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DESCRIPTION OF THE GAS AND SERVICE WATER CONSUMPTION PROCESS BY MEANS OF PROBABILITY CALCULATION

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Abstract

The characteristics of random gas and service water consumption of individual consumer groups (quantity, intensity, simultaneity, etc.) are probability variables in the mathematical sense. In scheduling gas and service water consumption, the chances of the realisation of the characteristics and the probability distribution also have to be determined. The authors relate the determination of scheduled values to the extent of assumption of risk. In the study they present new methods based on the theory of stochastic processes for describing the gas consumption of individual consumer groups and for determining the probability distribution of consumption values. They also show correlations to describe gas and service water consumption as the basis of the total heat load of gas appliances and cold or hot water fixtures.

Keywords: service water consumption, gas consumption, probability distribution.

1. Introduction

It is a well-known fact that the service water and gas consumption of a residential area or urban sector is generally not uniform, however, it shows very great fluctuations within a day and the consumption levels per days, weeks and months are different as well. Generally, the nature of consumption within a day shows substantial fluctuation, including a.m. and p.m. peak periods, low consumption level between these periods and modulations superimposed on the peaks and valleys.

The actual causes of consumption fluctuations by parts of the day and week in the domestic sector are well known and are to be shared for in the hygiene and organisation of work and living conditions of the population.

The pattern of consumption fluctuation may follow certain constancy and regularity but the superimposed modulations are entirely of random character and cannot be predicted with accuracy.

Finally, the case is that consumers will generally be involved in the service water and gas consumption at random times and will consume water or gas with varying intensity and for random periods. Therefore, both the resulting consumption intensity and its volume show random variations.

For this reason, when designing and operating service water or gas supplying installation, the random character of the consumption cannot be neglected. At the same time it should be pointed out that up to the present, no comprehensive theoretical study of this subject has been carried out.

The following should be studied in the course of selecting and designing the operation of service water or natural gas supplying equipment:

- the consumption in a certain period (peak period), consumption per minute, consumption per hour or the total volume of the daily consumption,
- the whole daily pattern of the consumption intensity, the pattern of workday and weekday consumption,
- certain derived characteristics, the tendencies of the nonuniformity and simultaneity factors, etc. The process of service water and natural gas consumption as a phenomenon will be described by means of the above characteristics.

2. Currently Used Methods for the Sizing Calculation of Service Water and Natural Gas Consumption

Methods for the determination of service cold and hot water as well as natural gas consumption provide a deterministic value for the sizing – applicable for the design of the connection pipeline – calculation of the volume flow and do not give any information as regards what risks are attached to these values.

For the *calculation of short time cold water consumption* as regards the concept of unit of water out-flow concerning the technical regulation MSZ 04-132-91 is introduced, but the previous technical directive and the application of technical instructions also include this expression. According to the definition a volume mass of 0,2 l/s water flows out as a result of the pressure difference of 5 kPa (approx. a 5 m high water jet) on the unit of water out-flow ($N = 1$). By applying this the sizing volume flow is:

$$\dot{V} = 0.2\sqrt{\Sigma N + k \cdot \Sigma N}. \tag{1}$$

The relation shows that the water out-flow equivalent value expressed by almost square root operates simultaneously out of the analysed user group characterised by the total $\sum N$ water out-flow equivalent value. In order to determine the volume flow the simultaneously operating water out-flow equivalent values have to be multiplied by the volume flow of the unit of water flow (0.2 l/s). As the sizing consumption derived from the consumption per person, which linearly increases with the number of consumers, in case of a certain number of consumers it will be higher than the simultaneous consumption close to square root. Therefore, the creators of the regulation have applied a correction ($k \cdot \Sigma N$). Thus the basis of the method is the determination of the simultaneous consumption.

In practice the calculation of *daily service water consumption* is often based on the daily consumption of one person ('unit consumption per person') ($\dot{q}_1$). The value of daily consumption per person multiplied by the number of people (n) will give the degree of daily consumption.

The *consumption of service cold and hot water* in respect of a short time – usually an hour – can be determined in relation to the daily average consumption. The consumption in the peak periods can be calculated with the help of the coefficient Z of irregularity dependent on the number of users (number of flats), which is defined as the ratio of peak consumption and average daily consumption:

$$\dot{V} = Z \cdot \frac{\dot{q}_h \cdot n}{24}. \tag{2}$$

In case of pipe gas supply the basis of the deterministic method is the determination of the total possible gas consumption (gas load) decreased with the factor of simultaneity. For this the gas load of the units of the surveyed user groups has to be summarised first, then this sum has to be multiplied by the factor of simultaneity:

$$\dot{V}_m = e \cdot \sum_{i=1}^{n} \dot{V}_{Ni}, \tag{3}$$

where

$\dot{V}_m$ – is the designed gas load,
e – is the factor of simultaneity,
n – is the number of gas units joining to the analysed pipe section,
$\dot{V}_{Ni}$ – is the nominal gas load of the gas consumer i.

