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HEAT SENSITIVITY OF WATER-HEATING SYSTEMS

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Abstract

The most characteristic feature, the heat sensitivity of a water-heating system is expressed by the way in which the effects of some disturbing factors occur. This is especially interesting in the case of systems having simple or double lines, because not being aware of their different characteristics may lead to operating troubles.

Keywords: heating process, double line water-heating system, single line water-heating system, heat sensitivity.

1. Introduction

Water-heating systems are applied most frequently for the heating of houses, generally using hot water with a maximum temperature of 100 °C. The main elements of the system are the heat-generating boiler or the heat exchanger, the pipe network and its fittings, as well as the radiators inside the rooms. Heating itself is an elaborate process of heat transportation, during which the temperature loss of the rooms depending on the weather conditions, basically on the outside temperature, must be compensated for by the heat loss of the radiators, that is, by the change of the heat content of the heat transporting water, circulated by a pump, and eventually by the output of the heat-generator.

Obviously, those factors have a role in this process that are in connection with planning and operation. The characteristics of the heating system are described by the influence of the factors mentioned on the temperature of the heated rooms, in other words, we have to define which factors influence the room temperature in what degree. Heat sensitivity expresses this complicated relation in a concise way.

This phenomenon was considered important, because different heating systems were applied, but some characteristic features of these have not been properly investigated yet. During the course of application of the most wide-spread double line systems with pump (*Fig. 1*, e.g. bottom distribution) it has become quite obvious, that overheating resulting from a securely calculated heat loss or the securely designed dimension of the radiators can be compensated for by decreasing the planned temperature of the heating water, since the radiators connected in a line get a heating water of almost identical temperature. This does not apply to single line heating systems with a pump (*Fig. 2*, top distribution, vertical, by-pass, in the case of which the interactions occur among the radiator groups connected in one line. Consequently, the application of the dimensional principles of double line systems in this case may cause disturbances concerning the planned temperature of the rooms.

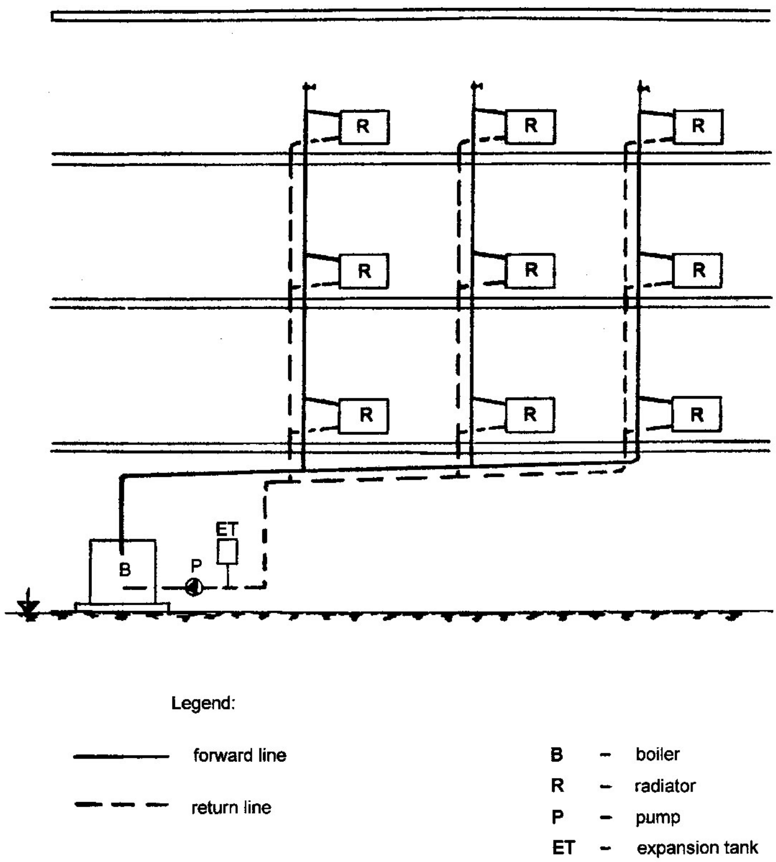

Fig. 1. Circuit diagram of a double line, bottom distribution system with a pump

2. Causalities Concerning Heat Transportation

Simplifications are needed for the mathematical description of the elaborate phenomenon of heat transportation. These simplifications are not only necessary, but they may also be applied, because the differences among the characteristics of the individual systems are shown univocally this way.

