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COUPLING OF GENERALIZED HEAT AND MOISTURE TRANSFER

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Abstract

The paper deals with the interdisciplinary problem of coupled diffusion and convention of cross-coupled heat and moisture. After a summary on the well-known and often used cases, the general governing equations are given with examples.

Keywords: heat, moisture, diffusion, convection.

1. Introduction

The problem of heat and moisture transfer has two basic roots. One of them is very practical. Namely, in case of hygroscopic materials, e.g. composites, one has to handle the moisture problems, because of the well-known reason, i.e. the hygroscopic property similarly to the temperature sensitivity may cause degradation and finally may lead to failure.

The other reason why to generalize the heat and moisture transfer problems is a very theoretical one. Namely, the most often used couplings of mechanics are between the displacement, temperature and moisture fields according to modified equation of Duhamel–Neumann, to Gough–Joule, Soret and Dufour effects.

The possibility of symbiosis is obvious: the theory needs the experimental, practical verification, the practice requires theoretical support.

On the other side, we are going to extend the well-known coupling, let us call it crosscoupling, among the objects of transfer (displacement, heat, moisture) to the modes of transfer (diffusion, convection). In this paper the radiation effects are excluded.

The reason is doublefold: according to our knowledge there is no sense to speak about moisture radiation. On the other hand, even though the moisture diffusion through the heat conduction influences the heat radiation, the consideration would be more philosophical than mathematical.

A good example for mode-coupling are the heat-exchangers, e.g. [1], where heat conduction and convection are applied. Another example for both the object coupling (heat and moisture) and mode coupling (conduction, convection and radiation) is the case of repairing buildings by fiber reinforced composite materials [2].

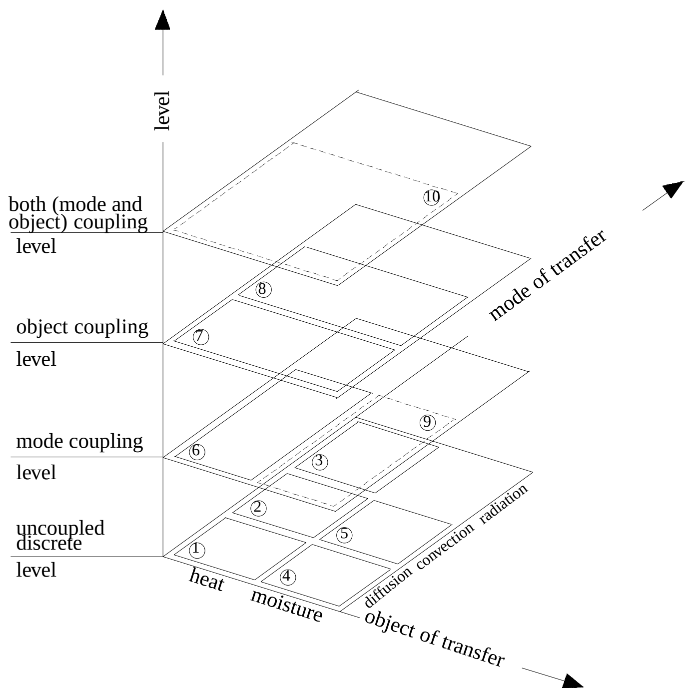

Fig. 1.

Our intention in this study covered by the following pages is summarized on the *Fig. 1.* The *Table 1* contains the needed comments. In the boxes marked by 1 to 8 the wellknown, or at least known expressions and equations are collected in a brief form. The goal of our current investigations is to analyse the boxes 9 and 10, e.g. analysing the coupled diffusion and convection of moisture and coupled diffusion and convection of crosscoupled heat and moisture. The nonframed boxes are in connection to radiation; we are not going to deal with these cases, as mentioned before.

Table 1.

