second coil, respectively; x_1 and x_2 are the air gaps of the electromagnets. The inductance can be approximately calculated from the magnetoresistance (R_m):

$$L = \frac{N^2}{R_m} = N^2 \frac{\mu_0 A}{\delta}. \tag{6}$$

The second way is to utilize a bias current or bias voltage instead of permanent magnets. In this way we can write for the current:

$$i = i_b \pm i_c, \tag{7}$$

where i_b is the bias current, i_c is the control current, in the case of each coil. If the displacement of the rotor is assumed small enough, and the control current is much smaller than the bias current, we can write for the arising force [3]:

$$F_b = \frac{4\mu_0 i_b N^2 A}{\delta^2} i_c + \frac{4\mu_0 i_b^2 N^2 A}{\delta^3} x = hi_c + kx \tag{8}$$

which is similar to *Eq.* (5).

From the above it is obvious that in both of the two cases the dynamical model of the system will be similar, differences will occur only in the values of the coefficients.

The acting force accelerates the rotor, it can be described by the following differential equation:

$$m\ddot{x} = hi + kx. \tag{9}$$

There are two ways to excite the coils. The first is excitation by voltage source. In this case the input inductance must be taken into account, so the following can be written:

$$u_{in} = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + u_i = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + h\dot{x}, \tag{10}$$

where u_{in} is the input voltage, R is the resistance and L is the inductance of the coil and u_i is the induced voltage deriving from the flux changes. Taking into account all of the above mentioned, the following impedance network can be drawn (*Fig*3).

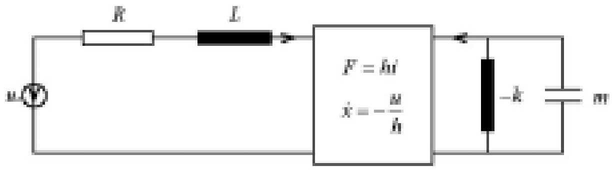

Fig. 3. Impedance network, in the case of voltage source

The transfer function is:

$$Y(s) = \frac{h}{s^3 \frac{Lm}{h} + s^2 \frac{Rm}{h} + s\left(h - \frac{Lk}{h}\right) - \frac{Rk}{h}}. \tag{11}$$

In the second case the coils are excited by a current source. Because the current is constrained to the circuit the inductance of the coil will not be an independent energy storage element, so the impedance network can be simplified, like in *Fig*4.

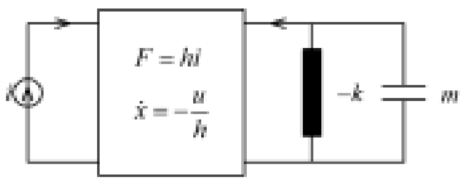

Fig. 4. Impedance network, in the case of current source

The transfer function will be:

$$Y(s) = \frac{h}{s^2 m - k}. \tag{12}$$

The above two methods of excitation have advantages and disadvantages. In the case of voltage fed regulating action the speed of the system can be affected by the inductance of the coil, because the rate of change of the control current is proportional to the inductance of the coil, according to *Eq.* (10). Disadvantage is the inductance dependency on the displacement of the rotor, derived from the dependency of the magnetoresistance on the air gap.

In the case of the current fed regulating action, the system is simpler; it is only second ordered, not third ordered. Another advantage is the independence of the inductance, so the dependency of inductance on the displacement is negligible. The speed of the system can be easily affected by the amplitude of the control current. As a disadvantage an additional voltage current converter is needed.

3. Developing the Control Circuit

It is obvious from *Eq.* (11) and *Eq.* (12) that the system is unstable in both of the current and voltage fed cases. Therefore, first the stabilization of the system has to be solved. According to the Hurwitz's stability criterion the system is stable when the coefficients of the denominator of the transfer function – the characteristic polynomial – are not equal to zero and their signs are similar. Furthermore the Hurwitz determinants, composed from the coefficients of the characteristic polynomial, has to be larger than zero.

Assuming that the characteristic polynomial have the form of:

$$K(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0. \tag{13}$$

In the voltage fed case the controller could be a single proportional controller, when a_1 is larger than zero, depending on the parameters of the system. When a_1 is smaller than or equal to zero, a proportional derivative member is needed. The situation is the same in the current fed case, because in this case a_1 is equal to zero.

After the stabilization the quality of the controlling can be affected. The integral action control reduces the static error, and the derivative action control reduces the response time, a proportional integrating derivative (PID) controller can be the optimal controller.

The schematic figure of the controlled system can be seen in *Fig*.5.

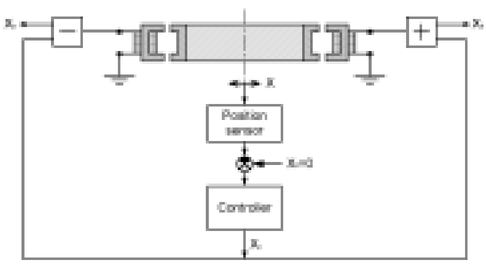

Fig. 5. The controlled system

In *Fig*.5 x_b is the bias signal – current or voltage – x_c is the control signal, x represents the changing of the air gap and x_0 is the reference input, which is equal to zero.

