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DETERMINING THE OPTIMAL SCHEDULE OF DISTRICT HEATING

Zoltán SZÁNTHÓ

Assistant lecturer

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Abstract

The heat demand in a given district heating system can be satisfied with discretional primary forward temperature and the appropriate primary mass flow within the limitations of the system. The task of the paper is the satisfaction of the heat demand with minimal operation cost. In the paper the objective function of control will be set up, and a method will be described that is used to define the minimum of the objective function. The minimal cost resulted primary forward temperature is the optimal primary schedule as a function of the other temperature. The secondary forward schedule plays a great part in optimal primary schedule. In the paper some results of the research with computing method will be presented.

Keywords: district heating; optimal primary schedule; minimal operation cost; objective function; computing method.

The competing district heating companies have to apply all possible tools in schedule to decrease the costs of heat supply. After revealing the losses of the Hungarian district heating companies the further decrease of costs usually involves considerable investments. In the present paper we would like to describe a method for costs decreasing, with the help of which considerable savings can be achieved with minimal input when applied to an operating network.

An analysis carried out in 1997–1998 at the Department of Building Services of the Technical University of Budapest showed that although the main district heating companies in Hungary converted from the system of continuous mass flow hot water circulation to that of variable mass flow, the economical advantages of the conversion are not apparent in the financial results of the companies. One of the reasons for this is that the conversion to variable mass flow was carried out with other investments aiming at decreasing the costs, therefore the financial profit of the conversion to variable mass flow cannot be clearly separated. Another possible reason is that the advantages of variable mass flow circulation were not managed to be exploited in all cases.

The conversion to variable mass flow was carried out with keeping the unchanged schedule of the primary forward temperature in the known cases. It is not likely that under the changed conditions the previous schedule would result in minimal operation costs in the system. Our target is to describe such a method, with which the operational characteristics, especially the $t_1'(t_k)$ primary temperature schedule can be determined, which results in the minimal operation costs of the district heating system regardless to their solution.

1. The Schedule of the Secondary System

For the determination of the primary schedule we have to know the $t_2'(q)$ schedule of the secondary system. This is practically set up by experimenting with the control system within its possibilities. The schedule necessary for the operation of the secondary system can be determined with a theoretical method.

The calibrating position of the secondary system is known. In case of partial loads of $\dot{Q}(t_{out}) \neq \dot{Q}_0$ considering the function of heat output of the heaters the following equation can be set up:

$$q = \frac{\dot{Q}(t_{out})}{\dot{Q}_0} = \frac{\dot{m}c(t_2' - t_2'')}{\dot{m}_0 c(t_{2,0}' - t_{2,0}'')}$$
$$= \left(\frac{\Delta t_{\log}}{\Delta t_{\log,0}}\right)^{1+M} = \left(\frac{t_2' - t_2''}{\ln \dfrac{t_2' - t_{in}}{t_2'' - t_{in}}} \bigg/ \frac{t_{2,0}' - t_{2,0}''}{\ln \dfrac{t_{2,0}' - t_{in,0}}{t_{2,0}'' - t_{in,0}}} \right)^{1+M}, \tag{1}$$

where q is the specified heat demand as regards to the $\dot{Q}_0$ calibrating heat demand. In buildings of district heating in Hungary generally and nominally a forward temperature of $t_{2,0}' = 90\,°\mathrm{C}$ and a heat gradation of $\Delta t_0 = 20\,°\mathrm{C}$ are applied; the interior nominal temperature of the rooms is 20 °C. For our analysis we have assumed that the heating equipments are accurately calibrated in accordance with the above data. The question is what forward temperatures are needed as regards to the partial loads. We assume that the heat demand of the rooms to be heated will decrease relatively to the increase of the outside temperature, thus $q \sim (t_{in} - t_{out})$. If we assume a continuous secondary mass flow then due to $\dot{m} = \dot{m}_0$ and $c \approx$ constant:

$$(t_2' - t_2'') = q(t_0' - t_0'') = q\Delta t_0. \tag{2}$$

Substituting this:

$$q = \left(\frac{\Delta t_{\log}}{\Delta t_{\log,0}}\right)^{1+M} = \left(\frac{q\Delta t_0}{\ln \dfrac{t_2' - t_{in}}{t_2' - q\Delta t_0 - t_{in}}} \bigg/ \frac{\Delta t_0}{\ln \dfrac{t_{2,0}' - t_{in,0}}{t_{2,0}'' - t_{in,0}}} \right)^{1+M}$$
$$= \left(\frac{q}{\ln \dfrac{t_2' - t_{in}}{t_2' - q\Delta t_0 - t_{in}}} \bigg/ \frac{1}{\ln \dfrac{t_{2,0}' - t_{in,0}}{t_{2,0}'' - t_{in,0}}} \right)^{1+M}; \tag{3}$$

from this

$$q = \left(q \ln \frac{t'_{2,0} - t_{\mathrm{in},0}}{t''_{2,0} - t_{\mathrm{in},0}} \Big/ \ln \frac{t'_2 - t_{\mathrm{in}}}{t'_2 - q\Delta t_0 - t_{\mathrm{in}}} \right)^{1+M}. \tag{4}$$

This equation explicitly containing q can be solved for q as regards to a defined t'_2 parameter, thus it can be determined what heat demand the given t'_2 forward temperature can satisfy. Our aim is to define the 'secondary forward' temperature, that is $t'_2 = f(q)$. For the purpose of this rearranging the above expression:

$$\left(q \ln \frac{t'_{2,0} - t_{\mathrm{in},0}}{t''_{2,0} - t_{\mathrm{in},0}} \Big/ \ln \frac{t'_2 - t_{\mathrm{in}}}{t''_2 - q\Delta t_0 - t_{\mathrm{in}}} \right)^{1+M} - q = f(t'_2); \tag{5}$$

and looking for the solution of $f(t'_2)$ with determining $q = q(t_{\mathrm{out}})$. By determining t'_2 for the discrete q values, we can define an arbitrary number of points of the $f(t'_2) = f(t_{\mathrm{out}})$ diagram (*Fig. 1*).