The method of the determination of *the factor of simultaneity* according to the practice applied in gas technology is as follows:

– in case of units used in the kitchen:

$$e = \frac{0.9}{\sqrt[4]{n}} \tag{4}$$

– in case units used in the kitchen + bathroom:

the factor of simultaneity up to 20 flats is provided in tabular form,
over 20 flats the relation as regards the kitchen is applicable leaving the bathroom users out of consideration.

The weakness of the above methods is not only that they do not provide any information as regards the degree of risk in respect of the given results, but it is also a concern that they were developed decades ago and no changes have been made to the relations ever since, while due to the changed life style and economy in the past period – especially in the last ten years following the change of the political system – the consumption habits have considerably changed, probably decreased. Therefore, an indefinable degree of over-designing can be observed in the design of the service water and gas systems, which also has economical consequences: bigger size of pipes is necessary and the size of the units and equipment planned to be installed onto the analysed pipe section is also larger than desired.

Due to this it seems necessary to develop and implement modern designing methods that also take the tools of probability theory into consideration.

3. Probability Calculation Models of the Consumption Process

The description of the service water and gas consumption process by means of probability calculation may be carried out in two ways, i.e. the following may be studied:

– the process nature of consumption on the basis of the theory of stochastic processes in the sense of which the following distribution is searched for

$$P\left(Q(\tau)\right); \qquad P\left(\dot{Q}(\tau)\right); \tag{5}$$

where:

where:	$P()$	–	Probability distribution function and
	τ	–	Time, as a mathematical variable
	$Q(\tau)$	–	Consumed water or gas volume, as a function of time
	$\dot{Q}(\tau)$	–	Consumption intensity, as a function of time

– the state character of the consumption based on 'static' models as well as the probability distribution of consumption in a finite period or the distribution of the simultaneity of consumption. Therefore, in this case the following relationships should be disclosed

$$P\left(Q\left(\tau_0, \tau\right)\right) \tag{6}$$

and

$$P\left(\dot{Q}\left(\tau_i\right)\right). \tag{7}$$

The most up-to-date inspection method which provides most information and which is at the same time the most complicated one, is based on the theory of the stochastic processes. The probability distribution of service water and gas consumption as a stochastic process is a function of time and includes it explicitly. The consumption fluctuations by parts of the day can be described only on the basis of this conception. In the sense of probability the model will give information concerning the consumption process at any time.

The static models are not suitable for the above purposes. The main difference between the static models and the stochastic process models is that a distribution below

$$P\left(Q\left(\tau_0, \tau_1\right)\right) \tag{8}$$

discloses nothing on the process behaviour beyond the particular period, moreover the behaviour within this period can only be described in the case of a process which is homogeneous in time with the use of the conclusions to be drawn from the static model ($Q\left(\tau_0; \tau_1\right)$ – Consumed water or gas volume in the time interval $(\tau_0; \tau_1)$).

In course of the description of service water and gas consumption by means of probability calculation, the theory of stochastic processes will be taken as a basis in our paper and the random water and gas consumption will be treated as a process. The employment of the static models is an established field, which generally covers the application of well-known statistic means.

3.1. Analysis of Consumption Process if the Consumers Consume at Random Times, Have Identical Consumption Intensities, and Can Be Characterised by '0–1' Connection State

3.1.1. Probability Distribution of the Gas Volume Consumed at Any Period

The process of random water or gas consumption is a so-called secondary stochastic process. In the process of consumption in any time interval the consumers will enter into consumption independently of each other.

The probability of the events of entering into consumption is described by the *Poisson* process as follows, on the basis of the principle of independence

$$P(k,t) = \frac{\Lambda(t)^k}{k!} e^{-\Lambda(t)}, \tag{9}$$

where

$$\Lambda(t) = \int_0^t \lambda(u)\mathrm{d}u \tag{10}$$

is the expected value of the connections occurring within the $(0,t)$ time interval. Therefore, the density of consumption (density of events) is generally a function of time.

