PERIODICA POLYTECHNICA SER. MECH. ENG. VOL. 44, NO. 1, PP. 31–38 (2000)

FULLY COUPLED ANALYSIS OF DUCTILE DAMAGE OF ELASTO-VISCO-PLASTIC MATERIAL AT FINITE STRAINS

Attila GARAI* and Pavel ÉLESZTŐS**

* University of Wales Swansea, Singleton Park, SA2 8PP, U.K.,
a.garai@swansea.ac.uk
** Strojnícka fakulta STU, Nám. slobody 17, Bratislava, Slovenko,
elesztos@sjf.stuba.sk

Received: Oct. 15, 1999

Abstract

This work addresses the computational aspect of a model for ductile damage at finite strains. Elasto-visco-plastic model including non-linear (time dependent) isotropic and kinematic hardening is extended with isotropic damage. The constitutive equations are numerically integrated using algorithm based on operator split methodology (elastic predictor visco-plastic corrector). The Newton–Raphson method is used to solve the discretized evolution equations in the visco-plastic corrector stage. The finite element method is used in the approximation of the incremental equilibrium problem and the resulting equations are solved by the standard Newton–Raphson procedure.

Keywords: finite strains, damage, visco-plasticity.

1. Introduction

During the past thirty years, issues concerning the linearization of continuum models for the description of general non-linear behaviour of solids have become subjects of intensive research in the computational mechanics.

To describe the gradual internal deterioration within the framework of continuum mechanics several continuum damage models, either phenomeno-logical or micro-mechanically based, have been developed. KACHANOV [1] was the first to introduce the *effective stress* concept to model creep rupture.

Isotropic damage formulations are extensively employed in the literature because of their simplicity, efficiency and adequacy for many practical applications. In this paper the constitutive equations introduced by LEMAITRE [2] for ductile plastic damage are incorporated.

In following we present the numerical integration of the constitutive equations. The used algorithm is based on an operator-split methodology. In the plastic corrector stage the N–R method is used to solve the system of discrete evolution equations. To stabilise the N–R method the trial stress is replaced by the solution obtained through a sub-incrementation of the original evolution equation [3].

2. Kinematics of Finite Strain Elasto-plasticity

The main hypothesis underlying the present approach to finite strain elasto-visco-plastic damage is the multiplicative split of deformation gradient F into elastic and plastic parts [3], [4]:

$$\mathbf{F} = \mathbf{F}^e \mathbf{F}^p. \tag{1}$$

This assumption, firstly introduced by LEE [4], admits the existence of an unstressed intermediate configuration obtained from the current configuration by a purely elastic unloading of the neighbourhood of a material point. The polar decomposition of the elastic deformation gradient leads to:

$$\mathbf{F}^e = \mathbf{V}^e \mathbf{R}^e, \tag{2}$$

where $\mathbf{V}^e$ is the elastic left stretch tensor and $\mathbf{R}^e$ is the elastic rotation. We note that the plastic flow is assumed to be isochoric, i.e., $\det \mathbf{F}^p = 1$. The Eulerian logarithmic elastic strain is defined by:

$$\varepsilon^e := \ln\left[\mathbf{V}^e\right] = \frac{1}{2}\ln\left[\mathbf{B}^e\right], \tag{3}$$

where $\ln[\cdot]$ above denotes the tensor logarithm of $(\cdot)$ and $\mathbf{B}^e = \mathbf{F}^e \mathbf{F}^{eT} = \mathbf{V}^{e2}$ is the elastic left Cauchy–Green strain tensor. The rotationally invariant stress measure, τ, is the Kirchhoff stress tensor [3].

3. Constitutive Relations of Elasto-visco-plastic Model Fully Coupled to a Ductile Damage

The constitutive equations for elasto-plasticity coupled with damage adopted here have been originally proposed for small strains by LEMAITRE [2]. For conciseness, only the essential relations are reviewed in this section. The yield function f is defined, according to the principle of strain equivalence, by:

$$f\left(\tau, r, \overline{\mathbf{X}}, D\right) = J_2\left(\frac{\tau}{1-D} - \overline{\mathbf{X}}\right) - R(r) - \tau_{yo}, \tag{4}$$

where τ_{yo} is the uniaxial yield stress of the virgin material and R is the isotropic hardening function. The spatial quantity $\overline{\mathbf{X}}$ is the rotation of the backstress tensor $\mathbf{X}$ to the spatial configuration $\overline{\mathbf{X}} = \mathbf{R}^e \mathbf{X} \mathbf{R}^{eT}$. The normal to yield surface in the stress space is defined by