This ideal secondary schedule cannot be executed in case of more simple adjusters, but only with DDC ones.

2. The Optimal Schedule of the Primary Systems

Let us assume that the parameters of the 'secondary side' of district heating system and the heat exchanger are given. From these the data of the primary side are not decidedly determined. The demand of the secondary side can be satisfied within the limitations of the system with a higher primary forward temperature, or in case of higher mass flow with a lower primary forward temperature. One has to choose the solution that provides minimal operation costs amongst the endless number of options. In case of higher forward temperature the heat loss of the system increases but due to the smaller mass flow the pumping work decreases. Its inversion: in case of decreasing forward temperature the heat loss also decreases but for the circulation of the necessary higher primary mass flow an increased amount of pumping work is needed. Depending on the outside temperature the necessary heat flow and thus the primary temperature and the mass flow also change.

Our task is the satisfaction of all the given heat demands in such a way that the cost of the operation is minimal. In case of a given network the minimal operational cost can be achieved with $t'_1(t_{\mathrm{out}})$, $t''_1(t_{\mathrm{out}})$, $\dot{m}(t_{\mathrm{out}})$ and $\Delta p(t_{\mathrm{out}})$, which are the primary forward and backward temperature, and the optimal schedule of the pressure difference to be created by the primary mass flow and the pump. However, it is enough to determine $t'_1(t_{\mathrm{out}})$, as the other characteristics of the system (schedule and mass flow of the secondary side; the given heat exchanger; the length of the network; the diameter and insulation of the pipeline, etc.) will give the rest. The mass flow is adjusted by the control valves of the heat centre, which demand exactly the same amount of mass flow from the primary system as it is necessary for the adjustment of the secondary forward temperature coming from the primary mass flow temperature to the heat exchanger.

Therefore the task is:

$$\dot{K}_\Sigma = k_{\text{heat}}(\dot{Q}_{\text{dem}} + \dot{Q}_{\text{loss}} - \dot{V}^*\Delta p) = k_{\text{el}}\dot{V}\Delta p, \tag{6}$$

to determine the minimum value of the diagram expressing the costs per one time unit of circulation. In the relation we have considered that the dissipation of the pumping work heats the agent. The task is to convert the expression to a form free of dimension

$$K'_{DR}(q) = \left(\frac{1}{\Phi_1(X_W)} - A_Q\right)\frac{1}{X_W} + A_P(q)X_w^3 \to \textbf{min}! \tag{7}$$

K'_{DR} is the reduced cost free of dimension and in

$$K'_{DR} = \frac{\dot{W}_{1,0}\dot{K}_\Sigma}{l(k_{l'} - k_{l''})k_{\text{heat}}\dot{Q}_0 q} - \frac{\dot{W}_{1,0}(t'_2(q) - t_{\text{out}})}{\dot{Q}_0^* q} - \frac{\dot{W}_{1,0}}{l(k_{l_e} + k_{l_v})}. \tag{8}$$

In the expression (7)

$$\Phi(X_W) = \frac{1 - e^{-\left(1 - \frac{\dot{W}_{1,0}}{\dot{W}_2}X_W\right)\frac{kA}{X_W\dot{W}_{1,0}}}}{1 - \frac{\dot{W}_{1,0}}{\dot{W}_2}X_W e^{-\left(1 - \frac{\dot{W}_{1,0}}{\dot{W}_2}X_W\right)\frac{kA}{X_W\dot{W}_{1,0}}}} \tag{8}$$

is the Bosnjakovic factor of the heat exchanger (here: countercurrent). Considering the form of the relations, the minimum of expression (7) can only be determined by numerical methods.

The minimum of expression (7) has to be determined in respect of X_w with the given parameter of $q = q(t_{\text{out}})$. By determining X_w for the discrete values of q we can determine the schedule of the mass flow and from this an arbitrary number of points of the $t'_1 = f(t_{\text{out}})$ diagram.

3. Changes of the Temperature of the Primary Agent on the Pipeline

When setting up the relation the temperature change of the primary agent on the pipeline was neglected. We have to see the legitimacy of this negligence.

On the basis of the energy balance of the pipeline section of a length dx [3]:

$$\dot{W}\mathrm{d}t = \frac{\rho}{2}w^2\lambda\frac{\mathrm{d}x}{\mathrm{d}}\dot{V} - \dot{q}\,\mathrm{d}x. \tag{9}$$

The equation expresses that the temperature of the mass flow in the pipeline increases as a result of the dissipation of the friction work and it decreases as a result of the heat loss of the pipe. The excessive temperature in the x profile of the pipeline is the one, resulting as a solution of the equation. Considering the boundary conditions

$$\nu = \frac{\lambda w^2}{2d}\frac{\dot{W}}{kU} + \left(\nu_0 - \frac{\lambda w^2}{2d}\frac{\dot{W}}{kU}\right) e^{-\frac{kU}{\dot{W}}x}; \tag{10}$$

where ν_0 will have to be replaced by the temperatures of ν_1' entering in the forward section of the heat exchanger and of ν_1'' exiting in the backward section. The heat centre can be found in the place of $x = 0$ taking on the negative value of $-l \leq x \leq 0$ in the forward section and a positive one of $0 \leq x \leq l$ in the backward section. The heat loss of the pipeline can be accurately determined from the relation (10).