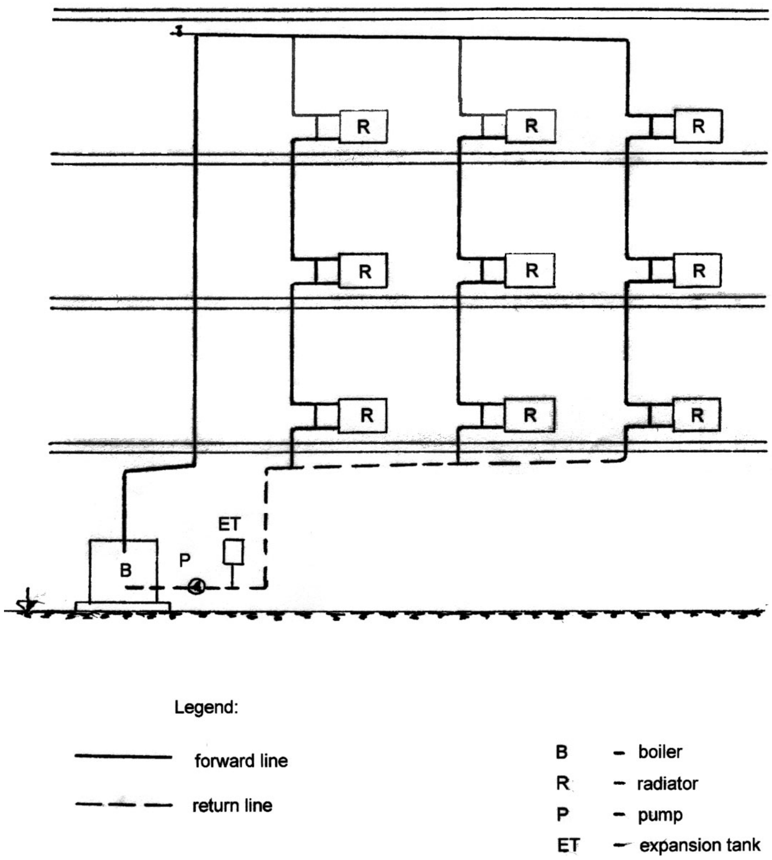

Fig. 2. Circuit diagram of a single line, top distribution by-pass system with a pump

The heat transportation process can be described with the following equation of balance:

$$\dot{Q}_h = \dot{Q}_A = \dot{Q}_f = \dot{Q}_k, \tag{1}$$

wherein

$\dot{Q}_h$ – is the heat loss of the rooms; W,
$\dot{Q}_A$ – is the heat dissipation of the radiators; W,
$\dot{Q}_f$ – is the change of the heat content of the heat carrier, W,
$\dot{Q}_k$ – is the heat output of the boiler; W,

Analysis of the above elements one by one.

Heat loss of the rooms.

As a first simplification let us assume, that in all the rooms of *Figs.* 1 and 2 the same even t_b room temperature is to be maintained continuously; that several rooms may be considered as one closed room. Furthermore, let us consider the cooling down of the heated room, that is the heat loss, as a stationary process, in which t_k, the outside temperature and the air exchange caused by the wind have a role; all other heat loss and heat gain must be left unconsidered. As a result:

$$\dot{Q}_h = N(t_b - t_k)\sum_{i=1}^{n} A_{hi}k_{hi} \qquad \text{W}, \tag{2}$$

wherein

N	–	is a factor concerning the effect of the wind, with a chosen value of	$\gtrless 1.0$
A_{hi}	–	is the inside surface of the closing structure;	m^2
k_{hi}	–	is the heat transmission coefficient of the closing structure;	W/m^2 °C.

