

MODEL FOR THE CALCULATION OF DEMOULDING FORCE FOR POLYURETHANE PARTS

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Abstract

The manufacturing of polyurethane (PU) parts has an ever-increasing trend due to the production of cars. The automation of the demoulding of polyurethane car seats is a current problem the suppliers of car manufacturers have to face. The automation of other processes of seat foam production has already been solved but the problem of demoulding is still a much-investigated subject. To design robot grippers for this process we need to know the value of the demoulding force. We present a mathematical model for the calculation of this force for polyurethane seat-like foams in general. A material model is proposed that uses only those parameters that are available for the factories, and this model is compared with test results.

Keywords: demoulding, polyurethane, material properties.

Introduction

The ever-increasing need of vehicle seats made of polyurethane implicates the automation of the whole manufacturing process of seat production. Parts of the process have already been automated, such as mixing the polyurethane components, pouring the PU mixture into the mould on a controlled path, operating the mould, etc., but according to our literature survey, the demoulding process has not yet been automated. Demoulding is the process when the finished foam part is taken out of the mould [3].

The problem encountered in this process is the different sizes and shapes of foam which often have large undercuts and inserts in them. The inserts are used to modify the characteristics of the seat to give it more rigidity, comfort, etc., to suit different car types (for example a sports car need to have a totally different characteristic seat than a family car or a truck or a train). The undercuts are used to give the seat a better look, a good fashion and a nice design. They also cover the insert (wrap them over) so the seat is much safer and softer for the user.

By making technologies for the automation procedure these characteristics of the foams must be kept as the designer has made them.

The automation of demoulding requires assistance of robots with different gripping strategies for different foams. This is necessary to keep the flexibility of the system that is required greatly in the automobile industry. To design such a system the demoulding force has to be calculated first for each foam in order to obtain the initial value for the design process.

In this paper we present a method for estimating an upper boundary for the demoulding force for a general foam from as few material parameters as possible. This upper boundary gives an initial safety value for the design.

1. The Task

The task is to determine the F force that is required to lift a foam out from the mould (see: *Fig. 1*).



Fig. 1. Model of foam in the mould



Fig. 2. The deformation of undercuts in the foam during demoulding

This means to calculate the force that is required to deform the undercuts of the foam to the size of the mould opening. Demonstration of the deformation of undercuts during lifting the foam out of the mould can be seen in *Fig.2*.

The deformation of the foam is the principle of the calculation.

The first assumption is that the deformation energy is constant in every foam. This means that no matter how and in what sequence the foam is deformed (e.g. first the small undercuts are taken out, then the large ones) the same energy is needed for the whole foam to be lifted out of the undercuts. A proper lifting path, however, can considerably reduce the force (see: *Fig. 3*). The force can be considered to be proportional to the amount of volume that is deformed at a given time,

so when the whole volume is deformed we get the largest force. We take this case for obtaining the maximum value for the demoulding force.

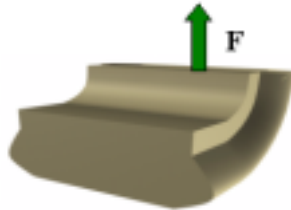


Fig. 3. A lifting method (path) to reduce demoulding force

In the case of controlled lifting path (as seen in Fig. 3) only a part of the foam is deformed at a given time, so the force is much smaller, but the overall energy is the same, because the foam has to be bent and this also causes deformations.

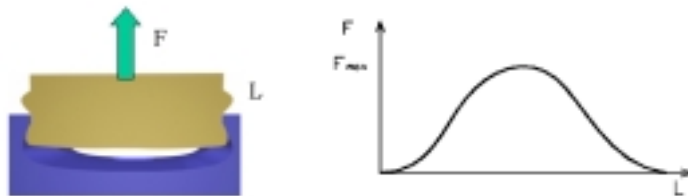


Fig. 4. The free height (L) of the foam being lifted out of the mould (a) and the change of the lifting force with the free height (b)

When the foam is lifted out of the mould the amount of volume that is deformed at a given time is determined by the free height of the mould seen in Fig 4.a. The change of force (F) along the lifting path (L) can be seen in Fig 4.b. This curve is different for each foam therefore it is just an approximation to show the characteristics of the lifting process.

