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BASIC FLOW DIFFERENTIAL EQUATIONS APPLIED IN BUILDING SERVICE ENGINEERING

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Abstract

In this article basic flow equations will be presented to describe the flow behaviour of various mediums, like gas, ideal gas, wet steam, ... etc. The equations constitute a system of simultaneous differential equations. The solution of this sytem is here not discussed. The Appendix gives a clearly arranged synopsis of the differential equations of flows under different conditions.

Keywords: district heating, gas supply,energy equation, flow equation.

1. Introduction

An essential knowledge of flow equations is required in process planning, like Building Service Engineering. There are several areas in practise where giving a detailed model description is needed. The main areas where flowing mediums are investigated are the followings

- gas supply
- water supply
- district heating
- central heating

Tests are often made of current taking place in

- diffusers and reducers
- valves and dampers
- nozzles

It is important for the designer and operating engineer to be able to determine as exactly as possible the changes in the thermo- and hydrodynamic parameters of the current medium. In our paper we will show how to write down the current simultaneous differential equation system for various current problems and how to solve those problems with the highest accuracy while using the simplest computer tools. In the studied apparatuses the currents can take place with mechanical losses, either negligible mechanical and heat loss, or simply negligible heat loss. A current can take place in pipes or ducts with constant or variable cross sections.

The most common mediums are

- hot water
- wet steam
- dry saturated steam or superheated steam
- natural gas

These mediums' behaviour approaches the properties of the ideal gas or real gas.

In our paper we test the above descriptions of currents. We will present those differential equations with wich we can determine, under stationary conditions, changes in the pressure, specific volume, and temperature of the current medium.

2. Basic Equations of Currents

The one-dimensional motion equation for stationary flows with friction:

$$w\frac{dw}{dz} = -\frac{1}{\varrho}\frac{dp}{dz} - \frac{\lambda}{2d}w^2.$$

The continuity equation:

$$A\varrho w = \text{const.}$$

The energy equation

$$\frac{d}{dz}\left(h + \frac{w^2}{2}\right) = \frac{\dot{q}}{\dot{m}}.$$

The equation of state for ideal and real gas:

$$pv = ZRT.$$

The condition equation for wet and dry saturated steam and the used approximations are:

$$\begin{array}{rclcrcl}
p & = & \delta_0 + \delta_1 T + \delta_2 T^2 + \dots & \rightsquigarrow & p & \approx & \delta_0 + \delta_1 T \\
h(v = \text{const.}, T) & = & b_0 + b_1 T + b_2 T^2 + \dots & \rightsquigarrow & h(v = \text{const.}, T) & \approx & b_0 + b_1 T \\
h(v, T = \text{const.}) & = & g_0 + g_1 v + g_2 v^2 + \dots & \rightsquigarrow & h(v, T = \text{const.}) & \approx & g_0 + g_1 T \\
v'' & = & \gamma_0 + \gamma_1 T + \gamma_2 T^2 + \dots & \rightsquigarrow & v'' & \approx & \gamma_0 + \gamma_1 T
\end{array}$$

3. Special Cases of Flows

The special cases of flows are deduced from the differential equations of politropic case with the appropriate substitutions:

- for isothermal current $\frac{dT}{dz} = 0$
- for isochor current $\frac{dv}{dz} = 0$
- for isobar currents $\frac{dp}{dz} = 0$
- for adiabatic currents $\dot{q} = 0$
- for isentropic currents $\frac{dS}{dz} = 0$
- constant cross section $\frac{dA}{dz} = 0$
- without friction $\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = 0$

4. The Solution of the System of Equation

For the solution of the system of equations we must specify the initial conditions. If $x = 0$, $p = p_0$,$T = T_0$,$h = h_0$ and $v = v_0$

$$\left(\frac{\partial h}{\partial T}\right)_{v_0} \approx b_1, \ \left(\frac{\partial h}{\partial v}\right)_{T_0} \approx g_1, \ \left(\frac{\partial p}{\partial T}\right)_{p_0} \approx \delta_1.$$

We need to know the functions

$$\begin{aligned} A &= A(z) \\ \dot{m} &= \text{const} \\ \dot{q} &= \dot{q}(z) \end{aligned}$$

The differential equations presented in the appendix can be solved by direct integration (after separation), by a method of successive approximation or with step-by-step finite difference method.

5. Summary

The tables summarize the main equations for special cases of flows. Engineers, scientists are often in need of having a table, wich contains of the describing equations of flows and their solutions. In civil building service engineering it is useful to have the main equations collected.

SYMBOLS

T	temperature
w	velocity
v	specific volume
h	enthalpy
p	pressure
z	space coordinate
$\dot{m}$	mass flow
A	cross section
d	diameter
R	specific gas constant
ϱ	density
λ	coefficient of friction
$\dot{q}$	specific heat loss
U	perimeter
Z	compressibility factor
S	entropy
κ	adiabatic exponent

Table 1. Ideal Gas Flow Equations I.