$$\mathbf{N} = \frac{\partial f}{\partial \widetilde{\tau}} = -\frac{\partial f}{\partial \overline{\mathbf{X}}} = \frac{3}{2}\,\frac{\left(\widetilde{\tau}^D - \overline{\mathbf{X}}\right)}{J_2\left(\widetilde{\tau} - \overline{\mathbf{X}}\right)}, \tag{5}$$

where $\widetilde{\tau}^D$ is the deviatoric effective stress tensor. The tensor $\overline{\mathbf{X}}$ is also deviatoric. According to hypothesis of normality of the evolution of the plastic flow, the spatial modified plastic stretching tensor [4], $\widetilde{\mathbf{D}}^p$, is assumed to be governed by the equation:

$$\widetilde{\mathbf{D}}^p = \dot{\lambda}\frac{\partial f}{\partial \tau} = \frac{\dot{\lambda}}{1-D}\frac{\partial f}{\partial \widetilde{\tau}} = \frac{1}{\eta}\left\langle \left(\frac{J_2\left(\widetilde{\tau} - \overline{\mathbf{X}}\right)}{R(r) + \tau_{yo}} \right)^N - 1 \right\rangle \mathbf{N}, \tag{6}$$

where η is the viscosity parameter (which is the relaxation time of mechanical disturbance T_m) and N is a material parameter. For the isotropic hardening variable, the simple equation $\dot{r} = \dot{\lambda}$ is adopted. The evolution of the kinematic hardening tensor (backstress tensor) is assumed to be given:

$$\dot{\mathbf{X}} = \dot{\lambda}\mathbf{R}^{eT}\left(K_1 \frac{\partial f}{\partial \overline{\mathbf{X}}} - K_2\overline{\mathbf{X}} \right)\mathbf{R}^e = \dot{\lambda}\mathbf{R}^{eT}\left(K_1\mathbf{N} - K_2\overline{\mathbf{X}} \right)\mathbf{R}^e, \tag{7}$$

where K_1 and K_2 are material constants. The continuum damage variable is assumed to be governed by the evolution law

$$\dot{D} = \frac{\dot{\lambda}}{1-D}\left(\frac{Y}{S_0}\right)^{s_1}, \tag{8}$$

where S_0 and s_1 are material parameters and $Y = \frac{1}{2}\varepsilon^e : \mathbf{D}^e : \varepsilon^e$ (see [2]).

4. Numerical Integration of Constitutive Equations

The algorithm summarised in Box 1, corresponds to the standard *elastic predictor visco-plastic corrector* (employed for plasticity by BENALLAL et al. [6]) in the small strain context. The evolution equations for r, $\mathbf{X}$ and D have been discretized by one step *backward* Euler scheme. The standard N–R method is used in the solution of the system of equations of the plastic corrector phase. The operations, on the kinematical level, resulting to integration of the constitutive equations at finite strain are summarised in Box 2.

Box 1. Stress updating procedure

(i) Elastic predictor

$$\tau_{n+1}^{trial} = (1-D)\mathbf{D}^e : \varepsilon_{n+1}^{trial}$$

(ii) Check plastic consistency condition

IF $f^{trial} = J_2\left(\frac{\tau_{n+1}^{trial}}{1-D} - \overline{\mathbf{X}}\right) - R(r) - \tau_{yo} \leq 0$ THEN

Set $(\bullet)_{n+1} = (\bullet)_{n+1}^{trial}$ and RETURN

ELSE go to (iii)

(iii) Plastic corrector (solve the system for τ_{n+1}, $\mathbf{X}_{n+1}$, D_{n+1} and $\Delta\lambda$)

$$\left\{ \begin{array}{c} J_2\left(\frac{\tau_{n+1}}{1-D_{n+1}} - \overline{\mathbf{X}}_{n+1}\right) - R\left(r_n + \Delta\lambda\right) - \tau_{yo} - \tau_{vp} \\ \tau_{n+1} - \left(1 - D_{n+1}\right)\mathbf{D}^e : \varepsilon_{n+1}^{e\,trial} + \Delta\lambda\mathbf{D}^e : \mathbf{N}_{n+1} \\ \overline{\mathbf{X}}_{n+1} - \overline{\mathbf{X}}_n - \Delta\lambda\left(K_1\mathbf{N}_{n+1} - K_2\overline{\mathbf{X}}_n\right) \\ D_{n+1} - D_n - \frac{\Delta\lambda}{1-D_{n+1}}\left(\frac{Y_{n+1}}{S_0}\right)^{s_1} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ \mathbf{0} \\ \mathbf{0} \\ 0 \end{array} \right\}$$