	Case	Law, equation, expression	Reference
1	Heat conduction	Fourier's law	For our
2	Heat convection	Newton's expression	purpose see,
3	Heat radiation	Stefan-Boltzmann's law	e.g. [3]
4	Moisture diffusion	Fick's law	E.g. Fick's orig.publ.: [4]
5	Moisture convection	KLaRa-1	[5]
6	Coupled conduction and convection of heat	Theory of heat-exchangers	E.g. [1]
7	Crosscoupled heat and moisture diffusion	See the reference	[6]
8	Crosscoupled heat and moisture convection	KLaRa-2,-3	[5]
9	Coupled diffusion and convection of moisture	?	
10	Coupled diffusion and convection of crosscoupled heat and moisture	?	

2. Brief Summary of Well-known Cases

According to Fourier's law of *heat conduction* the heat flux q (Wm^{-2}) can be calculated,

$$q = -k\nabla T, \tag{1}$$

where k (Wm^{-1} K^{-1}) is the conductivity, ∇T (Km^{-1}) is the temperature gradient and for further use we have also to define the diffusivity D_T (m^2 s^{-1})

$$D_T = \frac{k}{\rho c_p}, \tag{2}$$

where ρ (kg m^{-3}) is the density and c_p (J kg^{-1} K^{-1}) is the heat capacity.

The *heat convection* can be calculated by Newton's expression,

$$q = h\,(T_s - T_l)\,, \tag{3}$$

where h (Wm^{-2} K^{-1}) is the coefficient of heat transfer, T_s and T_l are temperature of solid surface and of liquid, resp.

The *heat radiation* is described by the Stefan–Boltzmann's law.

According to Fick's law of *moisture diffusion* the moisture flux f (kg m^{-2} s^{-1}) can be calculated,

$$f = -D_m \nabla m, \tag{4}$$

where D_m (m^2 s^{-1}) is the moisture diffusivity, ∇m (kg m^{-4}) is the moisture concentration gradient.

The *moisture convection* can be described by

$$f = h_m \left(m_s - m_l \right), \tag{5}$$

where h_m (ms^{-1}) is the coefficient of moisture transfer, m_s (kg m^{-3}) and m_l are moisture concentration on the surface of the porous material and in the bounding gas or liquid, resp. For further details see [5].

The problem of *coupled conduction and convection of heat* is well elaborated because of the theory of heat-exchangers. We refer only to the literature, e.g. [1] and recall the expression of coupled coefficient of heat transport H (Wm^{-2} K^{-1}) in case of a plane wall,

$$q = H \left(T_1 - T_2 \right), \qquad H = \frac{1}{\dfrac{\delta}{k} + \dfrac{1}{h_1} + \dfrac{1}{h_2}}. \tag{6}$$

Fig. 2 explains all the notations. The *crosscoupled heat and moisture diffusion* is

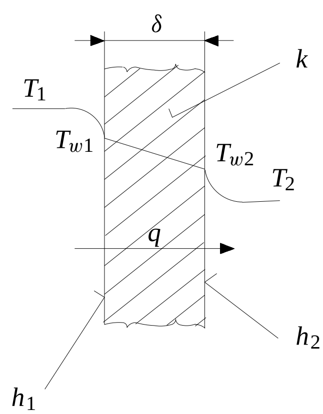

Fig. 2.

discussed in [6]. We recall only the constitutive law of the problem containing the second sound phenomenon, too.

$$\underline{F} + \underline{\underline{\tau}}\, \underline{\dot{F}} = -\underline{\underline{D}}\nabla, \tag{7}$$

where

$$\underline{F} = \begin{bmatrix} f \\ q \end{bmatrix}, \quad \underline{\underline{\tau}} = \begin{bmatrix} \tau_m & \tau_{Tm} \\ \tau_{mT} & \tau_T \end{bmatrix}, \quad \underline{\underline{D}} = \begin{bmatrix} D_m & D_{Tm} \\ D_{mT} & D_T \end{bmatrix}, \quad \underline{\nabla} = \begin{bmatrix} \nabla m \\ \nabla T \end{bmatrix} \tag{8}$$

are the flux matrix, the relaxation time matrix, the diffusivity matrix and the gradient matrix, resp. The elements τ_{Tm}, τ_{mT} and D_{Tm}, D_{mT} express the crosscoupling between heat and moisture.