	Cross-Section	Heat Loss and Friction	No Heat Loss
Polytropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = \frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot{m}^2}{A^2} v \frac{dv}{dz}$	
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot{m}^2}{A^2} v \frac{dv}{dz}$	
Adiabatic	Variable		$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = \frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot{m}^2}{A^2} v \frac{dv}{dz}$
	Constant		$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot{m}^2}{A^2} v \frac{dv}{dz}$
Isentropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = \left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot{m}}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \left(\frac{\dot{m}}{A}\right)^2 v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = \left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot{m}}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \left(\frac{\dot{m}}{A}\right)^2 v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = \left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot{m}}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \left(\frac{\dot{m}}{A}\right)^2 v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = \left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot{m}}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \left(\frac{\dot{m}}{A}\right)^2 v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$
Isothermal	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t}{v}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\dot{m}}{A}\right)^2 v \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t}{v}\right]\frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\dot{m}}{A}\right)^2 v \frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t}{v}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left(\frac{\dot{m}}{A}\right)^2 v \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left[\left(\frac{\dot{m}}{A}\right)^2 - \frac{R_t}{v}\right]\frac{dv}{dz}$ $0 = \left(\frac{\dot{m}}{A}\right)^2 v \frac{dv}{dz}$
Isochor	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz}$ $0 = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$
Isobar	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot{m}^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot{m}^2}{A^2} v\right]\frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot{m}^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot{m}^2}{A^2} v\right]\frac{dv}{dz}$

Table 1. Ideal Gas Flow Equations II.

	Cross-Section	No Friction	No Heat Loss No Friction
Polytropic	Variable	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot m^2}{A^2} v \frac{dv}{dz}$	
	Constant	$0 = \frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot m^2}{A^2} v \frac{dv}{dz}$	
Adiabatic	Variable		$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $-\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot m^2}{A^2} v \frac{dv}{dz}$
	Constant		$0 = \frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz} + \frac{\dot m^2}{A^2} v \frac{dv}{dz}$
Isentropic	Variable	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
	Constant	$0 = \left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$	$0 = -\left[-\frac{R_t}{v}\left(\frac{\partial T}{\partial v}\right)_S - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \left[\frac{\kappa}{\kappa-1} R_t \left(\frac{\partial T}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$ $\left(\frac{\partial T}{\partial v}\right)_S = -(\kappa-1)\frac{T}{v}$
Isothermal	Variable	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\dot m^2}{A^2} v \frac{dv}{dz}$	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\dot m^2}{A^2} v \frac{dv}{dz}$
	Constant	$0 = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \frac{\dot m^2}{A^2} v \frac{dv}{dz}$	$0 = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \frac{\dot m^2}{A^2} v \frac{dv}{dz}$
Isochor	Variable	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$
	Constant	$0 = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot q}{\dot m} = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$	$0 = -\frac{R_t}{v}\frac{dT}{dz}$ $0 = \frac{\kappa}{\kappa-1} R_t \frac{dT}{dz}$
Isobar	Variable	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$-\frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
	Constant	$0 = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$0 = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\frac{\kappa}{\kappa-1} p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$

Table 2. Wet Steam Flow Equations I.

	Cross-Section	Heat Loss and Friction	No Heat Loss
Polytropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \frac{dT}{dz} - \frac{\dot{m}^2}{A^2} v$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	
Adiabatic	Variable		$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant		$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isentropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isothermal	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isochor	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\delta_1 \frac{dT}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
Isobar	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$

Table 2. Wet Steam Flow Equations II.

	Cross-Section	No Friction	No Heat Loss No Friction
Polytropic	Variable	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	
	Constant	$0 = -\delta_1 \frac{dT}{dz} - \frac{\dot{m}^2}{A^2} v$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	
Adiabatic	Variable		$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant		$0 = -\delta_1 \frac{dT}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isentropic	Variable	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$0 = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $p = \delta_0 + \delta_1 T$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$0 = -\delta_1 \left(\frac{\partial T}{\partial v}\right)_S \frac{dv}{dz} - \left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isothermal	Variable	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$0 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$0 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
Isochor	Variable	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\delta_1 \frac{dT}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
	Constant	$0 = -\delta_1 \frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$0 = -\delta_1 \frac{dT}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
Isobar	Variable	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$
	Constant	$0 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}} = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$	$0 = -\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot{m}^2}{A^2} v\right] \frac{dv}{dz}$

Table 3. Superheated Steam Flow Equations I.

	Cross-Section	Heat Loss and Friction	No Heat Loss
Polytropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$	
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$	
Adiabatic	Variable		$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
	Constant		$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz} - \left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz} + \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$
Isentropic	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[-\frac{R_t}{v}\frac{dT}{dz} - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[-\frac{R_t}{v}\frac{dT}{dz} - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = \left[-\frac{R_t}{v}\frac{dT}{dz} - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = \left[-\frac{R_t}{v}\frac{dT}{dz} - \left(\frac{\dot m}{A}\right)^2 + \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_S + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
Isothermal	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\left[\left(\frac{\dot m}{A}\right)^2 - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_T - \frac{R_t T}{v^2}\right]\frac{dv}{dz}$
Isochor	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\frac{R_t}{v}\frac{dT}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot q}{\dot m} = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\frac{R_t}{v}\frac{dT}{dz}$ $0 = \left(\frac{\partial h}{\partial T}\right)_v \frac{dT}{dz}$
Isobar	Variable	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot q}{\dot m} + \dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} v \frac{dA}{dz} = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\dot m^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \left[\left(\frac{\partial h}{\partial v}\right)_p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$
	Constant	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $\frac{\dot q}{\dot m} = \left[\left(\frac{\partial h}{\partial v}\right)_p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$	$\frac{\lambda}{2d} v \left(\frac{\dot m}{A}\right)^2 = -\left(\frac{\dot m}{A}\right)^2 \frac{dv}{dz}$ $0 = \left[\left(\frac{\partial h}{\partial v}\right)_p + \frac{\dot m^2}{A^2} v\right]\frac{dv}{dz}$

Table 3. Superheated Steam Flow Equations II.