where

$$\tau_{vp} = \left(R\left(r_n + \Delta\lambda\right) + \tau_{yo}\right)\left[\left(\eta\frac{\dot{\lambda}_{n+1}}{\left(1 - D_{n+1}\right)} + 1\right)^{\frac{1}{N}} - 1\right],$$

$$\mathbf{N}_{n+1} = \frac{3}{2}\,\frac{\dfrac{\tau_{n+1}^D}{\left(1 - D_{n+1}\right)} - \overline{\mathbf{X}}_{n+1}}{J_2\left(\dfrac{\tau_{n+1}}{1 - D_{n+1}} - \overline{\mathbf{X}}_{n+1}\right)}$$

and

$$\overline{\mathbf{X}}_n = \mathbf{R}_{n+1}^e\mathbf{X}_n\mathbf{R}_{n+1}^{eT}$$

(iv) Update r, ε^e and $\mathbf{X}_{n+1}$

$$r_{n+1} = r_n + \Delta\lambda, \qquad \varepsilon_{n+1}^e = \varepsilon_{n+1}^{e\,trial} - \Delta\lambda\mathbf{N}_{n+1},$$

$$\mathbf{X}_{n+1} = \mathbf{R}_{n+1}^{eT}\overline{\mathbf{X}}_{n+1}\mathbf{R}_{n+1}^e$$

(v) RETURN

5. Numerical Examples

The applicability of the present model is illustrated by loading of a thin plate (in plane stress state) with a geometrical imperfection subjected to stretching along its longitudinal axis with constant velocity $v = 1 \times 10^{-4}$ m/s. The geometry and boundary condition are shown in *Fig. 1*. A thickness heterogeneity is introduced according to

$$x_1 = l\left(1 + a\left(\tanh\frac{m\left(x_2 - y_0\right)^2}{L^2} - 1\right)\right), \tag{9}$$

where $L = 2 \times 10^{-3}m$, $l = 1 \times 10^{-3}m$, $a = 0.002$, $m = 25$ and $y_0 = 2 \times 10^{-3}m$. The calculations have been made with following material parameters: $E = 2.1 \cdot 10^5$ MPa, $\nu = 0.3$, $\tau_{yo} = 102$ MPa, $\rho = 7850$ kg/m^3, $R(r) = Q\left(1 - e^{-br}\right)$, $Q = 100$ MPa, $b = 100$, $K_1 = 6.12 \cdot 10^4$ MPa, $K_2 = 800$, $\eta = 3.5(17.5)$ s,

$N = 3.5$ for 200, 800 and 1800 eight nodes (nine integration points) elements. The reaction force obtained on the restrained edge during the loading process is compared with the results of elasto-plastic damageable model without viscosity in *Fig. 5.* The influence of viscosity in the global behaviour of the structure is clearly shown in *Figs 3–5.*

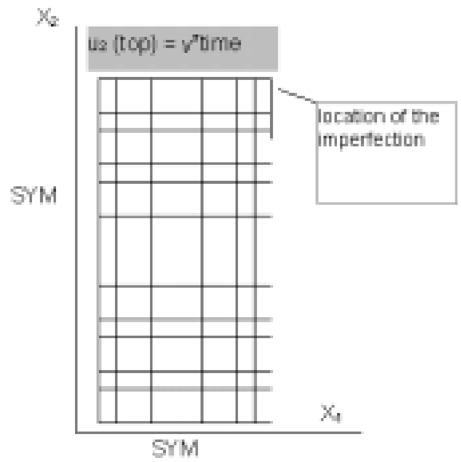

Fig. 1. Boundary and loading conditions

Box 2. Algorithm for integration of constitutive equations

(i) For given increment of displacement $\Delta\mu$, at configuration φ_n, evaluate incremental and total deformation gradient

$$\mathbf{F}_u := 1 + \text{grad}\varphi_n\left[\Delta\mu_{n+1}\right], \qquad \mathbf{F}_{n+1} := \mathbf{F}_u\mathbf{F}_n$$

