ANALYSIS OF HEAT SOURCE AND CONNECTED HEAT CONTAINER

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Abstract

Method for the complex dimensioning of the heat resource and energy accumulator, enabling an economical design regarding the implementation of an optimal heating technology.

Keywords: heat energy storing equipment, optimization, circulation, critical storing temperature, heating-up temperature.

It may be necessary to deploy an energy accumulator to satisfy the changing heat energy consumption. In the case of temporarily standard or changing consumer needs it is rather hard to design a heat source in such a way, that a relatively favourable solution can be obtained. This requires an analysis of the economical and heat technological optimum, which is made possible by the following method of investigation.

The block diagram of the investigated system is shown in *Fig. 1*. The heat agent is water.

The aim is to define a relationship suitable for dimensioning the system, which also makes it possible to prepare analyses and take decisions in order to achieve an economical design, furthermore it should be suitable for drawing conclusions regarding the heat technological optimum.

It is useful to choose the following simplifying conditions:

- the volume of the heat exchanger of the secondary side and the pipe section following it should be concentrated into the V container volume,
- the heat capacity of the other structural components of the system should be considered as negligible compared to the heat capacity of the water of the container,
- let us assume, that the system is surrounded by perfect insulating material with no heat capacity ($\eta_{sz} = 100\%$),
- let us assume, that the temperature of the water in the container levels out immediately with infinite mixing speed

$$\text{grad } \vartheta_t = 0,$$

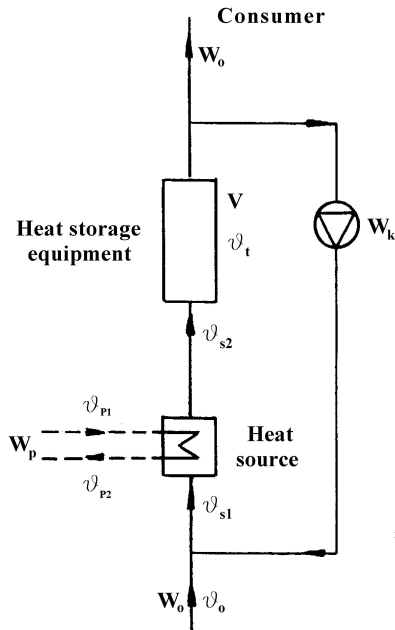- the amount of the water circulated should be considered as standard,

Fig. 1.

- the heat equivalent of the output of the circulation pump should be considered as negligible,
- the temperature dependence of the specific heat and the density should be considered negligible,
- let us assume, that the ϑ_{p1}, W_p and W_k values are temporarily constant,
- let us consider the heat transmission factor of the heat exchanger within a given time interval a constant value,
- all temperature values should be related to the secondary water temperature entering the system.

With the above conditions the equations of thermal equilibrium can be given. The equation of water mixing:

$$\vartheta_{sl} = \frac{W_k}{W_o + W_k}\vartheta_t. \tag{1}$$

Equation system of equilibrium of the heat exchanger:

$$Q = (W_o + W_k)(\vartheta_{s_2} - \vartheta_{s_1}). \tag{2}$$
$$Q = W_p(\vartheta_{p_1} - \vartheta_{p_2}). \tag{3}$$
$$Q = k \cdot A \cdot \Delta\vartheta_k. \tag{4}$$

Equation expressing the momentary equilibrium of the heat container:

$$\frac{d\vartheta_t}{d\tau} = \frac{W_o + W_k}{c \cdot \rho \cdot V}(\vartheta_{s_2} - \vartheta_t).$$ (5)

Let us consider the superheat of the container a variable.