Heat dissipation of the radiators.

We assume that in the first line convectional heaters are used, and for the heat output of these the following equation can be used:

$$\dot{Q}_A = Ak_f\Delta t_k = Ak_{fo}\Delta t_k^{1+M} \qquad \text{W} \tag{3}$$

wherein:

A	–	the heat dissipating surface of the radiators,	m^2
$k_f = k_{fo}\Delta t_k^M$	–	heat transmission coefficient of the radiator,	W/m^2 °C,
k_{fo}	–	heat transmission constant of a given radiator,	
M	–	heat transmission coefficient of a given radiator,	
Δt_k	–	average overheating temperature of the heat transferring material of the radiator, with t_e input and t_v output water temperatures:	

$$\Delta t_k = \frac{t_e - t_v}{\ell n\dfrac{t_e - t_b}{t_v - t_b}}. \tag{4}$$

Heat content change of the heat carrier.

The heat carrier pipeline between the boiler and the radiators may also be considered as a radiator itself, so its loss of heat, that is the cooling down of the heat carrier circulating between the boiler and the radiators may be left unconsidered in the calculations. In other words, the t_e temperature of the water leaving the boiler and entering the radiator is identical, as well as the t_v temperature of the water returning into the boiler and leaving the radiator. So the heat output of the boiler is also expressed by the heat content change of the heat carrier:

$$\dot{Q}_f = \dot{Q}_k = \dot{m}c(t_e - t_v) \qquad \text{W} \tag{5}$$

wherein:

$\dot{m}$	–	is the mass flow of the heat carrier circulated by the pump;	kg/h
c	–	is the specific heat of the heat carrier;	W/kg °C.

Since the output of the boiler is also defined by the mass flow and the heat change of the heat carrier ($t_e - t_v$), this needs no further explanation.

3. Exposition of the Equation of Balance, Disturbing Factors

Let us first accept the assumption, that the water heating system designed on the basis of the above causality meets the requirements, that is the heated rooms have the temperature planned.

Accordingly, we may relate the operational state belonging to some outside temperature to the dimensioning state ('o' index), and so we gain the H quotient equations, which we can simplify by using the constants. That is, the quotient equation of the heat loss:

$$H_h = \frac{N(t_b - t_k)\sum A_{hi}k_{hi}}{N_o(t_{bo} - t_{ko})\sum A_{hi}k_{hi}} = \frac{N(t_b - t_k)}{N_o(t_{bo} - t_{ko})}; \tag{2a}$$

the quotient equation of the radiator heat dissipation:

$$H_A = \frac{Ak_{fo}\Delta t_k^{1+M}}{Ak_{fo}\Delta t_{ko}^{1+M}} = \left(\frac{\Delta t_k}{\Delta t_{ko}}\right)^{1+M}; \tag{3a}$$

the quotient equation of the change of heat content in the heat carrier:

$$H_f = \frac{\dot{m}c(t_e - t_v)}{\dot{m}_o c(t_{eo} - t_{vo})} = \frac{\dot{m}}{\dot{m}_o}\frac{(t_e - t_v)}{(t_{eo} - t_{vo})}. \tag{5a}$$

According to our previous assumption, from the equivalence of the *Eqs.* (2a) and (5a) $t_v = t_{vo}$, and placing this into the *Eqs.* (3a) and (5a) leads to the result: $t_b = t_{bo}$. Now we can investigate what kind of mistakes may occur during the designing, constructing and operational phases, that is, how can the required, faultless heating process be disturbed.