(ii) Compute elastic trial state

$$\mathbf{B}_{n+1}^{e\,trial} := \mathbf{F}_u\mathbf{B}_n^e\mathbf{F}_n^T$$

$$\mathbf{F}_{n+1}^{e\,trial} := \mathbf{F}_{n+1}\left(\mathbf{F}_n^p\right)^{-1}$$

$$\mathbf{R}_{n+1}^{e} := \mathbf{R}_{n+1}^{e\,trial} = \left(\mathbf{V}_{n+1}^{e\,trial}\right)^{-1}\mathbf{F}_{n+1}^{e\,trial}$$

$$\varepsilon_{n+1}^{e\,trial} = \ln\left[\mathbf{V}_{n+1}^{e\,trial}\right] = \frac{1}{2}\ln\left[\mathbf{B}_{n+1}^{e\,trial}\right]$$

(iii) Perform stress updating procedure for small strain – go to Box 1

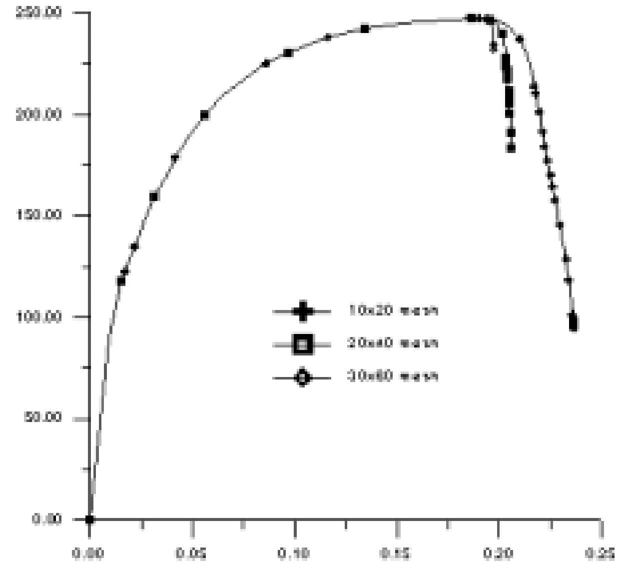

Fig. 2. Mesh dependence of the load-displacement curves for damageable material without viscosity

(iv) Update Cauchy stress and internal variable

$$\sigma_{n+1} := \det\left[\mathbf{F}_{n+1}\right]^{-1} \tau_{n+1}$$

$$\mathbf{V}^e_{n+1} := \exp\left[\varepsilon^e_{n+1}\right], \qquad \mathbf{B}^e_{n+1} := \left(\mathbf{V}^e_{n+1}\right)^2$$

$$\mathbf{F}^p_{n+1} := \left(\mathbf{R}^e_{n+1}\right)^T \left(\mathbf{V}^e_{n+1}\right)^{-1} \mathbf{F}_{n+1}$$

6. Conclusion

In the presented model due to the use of an elastic potential in terms of logarithmic stretches along with an exponential map in the approximation of the plastic flow rule, the finite strain extension effectively appears on the kinematic level only, independently of the characteristics of the material model. Therefore the form of small strain integration algorithm is preserved, resulting in a straightforward computational implementation.

The progressive softening during the loading process reflects the effect of internal deterioration on the material response. Its has also been shown, that viscosity might be used as a regularisation method for damage induced softening.