Within the $(0,t)$ time interval the consumers enter into consumption at random times t_k having uniform distribution. All the consumers entering into consumption at t_k times, will consume for random χ_k periods. The distribution of the period of connection is

$$H(x) = P\left(\chi_k \le x\right). \tag{11}$$

As a probability variable, the consumption of a consumer until t time entering into consumption at any time $(0 \le t_i \le t)$, is

$$f\left(t - t_k, \chi_k\right) \tag{12}$$

and its distribution function is

$$P\left(f\left(t - t_k, \chi_k\right) \le x\right) = h(t,x). \tag{13}$$

If in the time interval $(0,t)$ number k of consumers entering occurred, the value of consumption, i.e. the volume of consumption within the $(0,t)$ period at time t will obviously be

$$Q(t) = \sum_k f\left(t - t_k, \chi_k\right). \tag{14}$$

It is our objective to formulate the distribution function

$$P\left(Q(t) \le x\right) = G(t,x). \tag{15}$$

One way of formulating the distribution function (15) is to examine the characteristic functions.

Under the condition that the connection of the consumer into the water or gas supply occurred at time u $(0 \leq u \leq t)$, according to TAKÁCS's theorem, the characteristic function of the distribution function (13) is as follows [1], [2], [3]:

$$\varphi(t-u,\omega) = \int_{-\infty}^{\infty} e^{i\omega f(t-u,x)} \mathrm{d}H(x). \tag{16}$$

As the distribution function of u is $\Lambda(u)/\Lambda(t)$, therefore, utilising the theorem for the hypothetical expected values, the characteristic function of the consumption which started at random time and measured at time t, is

$$\psi_t(\omega) = \frac{1}{\Lambda(t)} \int_0^t \varphi(t-u,\omega)\mathrm{d}\Lambda(u). \tag{17}$$

If in the time interval $(0,t)$ the number of connections into the supply system was 'k', the sum of the consumption values would be the sum of number 'k', of independent probability variables, the characteristic function of which is

$$(\psi_t(\omega))^k$$

and finally the characteristic function of the searched for distribution function (15) is

$$\Phi(t,\omega) = \sum_{n=0}^{\infty} e^{-\Lambda(t)} \frac{(\Lambda(t))^n}{n!} (\psi_t(\omega))^n . \tag{18}$$

From the complex range the characteristic function can be transformed back into the real range by using of the so-called inversion integral. Through the inversion of the characteristic function (18) the probability distribution (15) of the consumption process will be obtained.

The distribution function (15) can be formed directly by means of set and probability algebraic means.

If the examination of the consumption process is started at a time τ_0 when

$$\dot{Q}(\tau_0) = 0$$

and, as has already been mentioned the connections to the water or gas supply occur as a *Poisson* process, it can be written reasonably as follows:

$$G(t,x) = P(Q(t) \leq x) = e^{-\Lambda(t)} + \frac{\Lambda(t)}{1!} e^{-\Lambda(t)} \cdot P\left(\overline{\chi}_1 \leq \frac{x}{1 \cdot \dot{q}}\right) +$$

$$+\frac{\Lambda(t)^2}{2!}e^{-\Lambda(t)}\cdot P\left(\overline{\chi}_2 \le \frac{x}{2\cdot\dot{q}}\right)+\ldots+\frac{\Lambda(t)^k}{k!}e^{-\Lambda(t)}\cdot P\left(\overline{\chi}_{k1} \le \frac{x}{k\cdot\dot{q}}\right)+\ldots\infty, \tag{19}$$

where $\dot{q}$ – the consumption intensity of a consumer.

As a probability variable, the average connection period of the consumers is

$$\overline{\chi}_k = \frac{\chi_{k,1}+\chi_{k,2}+\ldots+\chi_{k,k}}{k}. \tag{20}$$

The density function (10) of the average connection period, as a probability variable, is

$$g_k(y) = \frac{\mu^k\cdot y^{k-1}}{(k-1)!}e^{-\mu y}\cdot\frac{1}{k} \tag{21}$$

and its distribution function is

$$F_k\left(\overline{\chi} \le \frac{x}{k\cdot\dot{q}}\right) = \frac{1}{k}\int_0^{x/k\cdot\dot{q}}\frac{\mu^k\cdot y^{k-1}}{(k-1)!}e^{-\mu y}\cdot \mathrm{d}y. \tag{22}$$

By means of the distribution function, the expected value of the average connection period can be computed:

$$M\left(\overline{\chi}_k\right) = \frac{1}{\mu}. \tag{23}$$

By transforming same for water or gas consumption $\frac{1}{\mu}\cdot\dot{q}$, the expected value of distribution is

$$M\left(Q(t)\right) = \Lambda(t)\cdot\dot{q}\cdot\frac{1}{\mu}. \tag{24}$$

As concerning the actual consumption, there is a physical upper limit of consumption, i.e. this is the case where all of the consumers consume gas continuously within the inspected period, the following relationship is valid:

$$P\left(Q(t) \le Q_{\max}\right) = 1. \tag{25}$$

Therefore, by curtailing formula (16) the real distribution is

$$G^x(x) = \frac{G(x)}{G\left(Q_{\max}\right)} \quad \text{if} \quad x \le Q_{\max} \quad \text{and} G^x(x) = 1 \quad \text{if} \quad x > Q_{\max}. \tag{26}$$