	Cross-Section	No Friction	No Heat Loss No Friction
Polytropic	Variable	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\frac{R_t}{v}\frac{dT}{dz}-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}+\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}+\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$	
	Constant	$0=-\frac{R_t}{v}\frac{dT}{dz}-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}+\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$	
Adiabatic	Variable		$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\frac{R_t}{v}\frac{dT}{dz}-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}+\left[\left(\frac{\partial h}{\partial v}\right)_T+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$
	Constant		$0=-\frac{R_t}{v}\frac{dT}{dz}-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}+\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$
Isentropic	Variable	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left[-\frac{R_t}{v}\frac{dT}{dz}-\left(\frac{\dot{m}}{A}\right)^2+\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}+\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_S+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left[-\frac{R_t}{v}\frac{dT}{dz}-\left(\frac{\dot{m}}{A}\right)^2+\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_S+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$
	Constant	$0=\left[-\frac{R_t}{v}\frac{dT}{dz}-\left(\frac{\dot{m}}{A}\right)^2+\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}=\left[\left(\frac{\partial h}{\partial v}\right)_S+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$	$0=\left[-\frac{R_t}{v}\frac{dT}{dz}-\left(\frac{\dot{m}}{A}\right)^2+\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0=\left[\left(\frac{\partial h}{\partial v}\right)_S+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$
Isothermal	Variable	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}+\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$
	Constant	$0=-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}=\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$	$0=-\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$ $0=\left[\left(\frac{\partial h}{\partial v}\right)_T-\frac{R_t T}{v^2}\right]\frac{dv}{dz}$
Isochor	Variable	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}}+\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}$	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\frac{R_t}{v}\frac{dT}{dz}$ $\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}$
	Constant	$0=-\frac{R_t}{v}\frac{dT}{dz}$ $\frac{\dot{q}}{\dot{m}}=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}$	$0=-\frac{R_t}{v}\frac{dT}{dz}$ $0=\left(\frac{\partial h}{\partial T}\right)_v\frac{dT}{dz}$
Isobar	Variable	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left(\frac{\dot{m}}{A}\right)^2\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}+\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_p+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$	$-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}=-\left(\frac{\dot{m}}{A}\right)^2\frac{dv}{dz}$ $\dot{m}^2v^2\frac{1}{A^3}\frac{dA}{dz}=\left[\left(\frac{\partial h}{\partial v}\right)_p+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$
	Variable	$0=-\left(\frac{\dot{m}}{A}\right)^2\frac{dv}{dz}$ $\frac{\dot{q}}{\dot{m}}=\left[\left(\frac{\partial h}{\partial v}\right)_p+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$	$0=-\left(\frac{\dot{m}}{A}\right)^2\frac{dv}{dz}$ $0=\left[\left(\frac{\partial h}{\partial v}\right)_p+\frac{\dot{m}^2}{A^2}v\right]\frac{dv}{dz}$

Table 4. Solution of the System od Differential Equations for Ideal Gas with Heat Loss and Friction I.

	Cross-Section	Prepared Equations for Solution	Solutions
Polytropic	Variable	$a_{11}(z,v)\frac{dT}{dz}+a_{12}(z,v,T)\frac{dv}{dz} = b_{11}(z,v)$ $a_{21}\frac{dT}{dz}+a_{22}(z,v)\frac{dv}{dz} = b_{21}(z,v,T)$ $a_{11}(z,v)=-\frac{R_i}{v}, a_{12}(z,v,T)=\left[\left(\frac{\dot m}{A}\right)^2-\frac{R_i T}{v^2}\right]$ $a_{21}=\frac{\kappa}{\kappa-1}R_i, a_{22}(z,v)=\left(\frac{\dot m}{A}\right)^2 v$ $b_{11}(z,v)=\frac{\lambda}{2d}v\left(\frac{\dot m}{A}\right)^2-\frac{\dot m^2}{A^3}v\frac{dA}{dz}$ $b_{21}(z,v,T)=\frac{\dot q}{\dot m}+\dot m^2 v^2\frac{1}{A^3}\frac{dA}{dz}$	$\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}_z=\begin{bmatrix}a_{11}(z,v) & a_{12}(z,v,T)\\ a_{21} & a_{22}(z,v)\end{bmatrix}_z^{-1}\begin{bmatrix}b_{11}(z,v)\\ b_{21}(z,v,T)\end{bmatrix}_z$ $\begin{bmatrix}T(z+\Delta z)\\ v(z+\Delta z)\end{bmatrix}=\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}\Delta z\begin{bmatrix}T(z)\\ v(z)\end{bmatrix}$
	Constant	$a_{11}(z,v)\frac{dT}{dz}+a_{12}(z,v,T)\frac{dv}{dz} = b_{11}(z,v)$ $a_{21}\frac{dT}{dz}+a_{22}(z,v)\frac{dv}{dz} = b_{21}(z,v,T)$ $a_{11}(z,v)=-\frac{R_i}{v}, a_{12}(z,v,T)=\left[\left(\frac{\dot m}{A}\right)^2-\frac{R_i T}{v^2}\right]$ $a_{21}=\frac{\kappa}{\kappa-1}R_i, a_{22}(z,v)=\left(\frac{\dot m}{A}\right)^2 v$ $b_{11}(z,v)=\frac{\lambda}{2d}v\left(\frac{\dot m}{A}\right)^2$ $b_{21}(z,v,T)=\frac{\dot q}{\dot m}$	$\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}_z=\begin{bmatrix}a_{11}(z,v) & a_{12}(z,v,T)\\ a_{21} & a_{22}(z,v)\end{bmatrix}_z^{-1}\begin{bmatrix}b_{11}(z,v)\\ b_{21}(z,v,T)\end{bmatrix}_z$ $\begin{bmatrix}T(z+\Delta z)\\ v(z+\Delta z)\end{bmatrix}=\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}\Delta z\begin{bmatrix}T(z)\\ v(z)\end{bmatrix}$
		$\frac{dv}{dz}=\frac{-\frac{\dot q A^2}{\dot m^3}\frac{\kappa-1}{\kappa}v-\frac{\lambda}{2d}v^3}{\left(1+\frac{1}{2}\frac{\kappa-1}{\kappa}\right)v^2-\frac{\dot q A^2}{\dot m^3}\frac{\kappa-1}{\kappa}z-\frac{R_i A^2}{\dot m^2}T_0-\frac{1}{2}\frac{\kappa-1}{\kappa}v_0^2}$ If $\dot q=$ const.	$v(z+\Delta z)=\frac{dv}{dz}\Delta z+v(z)$ $T(z+\Delta z)=T(z)+\frac{\dot q}{\dot m}\frac{\kappa-1}{\kappa}\frac{\Delta z}{R_i}+\frac{\kappa-1}{\kappa}\frac{1}{R_i}\left(\frac{\dot m}{A}\right)^2[v(z)^2-v(z+\Delta z)^2]$