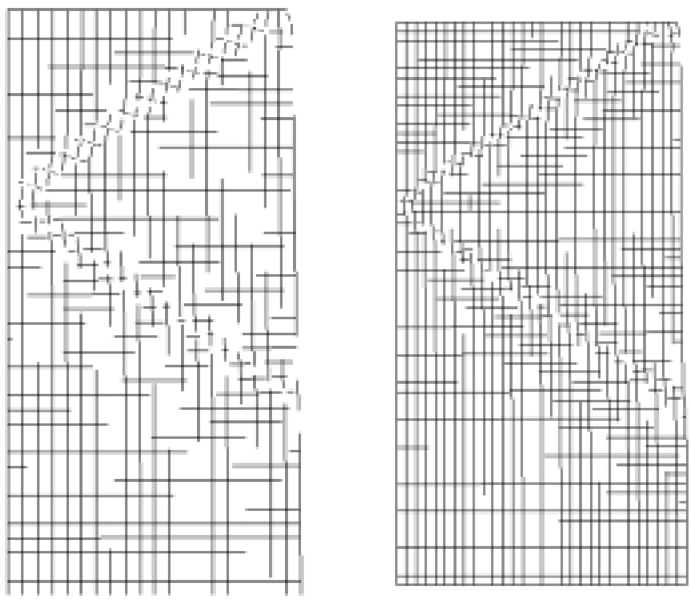

Fig. 3. Simulation of shear bands with a coarse and a fine mesh

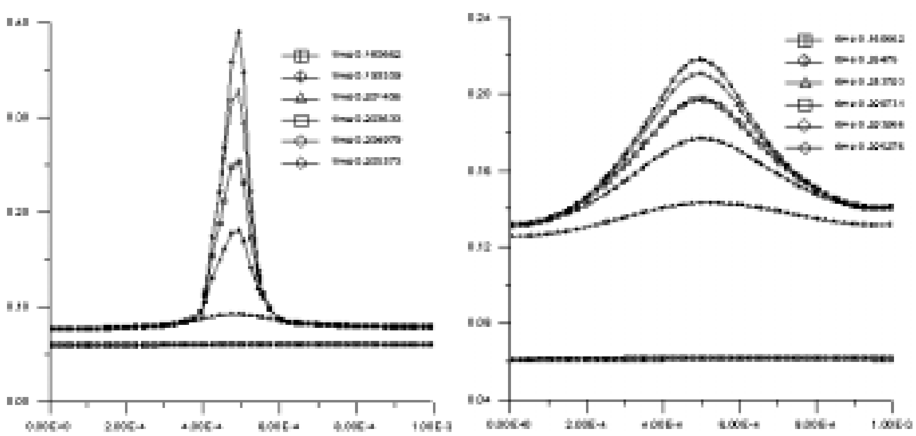

Fig. 4. Growth of a damage (also a shear band) with $\eta = 0$ (left) and $\eta = 3.5$ (right)

Acknowledgement

This work was partly supported by Science Grant Agency of Slovak Ministry of Education and Slovak Academy of Science through Grant No. 95/5195/247. This support is gratefully acknowledged.

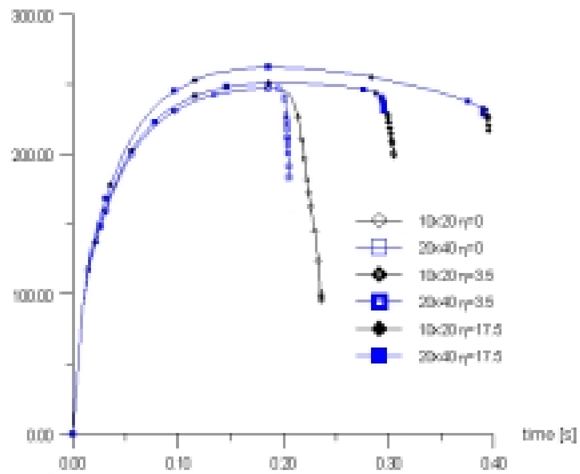

Fig. 5. Mesh dependence of the load-displacement curves for several values of a viscosity parameter ($\eta \in \{0; 3.5; 17.5\}$)

References

[1] KACHANOV, L. M. (1958): Time of the Rupture Process under Creep Conditions. *IVZ Akad. Nauk, S.S.R., Otd. Tech. Nauk*, No. 8, pp. 26–31.

[2] LEMAITRE, J. (1992): A Course on Damage Mechanics. Springer-Verlag.

[3] SUOZA NETO, E. A. – PERIC, D. – OWEN, D. R. J. (1994): A Model for Elasto-plastic Damage at Finite Strains: Algorithmic Issues and Applications., *Engineering Computations,* Vol. 11, Pineridge Press Ltd, pp. 257–281.

[4] LEE, E. H. (1969): Elastic-plastic Deformation at Finite Strains., *J. Appl. Mech.,* Vol. 36, pp. 1–6.

[5] NEMAT-NASSER, S. (1982): On Finite Deformation Elasto-plasticity., *Int. J. Solid Structures,* Vol. 18 (10), pp. 857–872.

[6] BENALLAL, A. – BILLARDON, R. – DOGHRI, I. (1988): An Integration Algorithm and Corresponding Consistent Tangent Operator for Fully Coupled Elasto-plastic and Damage Equations. *Commun. Appl. Numer. Methods,* Vol. 4, pp. 731–740.