We have also looked into the relation of the heat losses calculated accurately in the model to be described below and also in case of the calculation disregarding the cooling of the pipe. The results are shown in *Fig. 2*. We have analysed two types of insulation. The specific factor of heat transmission of a pipeline that complies with the current technological expectations having an appropriate insulation is 0.61 W/mK (the heat output of a one meter long ISOPLUS district heating pipe of a diameter of DN500 is 1K as a result of the temperature difference). The specific heat transmission factor of a pipeline the insulation of which is in bad condition, improperly designed and partly damp is assumed to be 6 W/mK. We relate the exact heat loss calculated on the basis of relation (10) in *Fig. 2* to the heat loss determined without considering the cooling of the pipeline. The boundary conditions are the entering and exiting temperatures of the heat exchanger: taking cooling into consideration the heater has to provide higher forward temperature and the temperature of the backward section is lower than it would be if cooling were considered. The total heat loss (the joint heat loss of the forward and backward pipelines) is higher than it was determined according to the expression (7). This difference is insignificant in the possible mass flow ranges in case of good insulation; in case of bad insulation, however, it cannot be neglected any more.

4. Results of the Computer Analyses

The minimum of expression (7) in case of arbitrary parameters can be determined for an arbitrary q. The solutions have been processed in nomograms. On the basis of these nomograms the optimal schedule of a section of district heating with the known parameters (diameter, length, insulation, heat exchanger, secondary schedule) providing a heat exchanger can be defined by point to point.

We have seen that the cooling of the heat carrier pipeline cannot be neglected in all situations. Therefore, the results revealed below are not determined partly by the minimum value according to expression (7) but by the elements of expression (6) considering the cooling of the primary agent by the pipeline.

The analyses have been carried out in a fictitious model, the parameters of which are determined on the basis of a real district-heating network. The heat is generated by a heating plant in our example. The length of the primary pipeline is

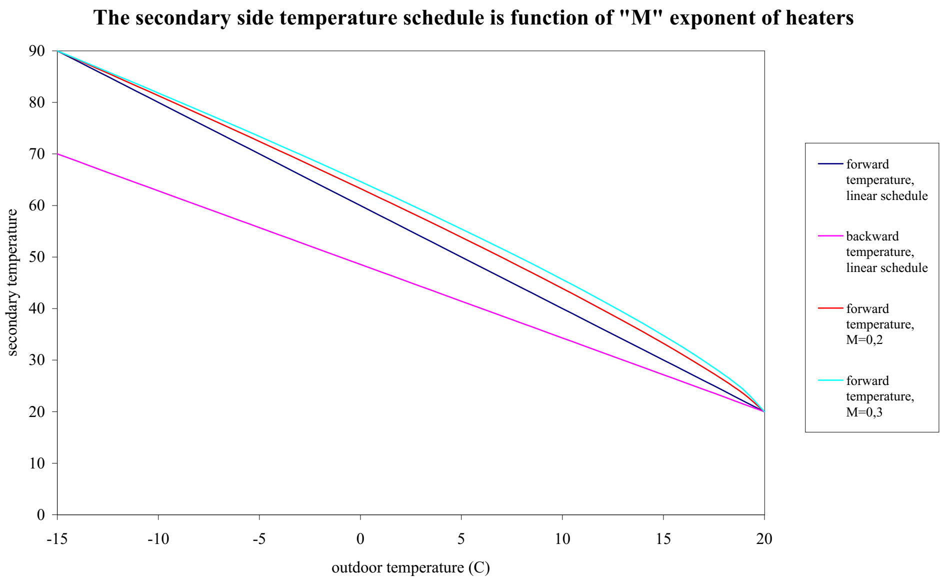

Fig. 1.

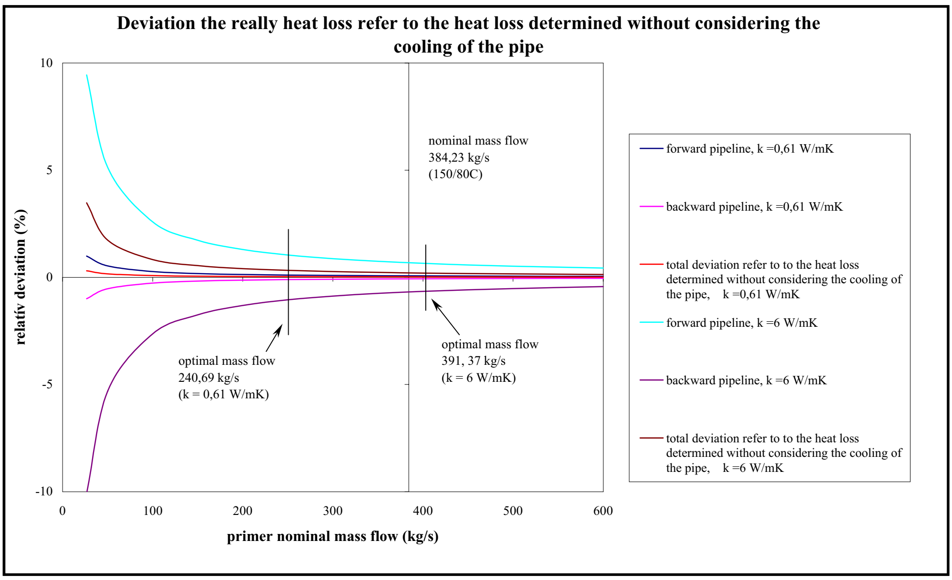

Fig. 2.