The disturbing factor (z) may be the quotient of the real values following the disturbance and the required values, that is:

the real heat loss is not identical with the required one:

$$z_h = \frac{\dot{Q}_{hv}}{\dot{Q}_{hsz}}; \tag{6}$$

the actual mass flow of the heat carrier differs from the required value, due to some kind of trouble in the flow:

$$z_m = \frac{\dot{m}_v}{\dot{m}_{sz}} = \frac{\dot{m}}{\dot{m}_o}; \tag{7}$$

the actual temperature of the heating water is not identical with the required value:

$$z_t = \frac{t_{ev}}{t_{esz}}; \tag{8}$$

there is a difference between the actual and the required surface area of the radiator:

$$z_A = \frac{A_v}{A_{sz}}; \tag{9}$$

finally, the actual heating output of the radiator differs from the required value calculated during the designing phase:

$$z_M = \frac{M_v}{M_{sz}}. \tag{10}$$

All of the disturbing factors might be

$$z_i \underset{<}{\overset{>}{-}} 1.0$$

and we may have $3^5 = 243$ variations of this. Only one of these is the $z_i = 1.0$ case, when the actual temperature of the room $t_{bv} = t_b = t_{bo}$. From the 242 further cases $z_i < 1.0$ and $z_i > 1.0$ the effect of the disturbances may be compensated for in about 160 cases, but in the remaining 80 cases the effect of the disturbances only gets stronger, that is $t_{bv} = t_b \neq t_{bo}$ comes into effect.

If we fill in the disturbing factors into the equations of heat carrying: (2a, 3a, 5a), as $H_h = H_A = H_f$, we get:

$$z_h \frac{N}{N_o} \frac{t_b - t_k}{t_{bo} - t_{ko}} = z_A \left(\frac{\ell n \dfrac{t_{eo} - t_{bo}}{t_{vo} - t_{bo}}}{t_{eo} - t_{vo}} \right)^{1+M} \left(\frac{z_t t_e - t_v}{\ell n \dfrac{z_t t_e - t_b}{t_v - t_b}} \right)^{1+M z_M} = z_m \frac{z_t t_e - t_v}{t_{eo} - t_{vo}}. \tag{11}$$

The t_b value can generally be expressed by the following double equation:

$$t_b = f(t_{bo}; t_{ko}; N_o; t_{eo}; t_{vo}; M; t_k; N; t_e; t_v; z_i). \tag{12}$$

4. The Calculation of the Room Temperature, t_b

The causality no.(12) can only be solved with iteration.

Since our primary goal is to find out more about the effects of disturbing factors (z_i), we take these as independent variables one by one, the other values are handled as parameters with a typical practical value.

The calculation process can look as follows:

- taking dimensional values, which are: t_{bo}; t_{ko}; N_o; t_{eo}; t_{vo}; M,
- choosing some kind of weather condition that can be interesting, like: $t_k = t_{kV}$; $N = N_V$,
  according to the relevant temperature controlling function of the heating water $t_e = t_{eV}$ is added to the above,

- z_i disturbing factors remain variables,
- taking $H_h = H_f$ part of the double *Eq.* (11) we get:

$$t_b = t_{kV} + \frac{z_m}{z_h}\frac{N_o}{N_V}\frac{t_{bo}-t_{ko}}{t_{eo}-t_{vo}}(z_t t_{eV} - t_v), \tag{13}$$

- then, from the $H_A = H_f$ part of the double equation we get:

$$t_v = z_t t_{eV} - \frac{z_A}{z_m}\frac{\left(\ell n\dfrac{t_{eo}-t_{bo}}{t_{vo}-t_{bo}}\right)^{1+M}}{(t_{eo}-t_{vo})^M}G, \tag{14a}$$

wherein:

$$G = \left[\frac{z_t t_{eV} - t_v}{\ell n\dfrac{z_t t_{eV} - t_{kV} - \dfrac{z_m}{z_h}\dfrac{N_o}{N_V}\dfrac{t_{bo}-t_{ko}}{t_{eo}-t_{vo}}(z_t t_{eV} - t_v)}{t_v - t_{kV} + \dfrac{z_m}{z_h}\dfrac{N_o}{N_V}\dfrac{t_{bo}-t_{ko}}{t_{eo}-t_{vo}}(z_t t_{eV} - t_v)}}\right]^{1+Mz_M} \tag{14b}$$