3.1. *Determining of the Parameters of the Controller*

First the voltage fed case will be analyzed. The coefficient a_1 was supposed larger than zero, because it can be affected by the parameters of the system, not only by the controller. According to the above mentioned a proportional controller is appropriate for the stabilization. Applying the Hurwitz's stability criterion, the coefficient of the proportional member (A) has to be between the following limits:

$$\frac{Rk}{h} < A < \frac{Rh}{L}. \tag{14}$$

In the current fed case a derivative member is also needed for the stable operation. The parameters have to meet the following criteria:

$$A > \frac{k}{h}, \tag{15}$$

$$T_D > \frac{k}{Ah}. \tag{16}$$

The optimal parameters of the controller can be determined for that case, when the phase margin is about 60° [1], because the phase margin will be large enough for the non-linear behavior caused instability effects. The method can be found in [1].

4. The Displacement Sensors

For the closed loop control an appropriate displacement sensor is needed. Only contact-free sensors can be applied, which can be inductive, eddy current and optical sensors. For this linearized system a sensor with an approximately linear characteristic curve should be appropriate. The inductive, contact-free sensors have a highly non-linear characteristic curve, while the eddy current sensors are highly linear. As a disadvantage an additional, difficult observer circuit is needed. The optical sensor can be a reflecting optical sensor, which contains a LED and a phototransistor, whose centerline subtend an angle between 0 and 180°. The characteristic curve has a relatively broad linear interval and its slope can be modified by the value of the load resistor, and by the angle between the centerlines, if the sensor is not armoured.

5. Developing the Test Machine

In the development a voltage fed case was chosen, because in this case the voltage current converter is unnecessary. The sensor was assumed to be a reflecting optical sensor, so the control circuit will not be difficult. The developed bearing was modeled with the following geometrical sizes:

The mass of the rotor is:

$$m = 0.2 \text{ g}.$$

The maximal acting force was chosen two times the earth attraction:

$$F_{\max} = 2 \cdot mg = 20 \text{ mN},$$

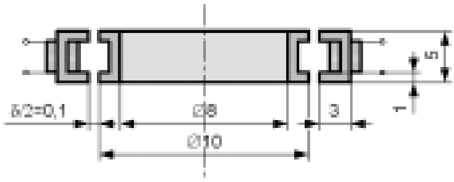

Fig. 6. The developed bearing

where g is the acceleration of gravity. In this way the system is also able to operate in vertical position. From the amount of the maximal force, the maximal excitation can be determined, assuming the bias current is the half of the maximal current. At the maximal displacement, which was chosen to be two times the air gap at the working point, only one of the coils works.

The air gap at the operating point was chosen:

$$\delta_0/2 = 0.1 \text{ mm}.$$

The bias voltage was chosen:

$$u_b = 6 \text{ V}$$

according to speed criteria. It could be enough, because the mass of the rotor is low. From the above parameters, the coil can be planned. The parameters of the coil:

$$N = 72,$$

$$R = 0.42 \; \Omega.$$

The inductance of the coil was checked by finite element analysis. The finite element analysis was made with a student version of Quickfield software. The inductance of the coil, near the operating point, derived from the analysis is:

$$L = 130 \; \mu\text{H}.$$

A PID controller was chosen. The optimal parameters of the controller are:

$$A = 2200,$$
$$T_i = 3.2 \cdot 10^{-3} \text{ s},$$
$$T_d = 3.2 \cdot 10^{-4} \text{ s}.$$

The closed loop response was simulated with the 20sim 2.02 software. An error displacement was set for initial condition. The results are shown in *Fig.7*.

Finally the system was checked also in non-linear case. The result can be seen in *Fig.8*.

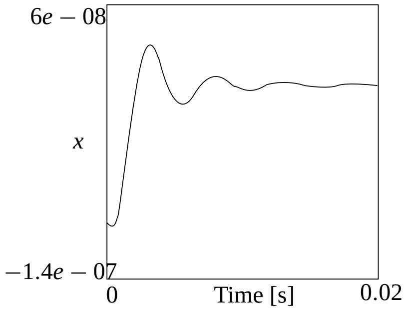

Fig. 7. The closed loop response of the linearized system

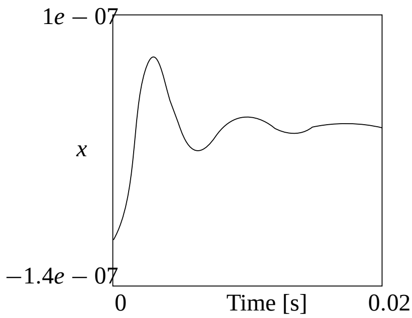

Fig. 8. Closed loop response in non-linear case

6. Conclusion

A guide was given to make stable and to choose the appropriate analogue control circuit for an active magnetic bearing. A sample controller was introduced, which was checked for non-linear case. The results of the simulations were advantageous, but extensive conclusions could be made only from measurement data.

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