The *crosscoupled heat and moisture convection* has been analyzed in [3]; [5]. The constitutive law in this case is the following:

$$\underline{F} = \underline{\underline{H}}_h \underline{\Delta}, \tag{9}$$

where

$$\underline{\underline{H}}_h = \begin{bmatrix} h_m & h_{Tm} \\ h_{mT} & h \end{bmatrix}, \qquad \underline{\Delta} = \begin{bmatrix} \Delta m \\ \Delta T \end{bmatrix} \tag{10}$$

are the transfer matrix and difference matrix, resp. Here again h_{Tm} and h_{mT} express crosscoupling.

It has to be mentioned, that due to the Onsager's reciprocity relations the following expressions are held:

$$\widehat{\tau}_{Tm} = \widehat{\tau}_{mT}, \qquad \widehat{D}_{Tm} = \widehat{D}_{mT}, \qquad \widehat{h}_{Tm} = \widehat{h}_{mT}. \tag{11}$$

In *Eq.* (11) the $\wedge$ indicates, that the reciprocity relations are valid only after dimensional fitting of the coefficients.

3. Coupled Diffusion and Convection of Moisture

The considerations are based on the same case of heat, e.g. on the theory of heat-exchangers and also the results are in analogy. Starting with the notations shown in *Fig. 3*, the derivation gives the following expressions for the moisture flux f driven by the moisture concentrations m_1 and m_2 on the two sides of a porous plane wall:

$$f = H_m \left(m_1 - m_2\right), \tag{12}$$

where $H_m \left(ms^{-1}\right)$ is the coupling coefficient of moisture transport for a plane wall:

$$H_m = \frac{1}{\dfrac{\delta}{D_m} + \dfrac{1}{h_m^1} + \dfrac{1}{h_m^2}}. \tag{13}$$

Even the analogy to the same case of heat is obvious (c.f. *Eqs* (12, 13 and 6)), there are basic differences. First of all it is worth to mention, that while heat transfer is related to the microstructure of material, the moisture transfer is characterized by the macrostructure. Consequently, the question of inertia arises, which gives a velocity limit of the process. There is another kind of limit, too, especially the temperature limits of phase change: freezing and evaporation. In both cases the moisture transfer problem disappears: either no pores exist, or there is no moisture any more. To make the practical application of the results easier, we refer to the

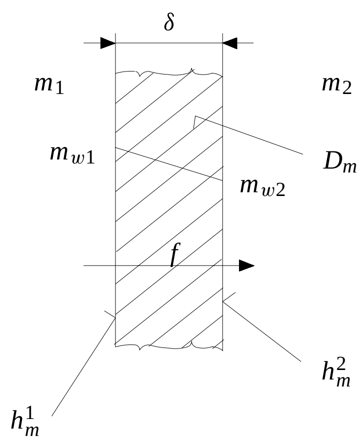

Fig. 3.

notion of moisture potential or normalized moisture concentration introduced earlier (see [5]). By this, instead of *Eq.* (12)

$$f^p = H_m \left(m_1^p - m_2^p\right) \tag{14}$$

applies. In case of a liquid, e.g. $m_1^p = 1$ and in case of a humid gas, e.g. m_2^p is equal to the relative humidity.

4. Coupled Diffusion and Convection of Crosscoupled Heat and Moisture

As it is apparent looking at the *Fig. 1*, this is the synthesis of the previous cases, as marked by 6, 7, 8, 9 on the figure and either referred to (6, 7, 8), or discussed (9) in details.