2. Model for the Calculation

When calculating the demoulding force the following components are taken into consideration:

$$F_{\text{dem}} := F_{\text{def}} + F_{\text{frict}} + F_{\text{str}} + F_g + F_{ad}. \tag{1}$$

Where:

F_{dem}	Demoulding force.
F_{def}	Deformation force. It is a projection of the compression force perpendicular to the wall of the mould issued from the compression of the foam in cross direction. (We use this for the calculation).
F_{frict}	Frictional force acting between the foam and mould.
F_{str}	Stretching force arisen from the elongation in the lifting direction.
F_g	Mg , where M is the mass of the foam, g is the gravitational constant 9.81 m/s^2 .
F_{ad}	The adhesion force acting between the foam and the mould when production parameters are not set properly. It is not taken into consideration in this model (see: Appendix 2).

If we presume that the foam is demoulded by making the whole undercut volume deformed to the size of the mould opening, as a first approach the frictional and the stretching force can be neglected. This can be achieved with the use of the CGM gripper [2].

This will be our first assumption. The method gives the smallest force for demoulding and may be used for special grippers only.

Available data for the calculation.

For all force calculations the geometrical foam parameters are the base data.

Usually in the factories the precise geometry of the foam is documented in a 3D CAD model (e.g. in such CAD systems as ProEngineer, Unigraphics, etc.). This model is used for designing the seat, making the mould for the foam, calculating storage space, transport, etc. The solid model of the foams can be used to derive features and size measurements for the calculations and may be the base for further finite element studies.

The other important parameters for the calculations are the foam material parameters. These are the tensile strength, ripping strength, compression strength, tearing strength, bending strength, etc.

The most important parameter for the factory is the compression strength, a kind of compression resistance, which is called hardness by the factory men. This is regularly measured in the factory for every series of foam. From this value the factory can tell whether a series of foam has a good quality or not. The hardness is a designer value that is important for the safety and comfort of the car and its passengers. It can be very different for different types of cars. A sports car has to have hard seats that do not let the driver slide out of the seat in narrow bends and corners. A family car or a truck has to have soft, comfortable seats for long voyages. Sometimes a foam has different hardness values at different parts. This type of seat has a softer inner part and a harder outer rim. The comfort and the safety is achieved in one foam, although the manufacture is more costly. In this paper we only deal with single hardness foam. The calculation for the variable hardness foams is not much different although twice as much material parameters are needed.

For a first approach of the calculations we use the parameter of compression strength and later check the results on a real life tests.

3. The Compression Strength

When a foam material is compressed by a universal tensile test machine a force-displacement curve is obtained as seen in *Fig. 5*.

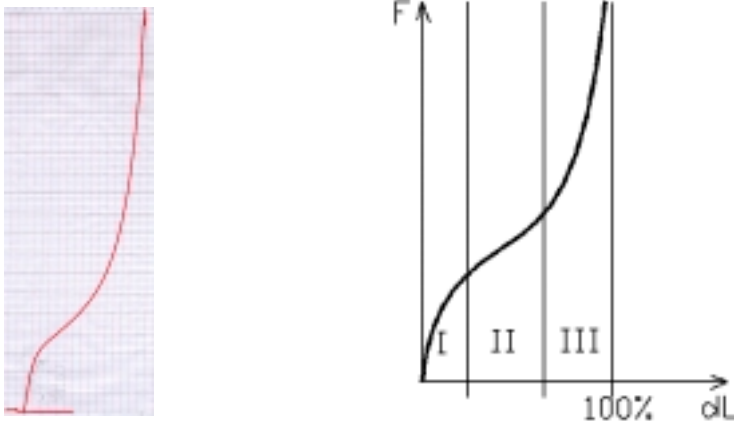


Fig. 5. The original curve from the test (a) and the principal main sections of that (b)

The curve can be divided into three sections. In sections I and II of the curve (see: *Fig. 5* I and II on the picture) a volumetric decrease occurs. The volume decreases in these sections because the foam can be considered a material that has a Poisson factor of relatively zero. So the foam decreases in height, the stress increases but the other two dimensions of the foam block do not increase. This is just an approximation but the change is so small that it does not concern the calculations in the range we are interested in.

The three sections of the curve can be explained by the physics of the foam.

First let us look at a foam at demoulding before the processing. When the foam has finished polymerising a lot of gas is trapped in it. These gas bubbles (in the foam cells) have to be ruptured to let the gas out, otherwise the foam collapses and this collapse ruins it. That is why after demoulding the foam has to be processed (beaten, rolled or put in vacuum) to rupture these cells. In the first part the foam can be considered as an entity that consists of lots of closed cells that has hot (approximately 55 deg. centigrade) gas in them. This is a residue of the polymerisation. When the compression of the foam part starts, the pressure in these cells begins to increase, as the volume of each cell decreases. This lasts till the end of section I. In the second part (section II) the pressure gets so great that it begins to rupture the cells. In this section the slope of the curve is much gentler than in the first section. It lasts till all the gas from the cells has escaped (till the end of section II). In the third part the slope of the curve increases radically. This is because at this time the polymer material itself, without any free volume, is being compressed. This is a very narrow section that happens in the last few mil-

limetres of the compression that ends at an infinitely large force when it reaches the 100% mark of the compression. That is why the part can never be compressed completely, the interaction forces between the atoms prevent that.