3657 m, the pipe diameter is DN500, the performance of the system consisting of

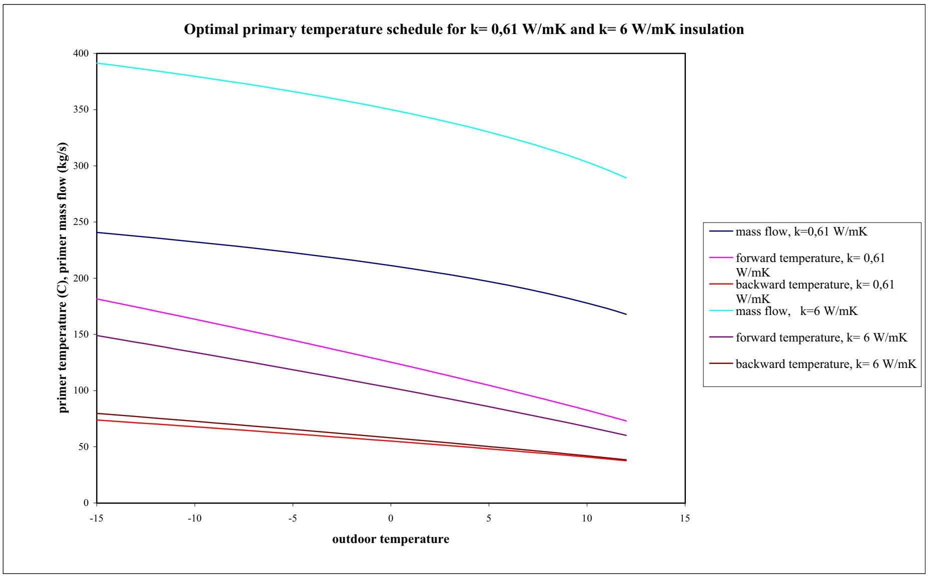
Optimal primary temperature schedule for k= 0,61 W/mK and k= 6 W/mK insulation
primer temperature (C), primer mass flow (kg/s)
outdoor temperature
mass flow, k=0,61 W/mK
forward temperature, k= 0,61 W/mK
backward temperature, k= 0,61 W/mK
mass flow, k=6 W/mK
forward temperature, k= 6 W/mK
backward temperature, k= 6 W/mK

Fig. 3.

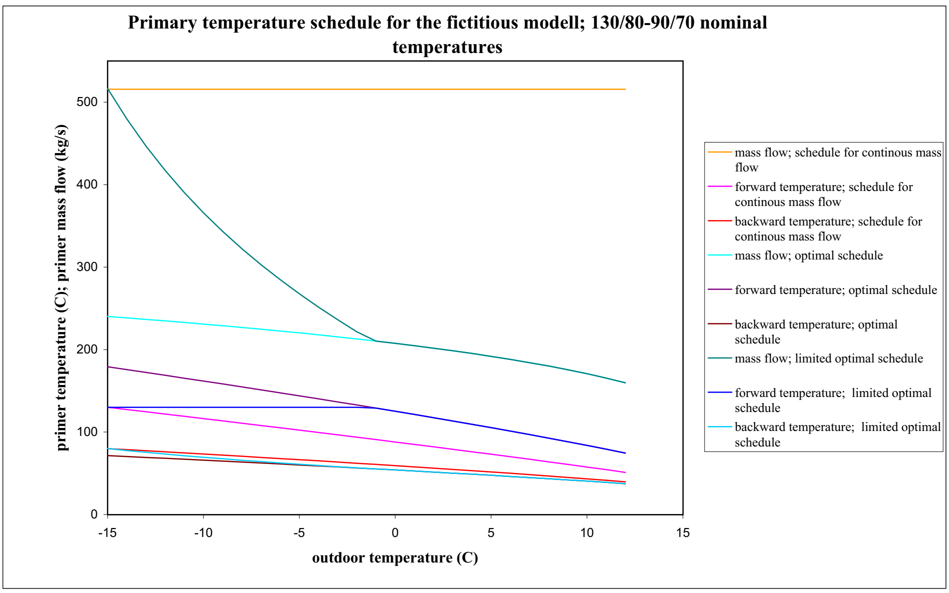
Primary temperature schedule for the fictitious modell; 130/80-90/70 nominal temperatures
primer temperature (C); primer mass flow (kg/s)
outdoor temperature (C)
mass flow; schedule for continous mass flow
forward temperature; schedule for continous mass flow
backward temperature; schedule for continous mass flow
mass flow; optimal schedule
forward temperature; optimal schedule
backward temperature; optimal schedule
mass flow; limited optimal schedule
forward temperature; limited optimal schedule
backward temperature; limited optimal schedule

Fig. 4.

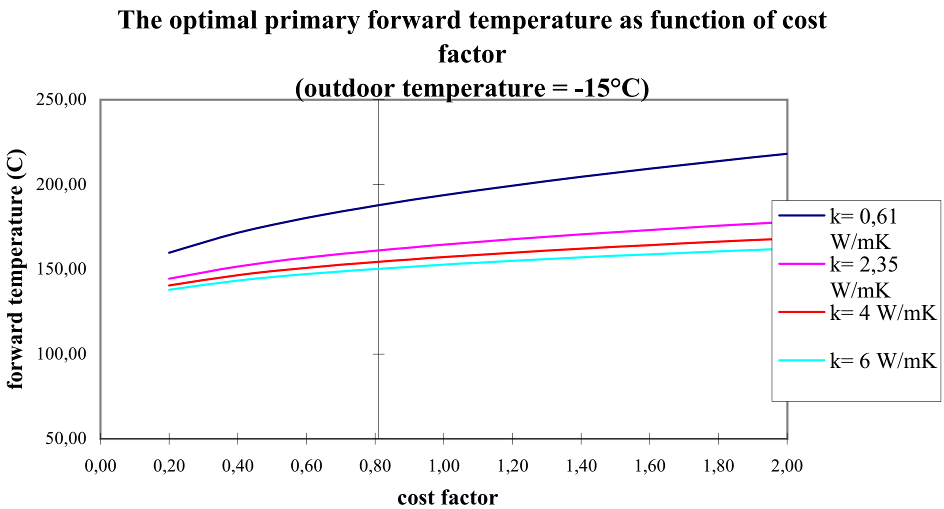

Fig. 5.