- after taking z_i value

$$P = \frac{z_m}{z_h}\frac{N_o}{N_V}\frac{t_{bo}-t_{ko}}{t_{eo}-t_{vo}} \quad \text{and} \quad B = z_t t_{eV},$$

with these:

$$t_b = t_{kV} + P(B - t_v), \tag{13a}$$

as well as

$$\begin{aligned} C &= \frac{z_A}{z_m}\frac{\left(\ell n\dfrac{t_{eo}-t_{bo}}{t_{vo}-t_{bo}}\right)^{1+M}}{(t_{eo}-t_{vo})^M}, \\ D &= B - t_{kV} - PB = B(1-P) - t_{kV}, \\ E &= -PB - t_{kV}; \qquad F = 1 + Mz_M, \end{aligned}$$

thus

$$t_v = B - C\left[\frac{B - t_v}{\ell n\dfrac{Pt_v + D}{(P+1)t_v + E}}\right]^F, \tag{14c}$$

- with the iteration of *Eq.* (14c) we take an estimated t_{v1est} value, substituting this we get the t_{v1res} result, then the mathematical average is the next estimate, and so on, or generally expressed:

$$t_{v(i+1)est} = \frac{t_{viest} + t_{vires}}{2}, \tag{15}$$

then we can prescribe a $t_{v(i+1)res} - t_{v(i+1)est} \to 0.00$ arbitrary requirement, the last t_v value is substituted in the (13a) causality, then we get the t_b final result.

5. Calculation Results for Typical Systems

In order to provide apartments on a large scale, houses with a medium height are quite wide spread. Numerous prefab houses of this kind have also been built in Hungary. With the primary aim of making the mounting of the heating network easier and typical in general, the systems applied have top distribution and crossbinding single pipes.

The calculation is applied for an $n = 5$ storey building, with a double line heating system or a single pipe, by-pass system, since these have been the typical solutions recently. A simplifying assumption is, that all radiators have an identical output.

Basic dimensional data:

room temperature: $t_{bo} = 22\ ^\circ\mathrm{C}$
outside temperature: $t_{ko} = -12\ ^\circ\mathrm{C}$
wind factor: $N_o = 1.0$
temperature of the forward heating water: $t_{eo} = 90\ ^\circ\mathrm{C}$
temperature of the returning heating water: $t_{vo} = 70\ ^\circ\mathrm{C}$
cooling down of the water in the radiators of the double line system:

$$\Delta t_{o;2} = t_{eo} - t_{vo} = 20\ ^\circ\mathrm{C}$$

in the single pipe by-pass system, with an inflow factor of $\beta = 0.35$, according to the relevant causality:

$$\Delta t_{o;1} = \frac{t_{eo} - t_{vo}}{n\beta} = \frac{90 - 70}{5 \cdot 0.35} = \frac{20}{1.75} = 11.43\ ^\circ\mathrm{C}$$

applying convection radiators: $M = 0.3$; the circumstances of the investigation should be identical with the dimensioning circumstances, thus:

$$\begin{aligned} t_{kV} &= t_{ko} = -12\ ^\circ\mathrm{C} \\ N_V &= N_o = 1.0 \\ t_{eV} &= t_{eo} = 90\ ^\circ\mathrm{C} \end{aligned}$$

which applies for all the radiators of the double line system, but in the case of the single line system it only applies for the first radiators of the downward line, but in the case of the others the incoming water is mixed and therefore has a different temperature.