The following equation can be set up on the basis of *Eqs.* (3) and (4):

$$kA\frac{\vartheta_{p_1} - \vartheta_{p_2} - \vartheta_{s_2} + \vartheta_{s_1}}{\ln\dfrac{\vartheta_{p_1} - \vartheta_{s_2}}{\vartheta_{p_2} - \vartheta_{s_1}}} = W_{p(\vartheta_{p_1} - \vartheta_{p_2})}$$ (6)

and the following on the basis of (2) and (3):

$$\frac{\vartheta_{s_2} - \vartheta_{s_1}}{\vartheta_{p_1} - \vartheta_{p_2}} = \frac{W_p}{W_o + W_k},$$ (7)

as well as

$$\vartheta_{p_2} = \vartheta_{p_1} - \frac{W_o + W_k}{W_p}\left(\vartheta_{s_2} - \frac{W_k}{W_o + W_k}\vartheta_t\right).$$ (8)

From the *Eqs.* (6) and (7) the following equation can be established:

$$e^{kA\left(\frac{1}{W_p} - \frac{1}{W_o + W_k}\right)} = \frac{\vartheta_{p_1} - \vartheta_{s_2}}{\vartheta_{p_2} - \vartheta_{s_1}}.$$ (9)

Let us assume that:

$$\varepsilon = \frac{\vartheta_{p_1} - \vartheta_{s_2}}{\vartheta_{p_2} - \vartheta_{s_1}}.$$ (10)

Expressing from (1), (7) and (10):

$$\vartheta_{s_2} = \frac{1 - \varepsilon}{1 - \dfrac{W_o + W_k}{W_p}\varepsilon}\vartheta_{p_1} + \frac{\left(\dfrac{W_k}{W_o + W_k} - \dfrac{W_k}{W_p}\right)\varepsilon}{l - \dfrac{W_o + W_k}{W_p}\varepsilon}\theta_t$$ (11)

(11) in a shorter form:

$$\vartheta_{s_2} = C_1 \cdot \vartheta_{P_1} + C_2 \cdot \vartheta_t.$$ (12)

From (5) and (12) the following can be established:

$$\frac{d\vartheta_t}{d\tau} = \frac{W_o + W_k}{c \cdot \rho \cdot V}[C_1\vartheta_{p_1} - (1 - C_2)\vartheta_t].$$ (13)

In a different form:

$$\frac{1}{C_1\vartheta_{p_1} - (1 - C_2)\vartheta_t}d\vartheta_t = \frac{W_o + W_k}{c \cdot \rho \cdot V}d\tau.$$ (14)

(14) with the following marginal condition: $[\vartheta_t]_{\tau=0} = \vartheta_{to}$

$$\int_{t_o}^{t} \frac{1}{\vartheta_t - \dfrac{C_1}{1 - C_2} \vartheta_{p1}} d\vartheta_t = -(1 - C_2) \int_{0}^{\tau} \frac{W_o + W_k}{c \cdot \rho \cdot V} d\tau. \tag{15}$$

As a result we get:

$$\vartheta_t = \frac{C_1}{1 - C_2} \vartheta_{p1} + \frac{1}{1 - C_2} \left[(1 - C_2)\vartheta_{to} - C_1\vartheta_{p1}\right] e^{-(1-C_2)\frac{W_o + W_k}{c\rho V}\tau}. \tag{16}$$

This makes the dimensioning of the system possible, if ϑ_{P1}, W_p, W_o, and $\vartheta_{t\,\min}$ are given. A, V, W_k may be chosen without restriction.

The temporal change in consumption is demonstrated graphically. Taking this into consideration it is useful to choose a time interval, within which the consumption can be considered constant. With the $W_k = 0$ condition of simplification it is easy to get a solution for the presentation of the following relation e.g. $V = f(A)$ (*Fig. 2*).
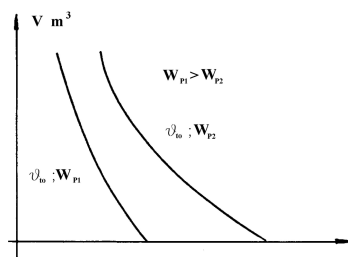

Fig. 2.