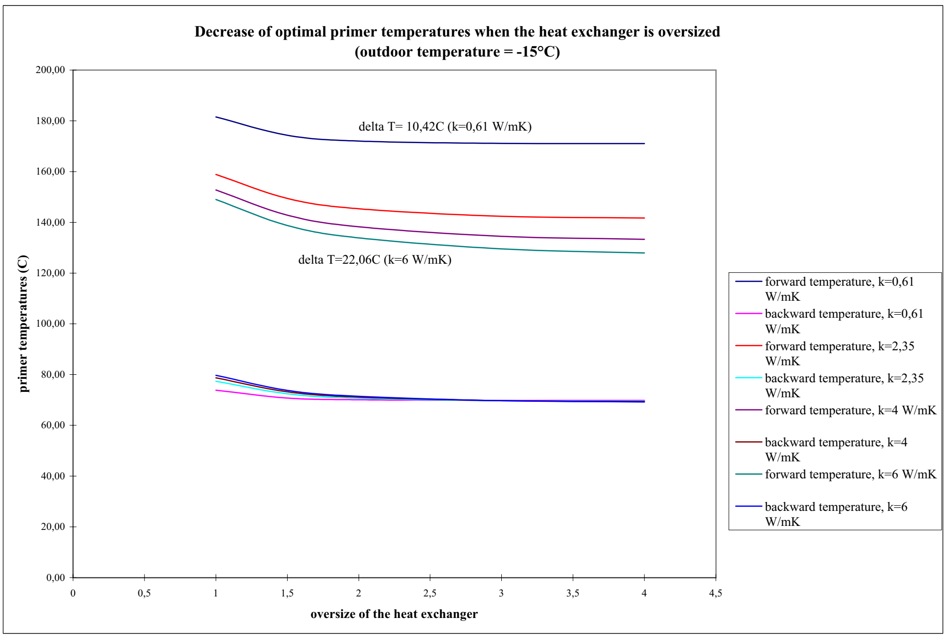

Fig. 6.

one heat exchanger and a pair of primary pipelines is 108.9 MW. The nominal heat

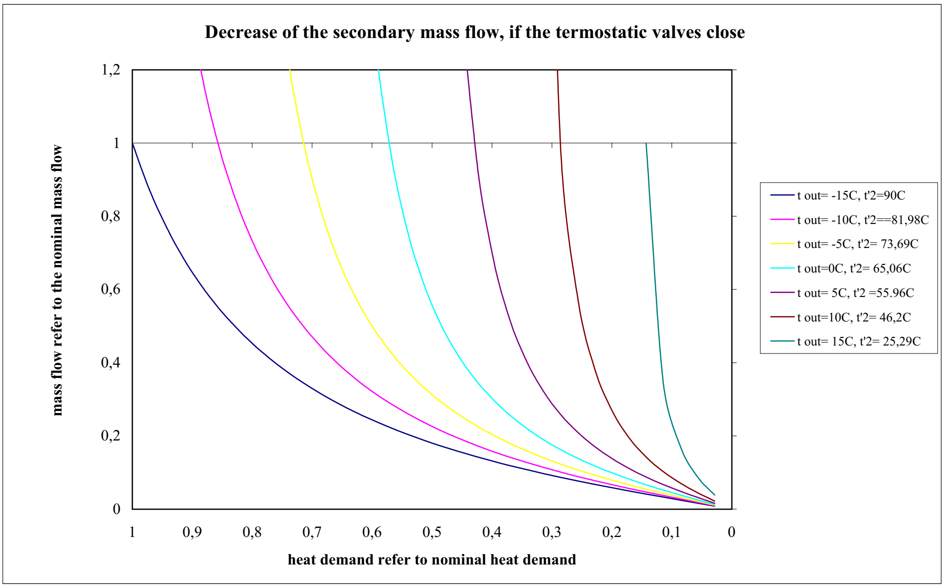

Fig. 7.

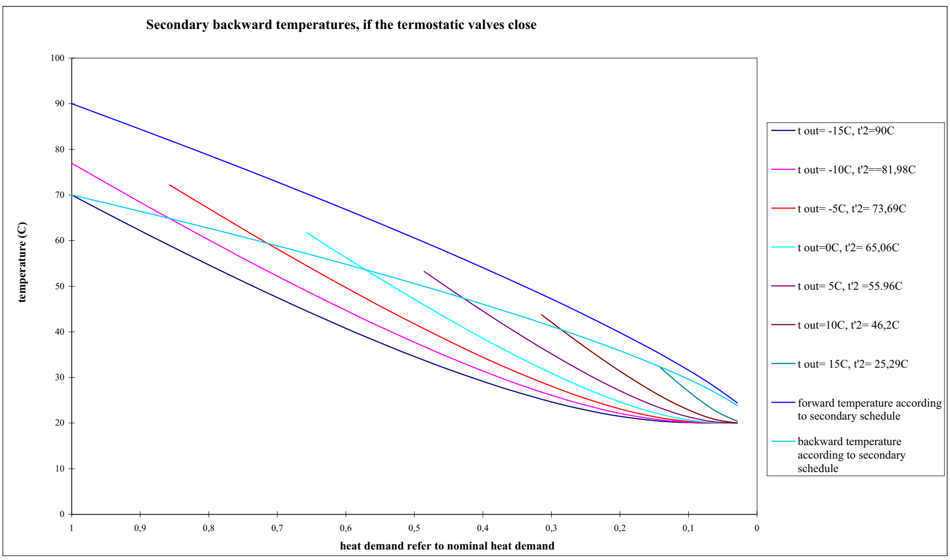

Fig. 8.