The t_b actual room temperature values are calculated with the above values in such a way, that only one disturbing factor is taken into consideration as a variable, the other values remain units. This way the two systems can be compared from the point of view of sensibility. In case there are more disturbing factors, the relevant calculations show, that if we calculate the overheating of the rooms by factors $(t_b - t_{bo})$, the total sum of these approximate the overheat temperature caused by several disturbing factors, that is:

$$t_{b;z_i} - t_{bo} = \sum [t_b = f(t_b; z_i) - t_{bo}]. \tag{16}$$

Some of the results gained in the above way are shown in a chart and also graphically.

Table 1. Chart of the calculated room temperatures: t_b

z_i	$t_{b;2}$	$t_{b;1;1}$	$t_{b;1;5}$	$t_{b;2}$	$t_{b;1;1}$	$t_{b;1;5}$
	if $z_h = z_i$			if $z_t = z_i$		
0.6	33.27	33.16	33.52	8.59	8.46	8.93
0.8	26.81	26.76	26.91	15.20	15.12	15.40
1.0	22.00	22.00	22.00	22.00	22.00	22.00
1.2	18.27	18.30	18.20	28.96	29.06	28.71
1.4	15.29	15.34	15.17	36.05	36.30	35.51
	if $z_m = z_i$			if $z_A = z_i$		
0.6	19.31	20.48	16.68	13.66	13.03	15.37
0.8	20.95	21.42	19.86	18.29	17.95	19.16
1.0	22.00	22.00	22.00	22.00	22.00	22.00
1.2	22.72	22.40	23.53	25.05	25.40	24.21
1.4	23.25	22.68	24.71	27.60	28.30	25.98
2.0	24.23	23.20	26.95	33.28	34.95	29.70
	if $z_M = z_i$					
0.6	13.72	12.97	15.75			
0.8	17.87	17.43	19.02			
1.0	22.00	22.00	22.00			
1.2	25.96	26.53	24.64			
1.4	29.65	30.87	26.94			

The chart shows the room temperatures in the case of some disturbing factors, for double line systems ($t_{b;2}$), as well as for the downward line of the single pipe system in the first ($t_{b;1;1}$) and fifth ($t_{b;1;5}$) rooms. In this last case the temperature of the intermediate rooms is an intermediate value on an exponential curve. (Here we mentioned a downward line, because we assume that a vertical connecting line is to be found between the radiators, but the results are the same in the case of horizontal connecting lines as well.)

Fig. 3 contains the diagrams of the above values.

6. Conclusions

Several important conclusions can be drawn from the results. The most important one is that the room temperature characteristics, i.e. the temperature sensitivity of the double line and single line systems are different. The other phenomenon found is that the room temperature in the case of a double line system falls between the temperatures of the first and last rooms of the single line system, it is about the mathematical average of those.

Situation according to disturbing factors:

The disturbing factor of the heat loss and the heating water temperature in the first and last rooms of the single pipe system causes only a very little temperature difference, which can hardly be recognized graphically. The lines coincide, thus they are identical with the curve of the double line system (curves signed z_h and z_t in *Fig. 3*). But in the case of values differing from the z_A, z_m, and z_M coefficients, the temperatures of the first and last rooms in the single line system (dashed line, numbers 1 and 5 in *Fig. 3*) are significantly different, which can be further aggravated by the parallel occurrence of several disturbing factors. This means that the effect of these three disturbing factors cannot be compensated for by the intentional modification of another disturbing factor, the resulting effect will occur in some other room anyway. For instance the oversized radiator ($z_A > 1.0$) will cause overheating in all rooms, and if the planned temperature of the heating water, or its mass flow is centrally reduced, the temperature difference between the rooms will remain, although, in a different temperature range. In other words, we get the planned temperature in an intermediate room, but the preceding rooms will be overheated and the rooms placed after it will not be heated sufficiently. The results show that many unfavourable situations might occur, which justify the decreasing popularity of the single line system, furthermore its application is questionable from a professional point of view, because of insufficient knowledge about its specific features.