Table 4. Solution of the System od Differential Equations for Ideal Gas with Heat Loss and Friction II.

	Cross-Section	Prepared Equations for Solution	Solutions
Adiabatic	Variable	$a_{11}(z,v)\frac{dT}{dz}+a_{12}(z,v,T)\frac{dv}{dz} = b_{11}(z,v)$ $a_{21}\frac{dT}{dz}+a_{22}(z,v)\frac{dv}{dz} = b_{21}(z,v,T)$ $a_{11}(z,v)=-\frac{R_i}{v}, a_{12}(z,v,T)=\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_i T}{v^2}\right]$ $a_{21}=\frac{\kappa}{\kappa-1}R_i, a_{22}(z,v)=\left(\frac{\dot{m}}{A}\right)^2 v$ $b_{11}(z,v)=\frac{\lambda}{2d}v\left(\frac{\dot{m}}{A}\right)^2-\frac{\dot{m}^2}{A^3}v\frac{dA}{dz}$ $b_{21}(z,v,T)=\dot{m}^2 v^2\frac{1}{A^3}\frac{dA}{dz}$	$\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}_z=\begin{bmatrix}a_{11}(z,v) & a_{12}(z,v,T)\\ a_{21} & a_{22}(z,v)\end{bmatrix}_z^{-1}\begin{bmatrix}b_{11}(z,v)\\ b_{21}(z,v,T)\end{bmatrix}_z$ $\begin{bmatrix}T(z+\Delta z)\\ v(z+\Delta z)\end{bmatrix}=\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}\Delta z\begin{bmatrix}T(z)\\ v(z)\end{bmatrix}$
	Constant	$a_{11}(z,v)\frac{dT}{dz}+a_{12}(z,v,T)\frac{dv}{dz} = b_{11}(z,v)$ $a_{21}\frac{dT}{dz}+a_{22}(z,v)\frac{dv}{dz} = b_{21}(z,v,T)$ $a_{11}(z,v)=-\frac{R_i}{v}, a_{12}(z,v,T)=\left[\left(\frac{\dot{m}}{A}\right)^2-\frac{R_i T}{v^2}\right]$ $a_{21}=\frac{\kappa}{\kappa-1}R_i, a_{22}(z,v)=\left(\frac{\dot{m}}{A}\right)^2 v$ $b_{11}(z,v)=\frac{\lambda}{2d}v\left(\frac{\dot{m}}{A}\right)^2$ $b_{21}(z,v,T)=0$	$\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}_z=\begin{bmatrix}a_{11}(z,v) & a_{12}(z,v,T)\\ a_{21} & a_{22}(z,v)\end{bmatrix}_z^{-1}\begin{bmatrix}b_{11}(z,v)\\ b_{21}(z,v,T)\end{bmatrix}_z$ $\begin{bmatrix}T(z+\Delta z)\\ v(z+\Delta z)\end{bmatrix}=\begin{bmatrix}\frac{dT}{dz}\\ \frac{dv}{dz}\end{bmatrix}\Delta z\begin{bmatrix}T(z)\\ v(z)\end{bmatrix}$
		$\frac{dv}{dz}=\frac{-\frac{\lambda}{2d}v^3}{\left(1+\frac{1}{2}\frac{\kappa-1}{\kappa}\right)v^2-\frac{R_i A^2}{\dot{m}^2}T_0-\frac{1}{2}\frac{\kappa-1}{\kappa}v_0^2}$ If $\dot{q}$ = const.	$v(z+\Delta z)=\frac{dv}{dz}\Delta z+v(z)$ $T(z+\Delta z)=T(z)+\frac{\kappa-1}{\kappa}\frac{1}{R_i}\left(\frac{\dot{m}}{A}\right)^2[v(z)^2-v(z+\Delta z)^2]$

Table 4. Solution of the System of Differential Equations for Ideal Gas with Heat Loss and Friction III.