gradient is 150/80 °C. The costs of the model system have been calculated on the

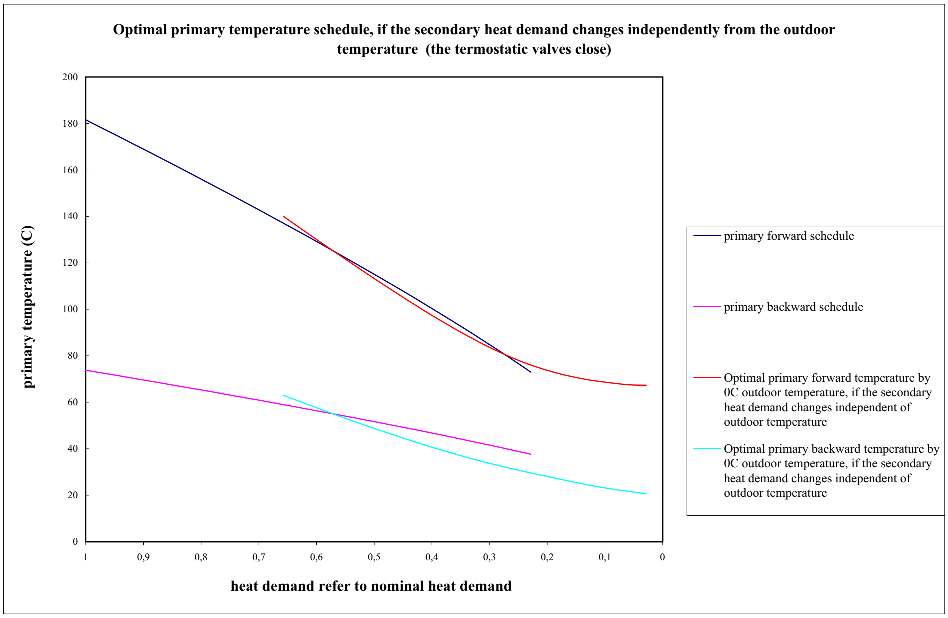

Fig. 9.

basis of the heat and electricity prices valid in 1999, which can be regarded to the Hungarian prices.

In case of a better insulation of the pipeline the optimal forward temperature tends to be higher (*Fig. 3*). The optimal forward temperature of a system of good insulation is higher, its mass flow is considerably lower (in case of an outside temperature of -15 °C it is 181.56 °C and 240.69 kg/s) in case of the parameters providing a basis for the relation and describing the situation before optimising (150 °C, 384.23 kg/s). In case of keeping the optimal schedule the calculated cost of the circulation is 27.36% less than in case of a schedule of a nominal temperature of 150/80 °C (with the temperature bridge of a year of average weather). Savings that can be achieved with a system of bad insulation are negligible, as the optimal schedule differs from the compared case schedule of continuous mass flow only to a small degree (nominal forward temperature of 149.02 °C and a mass flow of 391.37 kg/s). In case of a good insulation the expected yearly cost of circulation is certainly remarkably lower: in case of an insulation having a quality of $k = 0.61$ W/mK it is only 17% of the figure in respect of an insulation having a quality of 6 W/mK.

The optimal temperature (181.56 °C) as regards the pipelines of good insulation is considerably higher than the permitted value of the used insulation materials. We have also looked into the savings options in case of systems of a schedule nominally 130/80 °C (for this a different heat exchanger and more mass flow are

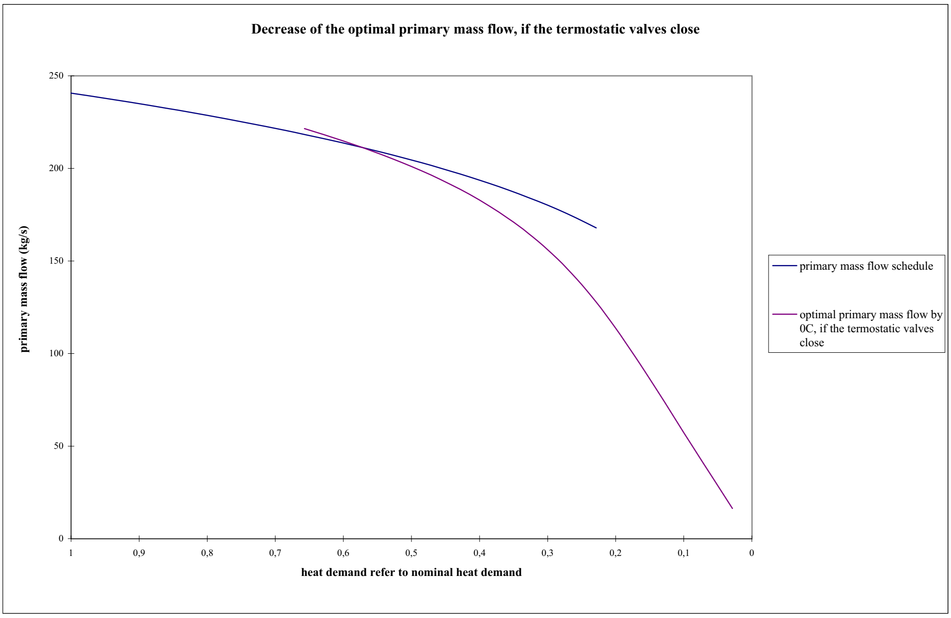

Fig. 10.