7. Summary

The above investigation, applying permissible estimations, provides theoretical proof for the different heat sensitivity characteristics of single line and double line water-heating systems. The operational characteristics of the single line system are known to be more elaborate, and according to the results, the dimensional designing needs special care and a more profound method. On this basis we can get a properly

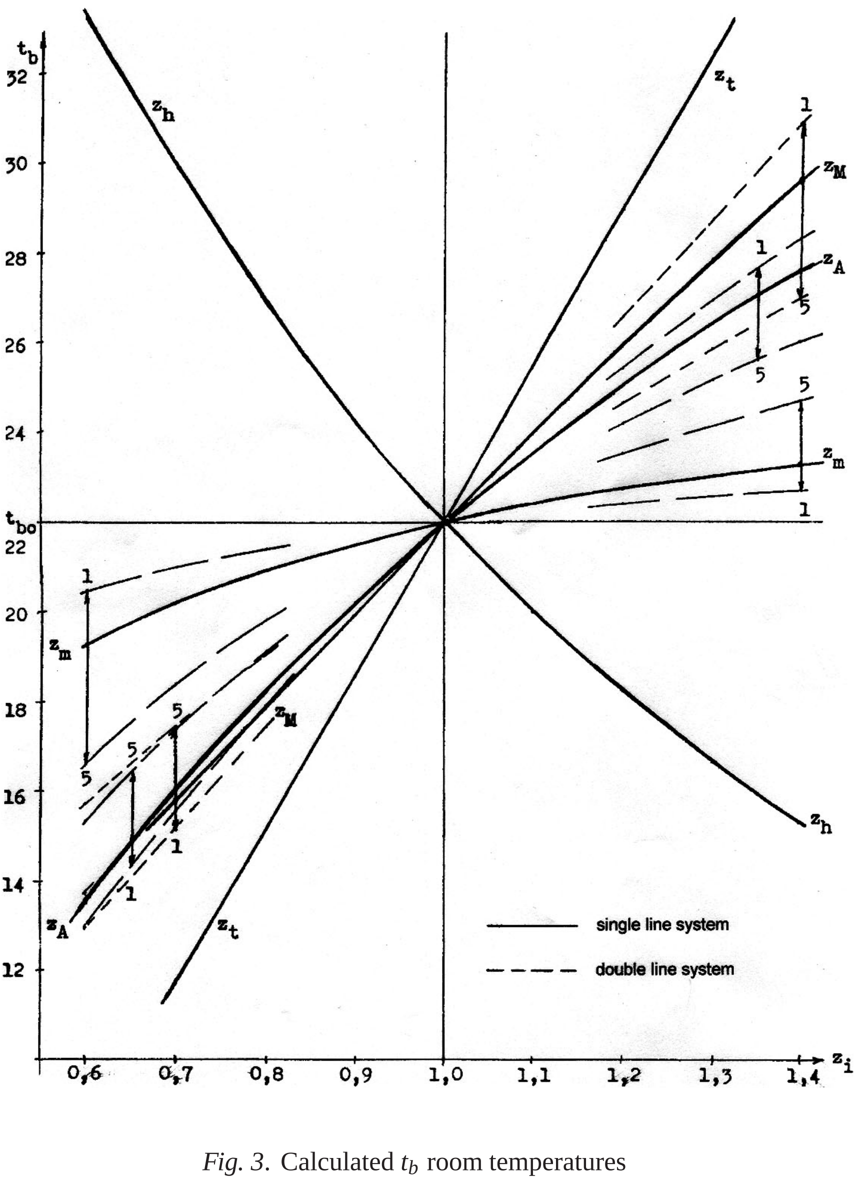

Fig. 3. Calculated t_b room temperatures

functioning system, nevertheless, the structural and productional inaccuracies, as well as the tolerances make the proper adjustment of the system necessary. In the case of single line systems this is rather complicated, special equipment and a lot of work is needed, the costs of which can be compensated for by the application of a thermostatic radiator valve, which beside being useful for eliminating some designing and operational defects, is also advantageous in other respects.

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