In this case:

$$\vartheta_t = C\vartheta_{p1} + (\vartheta_{t0} - C\vartheta_{p1}) \cdot e^{-\frac{W_0}{c\rho V}\tau}, \tag{17}$$

wherein $C = C_1$

On the basis of the figure it is possible to decide whether the system is economical or not. Obviously, the calculation is also possible in case Wk ....

It can also be interesting to find an answer for the following question: When would the heating speed of the container be maximal?

The following analysis is needed for finding an answer:

$$\frac{\partial^2 \vartheta_t}{\partial \tau \partial W_t} = 0, \tag{18}$$

or

$$\left| \frac{\partial^3 \vartheta_t}{\partial \tau W_k^2} \right|^{\frac{\partial^2 \vartheta_t}{\partial \tau \partial W_t}} \begin{array}{c} \leq \\ = \\ > \end{array} 0. \tag{19}$$

In order to make the deduction and finding the result more simple, it is reasonable to use the arithmetic mean temperature difference, but the result obtained should be interpreted from a physical point of view, also for the case of the logarithmic medium temperature difference.

The following equation can be set up with an arithmetic mean temperature difference on the basis of the equation systems (3) and (4) as well as the equations (1) and (8):

$$\vartheta_{s2} = \frac{2}{1 + (W_o + W_k)\left[\dfrac{1}{W_p} + \dfrac{2}{KA}\right]}\vartheta_{p1} + \frac{W_k\left[\dfrac{1}{W_p} + \dfrac{2}{KA} - \dfrac{1}{W_o + W_k}\right]}{1 + (W_o + W_k)\left[\dfrac{1}{W_p} + \dfrac{2}{kA}\right]}\vartheta_t.$$

(20)

Substituting this into the differential equation of the container, after throwing the equation into standard form, we get the following:

$$\frac{\mathrm{d}\vartheta_t}{\mathrm{d}\tau} = 2\vartheta_{p1} - \vartheta_t\frac{W_k}{W_o + W_k}\vartheta_t$$

$$-W_o\left(\frac{1}{W_p} + \frac{2}{kA}\right)\vartheta_t\left[\rho \cdot V\left(\frac{1}{W_o + W_k} + \frac{1}{W_p} + \frac{2}{kA}\right)\right]^{-1}.$$

(21)

The next task is to define the first derivative of the equation according to W_k. After simplification:

$$\frac{\partial^2\vartheta_t}{\partial\tau\,\partial W_k} = \vartheta_{p1}$$

$$-\vartheta_t\left(1 + \frac{2W_o}{kA} + \frac{W_o}{Wp}\right)\cdot\left[\frac{c\rho \cdot V}{2}\left(1 + \frac{W_p + W_k}{W_p} + 2\frac{W_o + W_k}{kA}\right)^2\right]^{-1}.$$

(22)

The condition (18) is fulfilled, if the denominator of the fraction is infinite, or its numerator is 0. The first case has no practical importance, and the next equation follows from the other one:

$$\vartheta_t = \vartheta_{t_{\text{crit}}} = \frac{\vartheta_{p1}}{1 + \dfrac{2W_o}{kA} + \dfrac{W_o}{W_p}}.$$

(23)

This is actually the average (ϑ_{sk}) temperature of the heated water in the heat exchanger, in case $W_k = 0$.

$$\vartheta_{t_{\text{crit}}} = \vartheta_{sk} = \frac{\vartheta_{s2}}{2} = \frac{\vartheta_{p1}}{1 + W_o\left(\dfrac{2}{kA} + \dfrac{1}{W_p}\right)}.$$

(24)

The equation should be examined in the assumed place of the marginal value. The equation should be differentiated for a second time according to W_k. In case the equation constituting the denominator of the equation is signed B:

$$\frac{\partial^3 \vartheta_t}{\partial \tau \partial W_k^2} = \left[-(\vartheta_{p1} - \vartheta_t) \left(1 + \frac{2W_o}{kA} + \frac{W_o}{W_p} \right) \frac{\partial B}{\partial W_k} \right] \cdot B^{-2}. \tag{25}$$