necessary, than in case of 150/80 °C; the other parameters are not changed). In case of an outside temperature of −15 °C the optimal primary schedule necessitates a forward temperature of 179.13 °C, which is still significantly higher than the limit of 130 °C. If this limitation was disregarded in the schedule of the temperature a saving of 56.45% could be achieved compared to the circulation costs of continuous mass flow operation of a nominal design of 130/80 °C. It is even more surprising that the limitation of the primary forward section to 130 °C only slightly decreases the possibilities for savings, to 51.14%. The reason for this is that those outside temperatures that are considerably different from the optimal temperature of the schedule are only apparent for a very short time, only for a few hours in a year of average weather. Over the characteristic temperature of −5 °C the difference is negligible, over −2 °C the optimal forward temperature is already below 130 °C (*Fig. 4*).

According to the observations of the analysis the optimal primary mass flow does not depend on the secondary schedule. A higher secondary temperature necessitates higher primary temperature, and this is due to the mass flow independent from the secondary temperature results in higher circulation costs.

Systems of good insulation are less sensitive to the changes of the price proportion of electrical and heat power than those of bad insulation. The changes of the price proportions can justify the re-determination of the optimal schedule. The optimal nominal forward temperature of a system of a gradient of 150/80 °C in case of continuous mass flow is shown in *Fig.* 5 as regards the cost factor. The value of the cost factor is 0.81 on the basis of the prices applicable at the time of the analyses. Compared to the Western European price proportions the value of heat in Hungary is underestimated. The cost factor decreases when the price of heat compared to electric power is increased; the heat loss is overestimated and the optimal primary temperatures decrease.

If the insulation of the system is good the occasional oversized heat exchangers provide little possibility for the decrease of the circulation costs. If the insulation of the system is bad the oversized heat exchangers make the considerable decrease of the primary forward temperature and the increase of the mass flow possible, and the costs of circulation can be remarkably decreased (*Fig.* 6).

In the previous analyses we assumed that the heat demand of the system only depends on the outside temperature. Due to the internal heat gain of the buildings the real heat demand can be smaller than the heat loss determined on the basis of the outside temperature. In these cases the thermostatic valves of the heaters intervene. The weather-dependent adjusting systems adjust secondary forward temperature according to the value determined by the outside temperature, the performance of the heater can be set to the real heat demand with the decrease of mass flow. The cooling of the decreased mass flow is larger, so at the closing of the thermostatic valves the secondary returning temperature is lower. In this case intervention in the primary schedule would also be necessary in schedule to achieve a minimal circulation cost.

Figs. 7 and 8 show how the secondary mass flow and the returning temperature change at the closing of the thermostatic valves. The q heat demand in respect of the designed heat demand is shown on the horizontal axis, but here this does not only depend on the outside temperature but also on the inside heat load. In case of a given outside temperature the heat demand is smaller than the designed one, and therefore the forward and the backward temperatures are also lower. If the real heat demand further decreases due to the heat generation inside the forward temperature – which depends on the outside temperature – it does not change, but because of the closure of the thermostatic valves the returning temperature decreases.

Figs. 9 and 10 show how the primary system would react to the changes of the heat demand of the secondary system independently from the outside temperature. In case of an outside temperature of 0 °C the adjustment of the primary side would have to decrease the forward temperature according to the diagram in *Fig.* 9 and as a result of this the mass flow would change according to *Fig.* 10. The forward temperature would certainly not do under the temperature adjusted by the secondary adjustment and determined by the secondary schedule. In case of optimal interference the adjustment of the heat source has to be informed about the decrease of the secondary heat demand. This could be possible by measuring the primary backward temperature – this method, which is a used adjustment method in air conditioning systems – is usually not applied in district heating currently. The decrease of the secondary heat demand can be directly measured in case of the application of DDC adjustment.

5. Determining the Optimal Schedule of Systems of Branched District Heating Pipelines

Let us take two adjacent heat centres of a district heating network, A and B, which are bisected in point C. The determination of the optimal schedule of a heat centre and its joining pipelines have been described above. What primary forward temperature is necessary in point C in schedule to have an optimal operation of the subsystem containing the heat exchangers and pipelines of A and B?

By knowing the heat demand dependent on the outside temperature, the heat exchanger and the entering and exiting temperatures of the secondary side the primary mass flow demand can be determined in respect of a given primary entering temperature; or the opposite case is possible, the necessary temperature of a given mass flow can be defined. On the basis of this the $t'_{1,A} = t'_{1,A}(\dot{W}_A)$, $t'_{1,B} = t'_{1,B}(\dot{W}_B)$, relation between the entering temperature and the flow of the water pipeline can be determined in respect of heat exchangers A and B. The geometry of the junction section and the insulation data are known; with the help of relation (10) we can determine that depending on $\dot{W}_A$ and $\dot{W}_B$ what t'_c temperature is needed for the forming of $t'_{1,A}$ and $t'_{1,B}$. In case of a primary temperature of t'_c heat exchangers A and B will necessitate a water pipeline flow of $\dot{W}_A$ and $\dot{W}_B$. Thus the $t'_{1,c} = t'_{1,c}(\dot{W}_A + \dot{W}_B)$ relation concerning the temperature and water pipeline flow of point C can be set up. The cost function according to relation (6) for the junction pipelines A and B has to be drawn up in such a way that $\dot{W}_A$ belongs to the $\dot{W}_B$ water pipeline flow according to $t'_{1,c} = t'_{1,c}(\dot{W}_A + \dot{W}_B)$. The joint minimum value of the two cost functions resulting from t'_{1c} is the optimal temperature as regards the given outside temperature.