The expected value of consumption on the basis of the curtailed distribution is

$$M^x\left(Q(t)\right) = \Lambda(t)\cdot q\cdot\frac{1}{\mu}\cdot\frac{1}{G\left(Q_{\max}\right)}. \tag{27}$$

3.2. The Determination of the Value of Consumption with the Given Risk from the Measurement Results

As the consumption of service water and pipe gas shows considerable and random fluctuation the question can also be put forward in the following form: what is the probability P of the consumption not exceeding a given Q value? As regards to the sizing consumption the question can also be asked as follows: what is the consumption Q that is not reached of the real consumption of probability P?

This way of asking also means that the consumption exceeds the sizing consumption Q of probability $1-P$, but this excess will happen with the degree of the planned probability that has been accepted.

In this case we can get the sizing consumption with the help of samples taken of the masses showing random fluctuation. The samples can be taken through a large number of measurements.

The mathematical process of the measurement results comprises the following steps:

– we assume the type of probability rule characteristic of the analysed mass,
– we define the characteristic parameters of distribution,
– we check the assumption with regard to the type of distribution.

It is proved that for the consumption samples gained from the masses of water and gas consumers the principles of normal distribution are applied. The normal distribution is determined by two characteristic parameters as independent variables, such as the expected value (m) and dispersion (σ).

The distribution function of the normal distribution in its general form is as follows:

$$P(t) = \frac{1}{2\pi} \int_{-\infty}^{t} e^{-\mu/2} \mathrm{d}u, \tag{28}$$

where $u = (x - m)/\sigma$.

After the statistical process of the measurement data the sizing (peak-) consumption value characterised by the given risk as regards one consumer group is:

$$Q(P) = m_1 \cdot n + t_p \cdot \sigma_1 \cdot \sqrt{n}, \tag{29}$$

where

m_1 – is the expected value on the basis of the results of the series of measurements carried out in n number of flats,

σ_1 – is the determined dispersion based on the measurements,

t_p – are the independent variable values belonging to the given probability p depending on normal distribution.

We have to mention that in case of shorter peak consumption not changing time-wise and consumption probability uniform as regards the different consumers determined for the peak consumption per hour are normative and can easily be calculated [4], [5].

4. Already Existing Methods for the Definition of Sizing Consumption Developed with the Help of Probability Theory Methods

Being aware of the necessity of the application of the modern methods there are such sizing methods already in two areas, which have been developed with the method of probability theory relying on the results of consumption measurements.

4.1. Calculation of the Sizing Heat Demand of Service Hot Water Consumption

In the 80's in the frame of the development of the modern district heating standards a calculating method was elaborated that has been created by the process of the large number of measurement data gained at different spots with the help of probability theory methods [6], [7], [8], [9]. With the relations resulting like this the heat performance data necessary for the sizing of the service hot water appliances larger than 25 flats joining to the district heating can be determined on the given level of risk.

In technical regulation MI 09.85.0006 the suggested relation as regards the calculation of the peak consumption per hour is as follows:

$$Q = A \cdot n + B \cdot \sqrt{n}, \tag{30}$$

where A and B are constant.

4.2. A Method Developed for the Definition of the Sizing Gas Consumption

In order to work out the sizing correlations, we carried out several measurements that involved various consumer groups at different locations in Budapest. In the examined buildings we looked at these kinds of consumers:

- only in the kitchen,
- in the kitchen and the bathroom,
- individual heating and kitchen and bathroom.

Looking at the obtained measurement data and taking into account several kinds of supply security, we established the correlations of standard gas load. These correlations describe gas consumption as the basis of the total heat load of gas appliances on the examined gas pipe section. According to our suggestion, the value of the used volume flow, taken into account at sizing, should be the ten-minute peak consumption period [10], [11].

For the calculation of this we suggest the following equations for the various consumer types:
for kitchens

$$\dot{Q}(10\text{ min}) = 0.009\Sigma\dot{q}_H + 0.16\sqrt{\Sigma\dot{q}_H} \qquad \text{m}^3/\text{h} \tag{31}$$

for kitchen and bathroom

$$\dot{Q}(10\ \text{min}) = 0.007\Sigma\dot{q}_H + 0.09\sqrt{\Sigma\dot{q}_H} \qquad \text{m}^3/\text{h} \tag{32}$$

where: $\sum \dot{q}_H$ – the total heat output of the installed gas appliances.

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