	Cross-Section	Prepared Equations for Solution	Solutions
Isentropic	Variable	$\frac{dv}{dz} = \frac{\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)v^{\kappa+2}}{\frac{\kappa R_i T_0 v_0^{\kappa-1}}{\dot{m}^2} A^2 - v^{\kappa+1}}$ $\frac{dv}{dz} = \frac{\frac{\kappa-1}{R_i \kappa}\frac{\dot{q}}{\dot{m}} v^{\kappa} + \frac{\kappa-1}{R_i \kappa}\dot{m}\frac{1}{A^3}\frac{dA}{dz} v^{\kappa+2}}{(1-\kappa)T_0 v_0^{\kappa-1} + \left(\frac{\dot{m}}{A}\right)^2 \frac{\kappa-1}{R_i \kappa} v^{\kappa+1}}$	If $A(z)$ is given, the heat loss $\dot{q}(z) = -\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)\left(\frac{\dot{m}}{A}\right)^2 v^2 \dot{m}$ and $v(z)$ $\frac{dv}{dz} = \frac{\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)v^{\kappa+2}}{\frac{\kappa R_i T_0 v_0^{\kappa-1}}{\dot{m}^2} A^2 - v^{\kappa+1}}$ If $\dot{q}(z)$ is given $\left(\frac{\kappa R_i T_0 v_0^{\kappa-1}}{\dot{m}^2} A^2 - v^{\kappa+1}\right)\frac{dv}{dz} + \frac{1}{A} v^{\kappa+2}\frac{dA}{dz} = \frac{\lambda\sqrt{\pi}}{4\sqrt{A}} v^{\kappa+2}$ $\left[\left(\frac{\dot{m}}{A}\right)^2 v^{\kappa+1} - \kappa R_i T_0 v_0^{\kappa-1}\right]\frac{dv}{dz} - \frac{\dot{m}}{A^3} v^{\kappa+2}\frac{dA}{dz} = v^{\kappa}\frac{\dot{q}}{\dot{m}}$
	Constant	$\frac{dv}{dz} = \frac{\frac{\lambda}{2d} v^{\kappa+2}}{\frac{\kappa R_i T_0 v_0^{\kappa-1}}{\dot{m}^2} A^2 - v^{\kappa+1}}$ $\frac{dv}{dz} = \frac{\frac{\kappa-1}{R_i \kappa}\frac{\dot{q}}{\dot{m}} v^{\kappa}}{(1-\kappa)T_0 v_0^{\kappa-1} + \left(\frac{\dot{m}}{A}\right)^2 \frac{\kappa-1}{R_i \kappa} v^{\kappa+1}}$	The heat loss $\dot{q}(z) = -\frac{\lambda}{2d}\left(\frac{\dot{m}}{A}\right)^2 v^2 \dot{m}^2$ and$v(z)$ $\frac{R_i \kappa T_0 v_0^{\kappa-1}}{\dot{m}^2(\kappa+1)} A^2 \left(\frac{1}{V_0^{\kappa+2}} - \frac{1}{v^{\kappa+2}}\right) - ln\frac{V_0}{V} = \frac{\lambda}{2d} z$

Table 4. Solution of the System of Differential Equations for Ideal Gas with Heat Loss and Friction IV.

	Cross-Section	Prepared Equations for Solution	Solution
Isothermal	Variable	$\frac{dv}{dz} = \frac{\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)v^3}{\frac{R_i T}{\left(\frac{\dot m}{A}\right)^2} - v^2}$ $\frac{dv}{dz} = \frac{\dot q}{\dot m^3} A^2 \frac{1}{V} + v\frac{1}{A}\frac{dA}{dz}$	If $A(z)$ is given, the $\dot q$ is $\dot q(z) = \left[\frac{\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)v^3}{\frac{R_i T}{\left(\frac{\dot m}{A}\right)^2} - v^2} - v\frac{1}{A}\frac{dA}{dz}\right]\frac{\dot m^3}{A^2} v$ for $v(z)$ $\frac{dv}{dz} = \frac{\left(\frac{\lambda\sqrt{\pi}}{4\sqrt{A}} - \frac{1}{A}\frac{dA}{dz}\right)v^3}{\frac{R_i T}{\dot m^2}A^2 - v^2}$ If $\dot q(z)$ is given $\left(\frac{R_i T A^2}{\dot m^2} - v^2\right)\frac{dv}{dz} + \frac{1}{A}v^3\frac{dA}{dz} = \frac{\lambda\sqrt{\pi}}{4\sqrt{A}}v^3$ $\frac{dv}{dz} - \frac{v}{A}\frac{dA}{dz} = \frac{\dot q}{\dot m^3}A^2\frac{1}{v}$
	Constant	$\frac{dv}{dz} = \frac{\frac{\lambda}{2d}v^3}{\frac{R_i T}{\left(\frac{\dot m}{A}\right)^2} - v^2}$ $\frac{dv}{dz} = \frac{\dot q}{\dot m^3}A^2\frac{1}{v}$	If $A(z)$ is given, the $\dot q$ is $\dot q(z) = \frac{\frac{\lambda}{2d}v^4}{\frac{R_i T}{\left(\frac{\dot m}{A}\right)^2} - v^2}\frac{\dot m^3}{A^2}$ for $v(z)$_ $\frac{1}{2}\frac{R_i T}{\left(\frac{\dot m}{A}\right)^2}\left(\frac{1}{v_0^2} - \frac{1}{v^2}\right) + ln\frac{v_0}{v} = \frac{\lambda}{2d}z$
Isochor	Variable	$\frac{dT}{dz} = -\left[\frac{\lambda}{2d}\left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3}\frac{dA}{dz}\right]\frac{1}{R_i}v^2$ $\frac{dT}{dz} = \frac{\dot q}{\dot m^3}A^2\frac{1}{V} + v\frac{1}{A}\frac{dA}{dz}$	If $A(z)$ is given, the $\dot q$ is $\dot q(z) = \frac{\dot m^3}{A^3}v^2\frac{dA}{dz}\frac{1}{\kappa-1} - \frac{\lambda}{2d}\frac{\dot m^3}{A^2}\frac{\kappa}{\kappa-1}v^2$ If $\dot q(z)$ is given $\frac{dA}{dz} = \frac{\dot q(z)(\kappa-1)}{\dot m^3 v^2}A^3 + \frac{\lambda\sqrt{\pi}}{4}\kappa v^2\sqrt{A}$
	Constant	$\frac{dT}{dz} = -\frac{\lambda}{2d}v^2\left(\frac{\dot m}{A}\right)^2\frac{1}{R_i}$ $\frac{dT}{dz} = \frac{\dot q}{\dot m}\frac{\kappa-1}{\kappa R_i}$	The heat loss $\dot q(z) = -\frac{\lambda}{2d}\frac{\dot m^3}{A^2}\frac{\kappa}{\kappa-1}v^2$