Based on the *Fig. 4* we are able to express the crosscoupled moisture and heat fluxes driven by the moisture and heat differences $(m_1 - m_2)$ and $(T_1 - T_2)$ through the plane wall with thickness δ. The final results are as follows:

$$\underline{F} = \underline{\underline{H}}^* \underline{D}^*, \tag{15}$$

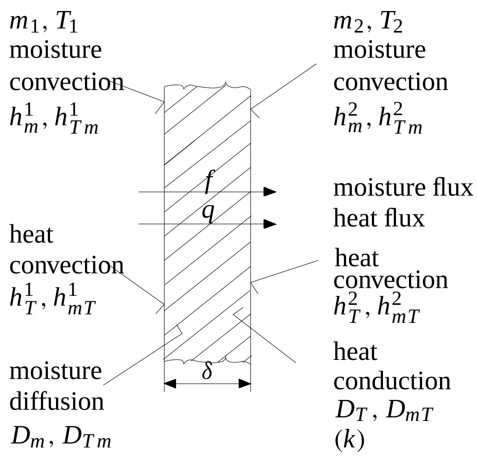

Fig. 4.

where

$$\underline{\underline{H}}^* = \begin{bmatrix} H_m & H_{Tm} \\ H_{mT} & H \end{bmatrix}, \qquad \underline{D}^* = \begin{bmatrix} m_1 & - & m_2 \\ T_1 & - & T_2 \end{bmatrix}, \tag{16}$$

$$H_{Tm} = \frac{1}{\dfrac{\delta}{D_{Tm}} + \dfrac{1}{h^1_{Tm}} + \dfrac{1}{h^2_{Tm}}}, \tag{17}$$

$$H_{mT} = \frac{1}{\dfrac{\delta}{D_{mT}} + \dfrac{1}{h^1_{mT}} + \dfrac{1}{h^2_{mT}}}. \tag{18}$$

Egs. (15 – 18) contain all the subcases mentioned before.

5. Example: Why Does an Amphora Keep the Water Cool?

In this section, as an application of the previous results, we try to explain what, e.g. the ancient Greek, European peasants, American Indians and Arab fellahs have known, or at least applied for centuries.

Let us start according to the *Fig. 5* and as a first approximation of the problem we neglect the curvature and model the wall of amphora as a plane one, even supposing thick walls. It is not a bad approximation compared to the gain in calculations.

We make the first model by pure physical considerations.

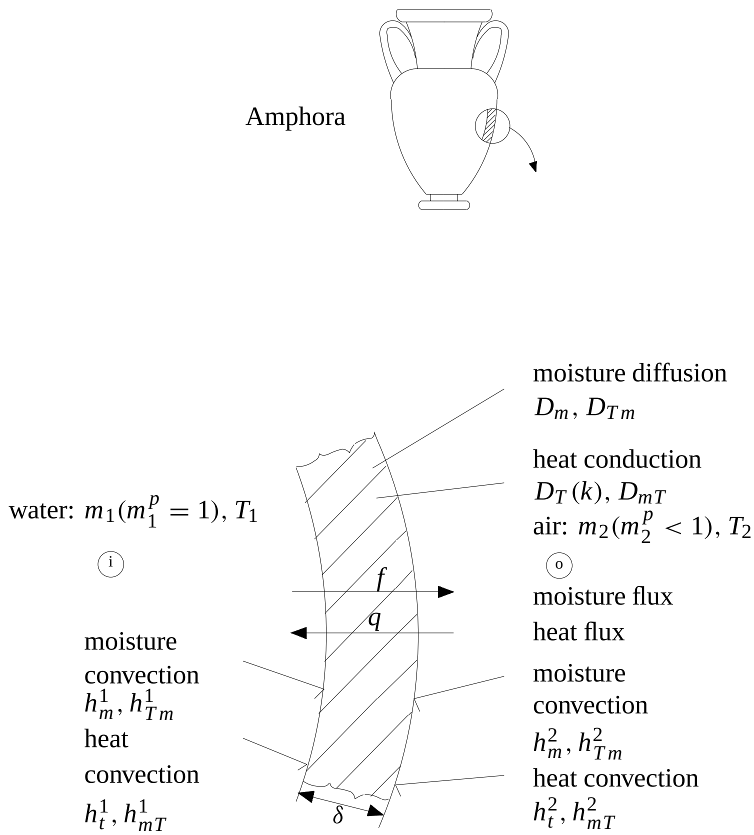

Fig. 5.