The $t'_{\mathrm{red}} = t'_{\mathrm{red}}\left(\sum_{r=1}^{n} \dot{W}_r\right)$ reduced forward temperature can be created in a recursive way with the previous method from the heat centres to the heater. In case of $t'_{\mathrm{red}} = t'_{\mathrm{red}}\left(\sum_{r=1}^{n} \dot{W}_r\right)$ forward temperature the mass flows of the heat centres are $\dot{W}_1, \dot{W}_2 \ldots \dot{W}_r \ldots \dot{W}_n$ and the heat and pressure loss of the given sections can be determined. Setting up the cost function according to (6) for all sections of the network the $t'_{\mathrm{red}} = t'_1 = t'_1(t_{\mathrm{out}})$ giving the minimum of the sum of these will be the optimal forward temperature as regards the given outside temperature.

District heating systems usually provide supply the users not only with heating but domestic hot water. This is difficulty in the determination of the optimal primary schedule because the hot water heat exchangers have to be regarded with a schedule that is independent from the outside temperature.

By neglecting the cooling on the primary network the described method can be considerably simplified. In this case the heat loss of the primary pipeline sections does not depend on the mass flow, but only on the temperature of the agent. The temperature of the forward pipelines is uniform on the whole network. By knowing the primary forward temperature the necessary mass flow and the returning temperature can be calculated in respect of all heat exchangers. On the basis of the returning temperatures the mixing temperature of the joining of the returning pipelines, from this the temperature and heat loss of the pipeline sections can be determined. In case of a fixed $q = q(t_{out})$, depending on t_1' we can determine the heat loss and the $\dot{K}(t_1')$ function expressing the cost of the performance of the circulation, the minimum of which can be defined with the help of numerical methods. By determining the minimum values for the discrete $q = q(t_{out})$ values an arbitrary number of points of the $t_1' = t_1'(t_{out})$ optimal schedule can be defined.

6. Limitations of the Application of the Model

The cooling of the agent by the primary pipeline in the expression (7) has been neglected. This negligence is only possible in case of systems of good insulation and appropriately loaded. If this condition is not provided the cost function according to function (6) has to be analysed for the determination of the optimal primary schedule.

In case of comprehensive systems it is possible that the heat centres are installed far away from the heat generator on the network. The result of this is that the primary agent reaches the closest heat centres in a few minutes, but gets to the farthest ones only in a few hours. The dead time due to the distance is different in respect of each heat centre. A separate analysis has to be applied in schedule to decide that in case of the given topology what economical schedule has to be applied to follow the changes of the outside temperature.

In our analysis we have determined only the optimal proportion of the heat lost of the district heating pipeline and the costs of the pumping work; we have not dealt with the impact of these changes on heat generation, e.g.: the changes of its efficiency. In case of a heat generation of a heat plant this does not influence the results considerably; in case of a joint heat generation, however, also considering its economy the optimal temperatures will shift to the lower values.

The method described for the branched networks is not or only partly applicable for the determination of the optimal schedule of loop or multiple feeding networks.

List of Symbols not Described in the Text

$\dot{K}_\Sigma$	total cost of circulation per one unit of time;
K'_{DR}	reduced, dimension-free cost;
t	temperature;
$q = q(t_{\text{out}})$	specific heat demand in reference to the nominal $\dot{Q}_0$ heat demand (dependent on the outside temperature);
$\Delta t_{\log}$	logarithmic median temperature difference;
$\dot{W}$	$= \dot{m}c$, the heat capacity flow;
kA	product of the heat transmitting factor and the heat output surface of the heat exchanger;
k_l	$= \dfrac{\Pi}{\dfrac{1}{\alpha_b d_b} + \sum_{n=1}^{n-1} \dfrac{1}{2\lambda} \ln \dfrac{d_{k+1}}{d_k} + \dfrac{1}{\alpha_k d_k}}$; the heat transmission factor on the pipeline in the k_l index, signs $'$ and $''$ refer to the forward and backward pipelines;
α	heat transmission factor;
$A_Q = \dfrac{k_{l'}}{k_{l'} + k_{l''}}$	number expressing the relation of the heat resistance of the forward and backward pipes;
l	length of the primary pipeline route;
X_w	specific heat capacity flow value in reference to the nominal primary heat capacity flow;
$A_P(q)$	$= \dfrac{k_{\text{el}} - k_{\text{heat}}}{k_{\text{heat}}} \dfrac{\dot{W}_{1,0}}{l^*(k_{l'} + k_{l''})} \dfrac{R_H}{(\rho^* c)^3} \dfrac{\dot{W}^3_{1,0}}{\dot{Q}_0} \dfrac{1}{q}$;
$\dfrac{k_{\text{el}} - k_{\text{heat}}}{k_{\text{heat}}}$	cost factor;
M	exponent of heaters;
k	specific cost of energy;
λ	factor of pipe friction;
R_H	hydraulic resistance of the pipe;
w	flow speed in the pipe;
ρ	density of the heating agent;
$\dot{q}$	specific heat loss of the pipeline;
kU	heat transmission factor in reference to the outline of the pipe multiple by the outline of the pipe;
$\mathrm{d}\dot{q}$	heat loss of $\mathrm{d}x$ long pipeline section;
$\dot{V}$	volume flow of the heating agent.

Indexes

1	primary;
2	secondary;
$'$	forward;
$''$	backward;
0	referring to the nominal position;
dem	demand;
out	outside;
in	inside;
H	hydraulic;
heat	heat;
l	along the pipeline;
loss	loss;
el	electric.

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