Table 4. Solution of the System of Differential Equations for Ideal Gas with Heat Loss and Friction V.

	Cross-Section	Prepared Equations for Solution	Solution
Isobar	Variable	$\frac{dv}{dz} = \left(\frac{1}{A}\frac{dA}{dz} - \frac{\lambda}{2d}\right) v$ $\frac{dv}{dz} = \frac{\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz}}{\frac{\kappa-1}{\kappa} p + \left(\frac{\dot{m}}{A}\right)^2 v}$	If $A(z)$ is given, the heat loss $\frac{\dot{q}}{\dot{m}} = \left(\frac{\kappa-1}{\kappa} p + \left(\frac{\dot{m}}{A}\right)^2 v\right) \frac{dv}{dz} - \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz}$ $v = v_0 e^{\frac{1}{A}\frac{dA}{dz} - \frac{\lambda}{2d} z}$ If $\dot{q}(z)$ is given, for $A(z)$ and $v(z)$ $\left(\frac{\dot{m}}{A}\right)^2 \frac{dv}{dz} - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz} = \frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2$ $\left(\frac{\kappa-1}{\kappa} p \frac{\dot{m}}{A^2} v\right) \frac{dv}{dz} - \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz} = \frac{\dot{q}}{\dot{m}}$
	Constant	$\frac{dv}{dz} = -\frac{\lambda}{2d} v$ $\frac{dv}{dz} = \frac{\frac{\dot{q}}{\dot{m}}}{\frac{\kappa-1}{\kappa} p + \left(\frac{\dot{m}}{A}\right)^2 v}$	$v = v_0 e^{-\frac{\lambda}{2d} z}$ $\dot{q} = -\frac{\lambda}{2d} \dot{m} \left[\frac{\kappa-1}{\kappa} p + \left(\frac{\dot{m}}{A}\right)^2 v_0 e^{-\frac{\lambda}{2d} z}\right] v_0 e^{-\frac{\lambda}{2d} z}$

Table 5. Solution of the System of Differential Equations for Wet Steam with Friction and Heat Loss I.