The equation has an inflexion in the assumed place of the marginal value, because
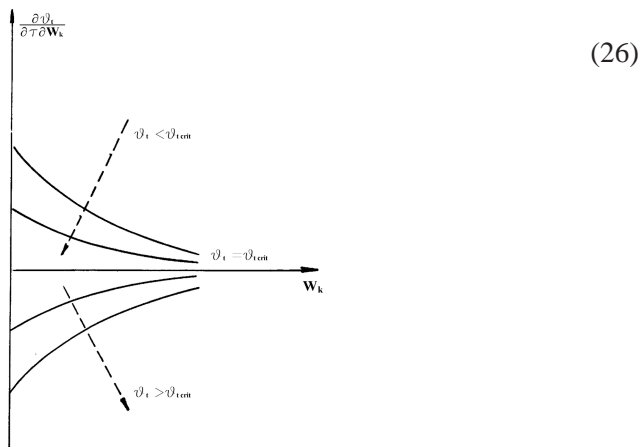
$$\tag{26}$$

Fig. 3.

Fig. 3 is showing the change of speed of overheating. The following conclusions can be drawn from the figure:

- if the temperature of the circulated water equals the critical temperature of the container, the amount of the circulated water is unimportant from the point of view of the speed change of the water temperature in the container

$$\vartheta_t = \vartheta_{t_{\text{crit}}} \longrightarrow W_k \qquad \text{indifferent,} \tag{27}$$

- if the temperature of the circulated water is lower than the critical temperature, in principle, it would be ideal to have an infinite amount of water circulated, because an increase in the amount of the circulated water increases the speed of heating beyond any limit:

$$\vartheta_t < \vartheta_{t_{\text{crit}}} \longrightarrow W_{k_{\text{opt}}} = \infty, \tag{28}$$

- if the temperature of the circulated water is higher than the critical temperature, the circulation of the water is unfavourable from the point of view of the heat technological optimum of the equipment.

$$\vartheta_t > \vartheta_{t_{\text{crit}}} \longrightarrow W_{k_{\text{opt}}} = 0. \tag{29}$$

Eventually, this means, that in case the temperature of the circulated water is higher than the average temperature of the consumed water in the heat exchanger, the circulation results in a worse average temperature difference of the heat exchanger, and vice versa.

In case the investigation is carried out using logarithmic mean temperature difference, the critical temperature would have occurred as the logarithmic mean temperature of the heated water. It can also lead to a change in the result, if we do not assume the heat transmission factor as a constant value. But this is not significant in the case of double sided turbulent flow, because the differential quotient

$$\frac{\partial k}{\partial W_k} \tag{30}$$

is low enough not to be taken into account.

Thus we can see, that depending on the temperature of the circulated water, the amount of water to be circulated is sometimes 0, and sometimes infinite, depending on the consumption.

The investigation of the effect of any further parameters is also possible on the basis of the above relationships.

Obviously, in a favourable situation the economic and heat technological optima coincide.

List of Symbols

ϑ super temperature considering the temperature of the water to be heated as reference temperature °C

W value of water flow $\frac{\mathrm{k\,W}}{\mathrm{K^\circ}}$

k heat transmission factor of the heat exchanger, W/m^2, K°

V volume of the container, m^3

A heating surface of the heat exchanger, m^2

c mean specific heat of the water, kJ/kg, K

τ time, s

ρ mean density of the water, kg/m^3

Q heat flow in the heat exchanger, kW

Δt_k mean, logarithmic temperature difference in the heat exchanger, K

Indexes

$p1$ primary heating water entering the heat exchanger
$p2$ primary heating water leaving the heat exchanger
$s1$ secondary water entering the heat exchanger
$s2$ water entering the container
t container
o consumed water
k circulated water
p primary heating water

References

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