a. There is a *moisture transport* between the inner (i) content (water) and outer (o) air through the porous wall of amphora. The moisture flux f is driven by the $(m_1 - m_2)$ moisture concentration difference and, of course, by the $(T_1 - T_2)$ temperature difference. The direction of f is $i \to o$ (*Fig. 5*).
b. There is a *heat transport* between the outer environment of amphora and its inner content through the wall. The heat flux q is driven by the already mentioned temperature and moisture concentration difference. The direction of q is $o \to i$ (*Fig. 5*).
c. There is an *evaporation process* on the outer surface that consumes heat. At this first approach: if this heat is bigger than q, then no heat gets in, by other words, the water inside the amphora won't be warmed up.
   In other words, this is nothing else, but the moisture convection on the outer surface, leaving aside the possible phase change.

d. The material properties, technology of manufacturing and usage regimes – all these define the *efficiency of amphora* and are collected in *Table 2.*
e. The *proper geometry* from a physical viewpoint requires small ratio of surface to volume. This is obvious because of the least heat loss. In addition, the wall should be thick because of better isolation.

Table 2.

	Requirement	Property	Practical hint
1	Good porosity	big D_m small D_{Tm}	Made of clay, faster burning → bigger pores
2	Good moisture convection* on the inner surface	big h_m^1 big h_{Tm}^1	Rough inner surface
3	Good moisture convection on the outer surface	big h_m^2 big h_{Tm}^2	Motion improves it, that is why it is handled more often than needed for drinking.
4	Pure heat conductivity	small $D_T(k)$ big D_{mT}	Made of clay with bigger pores
5	Pure heat convection on the outer surface	small h_T^2 small h_{mT}^2	Smooth outer surface
6	Pure heat convection* on the inner surface	small h_T^1 small h_{mT}^1	Marked by* are in contradiction, it needs compromise.
7	Even the radiation is excluded of our investigations we mention that amphora should be kept in shade		Keep in shade
8	Proper geometry	small A/V thick wall	Close to sphere with thick wall

The second approach is based on *mathematical modelling.* Based on the *Eqs.* (15 – 18), (13) and (6), using the parameters given in *Fig. 5,* the heat flux q can be calculated. If

$$q \cong 0, \tag{19}$$

the amphora keeps the temperature. If it is bigger than zero ($q \geq 0$), the warming up is only the question of time.

Let us collect again the requirements of the good working amphora.

– Material: good moisture diffusion, pure heat conduction.
– Geometry: sphere with thick wall.

– Manufacturing: fast burning (bigger pores), smooth outer, rough inner walls.
– Using: often shaken (to break the boundary layer), kept in shade (to prevent radiation).
– Summing up: sphere with thick wall, made of fast burnt clay, when using keep in shade and shake often.

Even we haven't mentioned yet, there are, of course, artistic requirements, too and these make the amphora an unforgettable treasure of the human culture.

6. Summary

Based on the previous results we extended the problem of heat and moisture transfer in both directions: coupling of modes, as diffusion and convection and crosscoupling of objects as heat and moisture. Finally, the coupled diffusion and convection of crosscoupled heat and moisture were formulated. The equations describing this extended process contain all subcases.

As an example an amphora was described as a container, that keeps the content cool.

Concerning the future of this investigation we have to mention the following possibilities. For numerical calculation, the approximation by spherical wall could be better compared with the plane wall. Taking into account the second sound phenomenon in the constitutive laws makes the result applicable to high-rate processes in special hygroscopic composite materials, e.g. in aircraft technology. The extension of the theory of similarity for this case may give a further opportunity to verify the theory by experiments.

To perform the numerical calculations, several new coefficients of material properties are needed, first of all the crosscoupling coefficients. Using the Onsager's reciprocity relations (see the expressions (11)), the number of coefficients to be determined is smaller, they can be obtained easier, but the dimensional fitting of coefficients in this case is needed.

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