	Cross-Section	Prepared Equations for Solution	Solutions
Polytropic	Variable	$\begin{bmatrix} \frac{dT}{dz} \\ \frac{dv}{dz} \end{bmatrix} = \begin{bmatrix} a_{11}(T) & a_{12}(z) \\ a_{21}(T) & a_{22}(z,v,T) \end{bmatrix} \begin{bmatrix} b_{11}(z,v) \\ b_{21}(z,v,T) \end{bmatrix}$	$\begin{bmatrix} T(z+\Delta z) \\ v(z+\Delta z) \end{bmatrix} = \begin{bmatrix} \frac{dT}{dz} \\ \frac{dv}{dz} \end{bmatrix} \Delta z + \begin{bmatrix} T(z) \\ v(z) \end{bmatrix}$
	Constant	$\begin{bmatrix} \frac{dT}{dz} \\ \frac{dv}{dz} \end{bmatrix} = \begin{bmatrix} a_{11}(T) & a_{12}(z) \\ a_{21}(T) & a_{22}(z,v,T) \end{bmatrix} \begin{bmatrix} b_{11}(z,v) \\ b_{21}(z,v,T) \end{bmatrix}$	$\begin{bmatrix} T(z+\Delta z) \\ v(z+\Delta z) \end{bmatrix} = \begin{bmatrix} \frac{dT}{dz} \\ \frac{dv}{dz} \end{bmatrix} \Delta z + \begin{bmatrix} T(z) \\ v(z) \end{bmatrix}$
		$\frac{dv}{dz} = \frac{\frac{\lambda}{2d} v - \frac{\dot q}{\dot m^3} A^2 \frac{\delta_1}{b_1}}{\frac{\delta_1}{b_1} v + \frac{A^2}{\dot m^2} \frac{\delta_1 g_1}{b_1} - 1}$	$v = v_0 + \frac{C_1}{B_0} z + \left(\frac{A_0}{C_0} + \frac{B_1}{B_0} \right) ln \frac{C_1 v_0 - A_0}{C_1 r - A_0}$
Adiabatic	Variable Constant		
Isentropic	Variable	$\frac{dv}{dz} = \frac{\left[\frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 - \frac{\dot m^2}{A^3} \frac{dA}{dz} \right] v}{-(\mu_1 + 2\mu_2 v + \dots) - \left(\frac{\dot m}{A}\right)^2}$ $\frac{dv}{dz} = \frac{\frac{\dot q}{\dot m} + \dot m v^2 \frac{1}{A^3} \frac{dA}{dz}}{(\gamma_1 + 2\gamma_2 v + \dots) + \left(\frac{\dot m}{A}\right)^2 v}$	$\dot q(z) = \frac{\left[\frac{\dot m^2}{A^3} \frac{dA}{dz} - \frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 \right] \left[(\gamma_1 + 2\gamma_2 v + \dots) + \left(\frac{\dot m}{A}\right)^2 v \right]}{(\mu_1 + 2\mu_2 v + \dots) + \left(\frac{\dot m}{A}\right)^2} \dot m - \frac{\dot m^3 v^2}{A^3} \frac{dA}{dz},$ If $A = A(z)$ is given. $\mu_1 ln \frac{v}{v_0} + \left[2\mu_2 + \left(\frac{\dot m}{A}\right)^2 \right] (v - v_0) + \frac{3\mu_3}{2} (v^2 - v_0^2) + \dots = \int_0^z \left[\frac{\dot m^2}{A^3} \frac{dA}{dz} - \frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 \right] dz$ $\frac{dA}{dz} = \left[\frac{\dot q \left[\mu_1 + 2\mu_2 v + \dots + \left(\frac{\dot m}{A}\right)^2 \right]}{[\gamma_1 + 2\gamma_2 v + \dots + \left(\frac{\dot m}{A}\right)^2] \dot m} + \frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 \right] \frac{A^3}{\dot m^2}$ $A(z + \Delta z) = \frac{dA}{dz} \Delta z + A(z), \quad \dot q = \dot q(z)$ is given
	Constant	$\frac{dv}{dz} = \frac{\frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 v}{-(\mu_1 + 2\mu_2 v + \dots) - \left(\frac{\dot m}{A}\right)^2}$ $\frac{dv}{dz} = \frac{\frac{\dot q}{\dot m}}{(\mu_1 + 2\mu_2 v + \dots) + \left(\frac{\dot m}{A}\right)^2 v}$	$\dot q(z) = -\frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 \frac{v[(\gamma_1 + 2\gamma_2 v + \dots) - \left(\frac{\dot m}{A}\right)^2]}{(\mu_1 + 2\mu_2 v + \dots) + \left(\frac{\dot m}{A}\right)^2}$ $v = v_0 - \frac{\lambda}{2d} \left(\frac{\dot m}{A}\right)^2 \frac{1}{\mu_1 - \left(\frac{\dot m}{A}\right)^2} z$, If $\mu_2 \equiv \mu_3 \equiv \dots \equiv 0$

Table 5. Solution of the System of Differential Equations for Wet Steam with Friction and Heat Loss II.

	Cross-S.	Comments and Notations
Polytropic	Variable	$a_{11}(T) \approx -(\delta_1 + 2\delta_2 T + \dots)$ $a_{21}(T, v) = \left(\frac{\partial h}{\partial T}\right)_V$ $b_{11} = \frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3} v \frac{dA}{dz}$ $h(v = \text{const.}, T) = b_0 + b_1 T + \dots$ $a_{12}(z) = -\left(\frac{\dot{m}}{A}\right)^2$ $a_{22}(z, T, v) = \left[\left(\frac{\partial h}{\partial v}\right)_T + \left(\frac{\dot{m}}{A}\right)^2 v\right]$ $b_{21}(z, v, T) = \frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3} \frac{dA}{dz}$ $h(T = \text{const.}) = g_0 + g_1 v + \dots$ $p = \delta_0 + \delta_1 T + \dots$
	Constant	$\frac{dA}{dz} = 0$ $a_{11}(T) \approx -(\delta_1 + 2\delta_2 T + \dots)$ $a_{21}(T, v) = \left(\frac{\partial h}{\partial T}\right)_V$ $b_{11}(z, v) = \frac{\lambda}{2d} v \left(\frac{\dot{m}}{A}\right)^2$ $a_{12}(z) = -\left(\frac{\dot{m}}{A}\right)^2$ $a_{22}(z, T, v) = \left[\left(\frac{\partial h}{\partial v}\right)_T + \left(\frac{\dot{m}}{A}\right)^2 v\right]$ $b_{21}(z, v) = \frac{\dot{q}}{\dot{m}}$
		$\frac{dA}{dz} = 0$, $\dot{q} = \text{const.}$, $A_0 = \frac{\dot{q}}{\dot{m}^3} \frac{\delta_1}{b_1} A^2$, $B_0 = \frac{\delta_1}{b_1}$, $B_1 = \frac{g_1}{b_1} \delta_1 \frac{A^2}{\dot{m}^2} - 1$, $C_1 = \frac{\lambda}{2d}$, $p \approx \delta_0 + \delta_1 T$
Adiabatic	Variable Constant	
Isentropic	Variable	In the motion equation $\frac{dp}{dz} = \left(\frac{\partial p}{\partial S}\right)_v \frac{dS}{dz} + \left(\frac{\partial p}{\partial v}\right)_S \frac{dv}{dz} = \left(\frac{\partial p}{\partial v}\right)_S \frac{dv}{dz}$ $p \approx \delta_0 + \delta_1 + \dots$ $p(v, S = \text{const.}) \approx \mu_0 + \mu_1 v + \mu_2 v^2 + \dots$ $T(v, S = \text{const.}) \approx \nu_0 + \nu_1 v + \nu_2 v^2 + \dots$ $h(v, S = \text{const.}) \approx \gamma_0 + \gamma_1 v + \gamma_2 v^2 + \dots$
	Constant	If $\mu_2 \equiv \mu_3 \equiv \dots \equiv 0$.

Table 5. Solution of the System of Differential Equations for Wet Steam with Friction and Heat Loss III.

	Cross-Section	Prepared Equations for Solution	Solutions
Isothermal	Variable	$\frac{dv}{dz} = -\left(\frac{\lambda}{2d} - \frac{1}{A}\frac{dA}{dz}\right)v$ $\frac{dv}{dz} = \frac{\frac{\dot{q}}{\dot{m}} + \dot{m}^2 \frac{1}{A^3}\frac{dA}{dz}v^2}{(\gamma_1+2\gamma_2 v+\dots)+\left(\frac{\dot{m}}{A}\right)^2 v}$	If $A = A(z)$ is given, $\dot{q}(z) = \dot{m}\left(\frac{1}{A}\frac{dA}{dz} - \frac{\lambda}{2d}\right)(\gamma_1 + 2\gamma_2 v + \dots) + \dots$ If $\mu_2 \equiv \mu_3 \equiv \dots \equiv 0$, $v(z) = v_0 \frac{A}{A_0} e^{-\frac{\lambda}{2d}z}$
	Constant	$\frac{dv}{dz} = -\frac{\lambda}{2d}v$ $\frac{dv}{dz} = \frac{\frac{\dot{q}}{\dot{m}}}{(\gamma_1+2\gamma_2 v+\dots)+\left(\frac{\dot{m}}{A}\right)^2 v}$	$\dot{q}(z) = -\frac{\lambda}{2d}v\dot{m}\left[(\gamma_1 + 2\gamma_2 v + \dots) + \left(\frac{\dot{m}}{A}\right)^2 v\right]$ $v(z) = v_0 e^{-\frac{\lambda}{2d}z}$
Isochor	Variable	$\frac{dT}{dz} = \frac{\frac{\lambda}{2d}v\left(\frac{\dot{m}}{A}\right)^2 - \frac{\dot{m}^2}{A^3}v\frac{dA}{dz}}{-(\delta_1+2\delta_2 T+\dots)}$ $\frac{dT}{dz} = \left(\frac{\dot{q}}{\dot{m}} + \dot{m}^2 v^2 \frac{1}{A^3}\frac{dA}{dz}\right)\frac{1}{(b_1+2b_2 v+\dots)}$	If $A(z)$ is given, $\dot{q}(z) = \left(\frac{\dot{m}}{A}\right)^3 v\frac{dA}{dz} - \frac{\lambda}{2d}v\left(\frac{\dot{m}^3}{A^2}\right)\frac{b_1+2b_2 T+\dots}{\delta_1+2\delta_2 T+\dots} - \dot{m}^3 v^2 \frac{1}{A^3}\frac{dA}{dz}$ $T_0 - T + \frac{\delta_2}{\delta_1}(T_0^2 - T^2) + \dots = \frac{1}{\delta_1}\int_0^z \left(\frac{\lambda}{2d}\dot{m}^2 v \frac{1}{A^2} - \dot{m}^2 v \frac{1}{A^3}\frac{dA}{dz}\right)dz$
	Constant	$\frac{dT}{dz} = \frac{\frac{\lambda}{2d}v\left(\frac{\dot{m}}{A}\right)^2}{-(\delta_1+2\delta_2 T+\dots)}$ $\frac{\dot{q}}{\dot{m}} = \frac{1}{(b_1+2b_2 v+\dots)}$	$\dot{q}(z) = -\frac{\lambda}{2d}v\left(\frac{\dot{m}^3}{A^2}\right)\frac{b_1+2b_2 T+\dots}{\delta_1+2\delta_2 T+\dots}$ $T_0 - T + \frac{\delta_2}{\delta_1}(T_0^2 - T^2) + \dots = \frac{1}{\delta_1}\frac{\lambda}{2d}\frac{\dot{m}^2}{A^2}vz$
Isobar	Variable	As Isothermal flow	
	Constant	As Isothermal Flow	

Table 5. Solution of the System of Differential Equations for Wet Steam with Friction and Heat Loss IV.

		Comments and Notations
Isothermal	Variable	$h(T = \text{const.}, v) = g_0 + g_1 T + g_2 T^2 + \ldots$
	Constant	$h(T = \text{const.}, v) = g_0 + g_1 T + g_2 T^2 + \ldots$ $\frac{dA}{dz} = 0$
Isochor	Variable	$h(v = \text{const.}, T) = b_0 + b_1 T + b_2 T^2 + \ldots$ $p = \delta_0 + \delta_1 T + \ldots$
	Constant	$h(v = \text{const.}, T) = b_0 + b_1 T + b_2 T^2 + \ldots$ $\frac{dA}{dz} = 0$
Isobar	Variable	As Isothermal flow
	Constant	As Isothermal Flow

References

[1] GARBAI, L.: *A hőveszteség számítása és mérése az ipari energetikában.* Műszaki Könyvkiadó. Budapest, 1988.
[2] GARBAI, L.–KROPE, J.: *Basic Flow Equations in Stationary Conditions for Building Service Engineering*, Gépészet '98 Proceeding, 1998.
[3] FŰZY, O.: *Áramlástechnikai Gépek és rendszerek*, Műszaki könyvkiadó. Budapest, 1991.