The division of the curve was done to explain what happens during compression. In real life these sections cannot be separated sharply. All three types of characteristics are present in all three sections, but the characteristics that are most typical at a part of the curve can be categorised in one section. The other two characteristics are present too in this section but in a much smaller scale.

After processing a foam part has a similar curve but a slightly different first section. This state of the foam is also interesting, because several times the tests have to be done on a processed foam (after pounding). There is just not enough time to make the test on a fresh foam before it collapses. The foam has air in its cells at the beginning and all the cells are already ruptured. When the compression starts these cells begin to contact and the air pressure in them begins to increase. Because the cell walls are already open they begin to let air out very soon, first at the open surface of the foam then gradually inside. So the first two sections cannot be separated well. The third section is the same as for the unprocessed foam.

To generate a function for the curve several methods can be chosen. The approximation of ideal elastomers or real elastomers is like in the Mooney–Rivlin approach from statistical physics. Or we can use mathematical approaches that are not supported by physics but give a good approximation for the curve. These can be spline approximations or analytical curves.

In this paper we try to give an analytical function for the curve from just the compression strength. The spline approximation will give a little better function but at a much larger calculation cost. For the statistical physics approach several other material parameters are needed.

To generate a function that approximates the curve properly the following criteria have to be taken into consideration:

1. At 0% of compression the stress has to be zero;
2. At 100% of compression the stress has to be infinite;
3. The function has to have an inflexion point;
4. From the 0 point to the inflexion point it has to be concave.

The proposed function that was found by us is as follows:

$$\sigma(\varepsilon) = a\varepsilon^b + c \tan(\varepsilon). \quad (2)$$

Where:

$\sigma = \frac{\partial F}{\partial A}$ is the stress;

$\varepsilon = \frac{dL}{L}$ is the strain;

a, b, c are constants for a given polymer material.

These parameters can be derived from the compression tests of a given material. For the determination of these parameters from the tests: see Appendix 1.

4. The Deformation Needed

The foam has to be compressed until all its parts are smaller than the undercuts. In *Fig. 6* a section of a sample seat cushion with real life parameters can be seen. Usually these values do not differ much on different seats therefore a general rule can be set. It means that the needed deformation on all seat-like foams may be less than 30% of the whole size of the foam section.

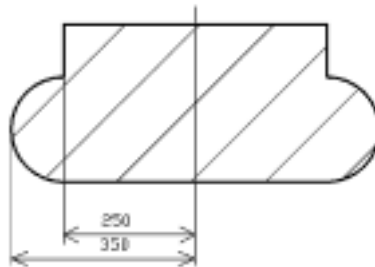


Fig. 6. The cross section of the seat foam

This deformation is well inside section I of the stress-strain curve. The deformation can go above section I till the end of section II, see *Fig. 5*. In section II we get a lot of deformation for just a little increase in deformation force! The deformation should never reach section III, because the force increases there very rapidly and could damage the foam structure. In real life situations, as we have seen earlier, there is no need to deform the foam into section III.

5. Simplifications for the Calculation

Let us summarise the simplifications that we have to make to set up the model for the demoulding force.

The most serious assumption is that σ acts in one direction only and this direction is collinear with the direction of the compression. On sharp corners the σ is three-directional. These corners, however, take up less than 1% of the foam and they are well rounded for proper foam design. At first approach we disregard them.

The other simplification used for the material is that the Poisson factor (ν) is zero, as was suggested earlier. In fact the Poisson factor in these foams is so small [3] that it may be neglected to make the model considerably simpler.

The next three assumptions are technical. The inserts in the foam that are parallel with the compression are not taken in consideration for the calculation. The inserts, that are collinear with the pulling out force (normal to the compression) do not affect the calculations. Although almost every seat has some inserts in them. Fortunately, these inserts never reach into the undercut parts of the seats,

otherwise the demoulding would be impossible. The compression causes a sliding stress along the surface of the inserts in the foam. The sliding stress is the largest at the part nearest to the side of the seat and zero at the middle. At present this stress is neglected, but further investigations are made to include it in the model later.

Sometimes the seat cover is polymerised onto the foam. This makes the production simpler, cheaper and gives a better securing for the cover. In the calculations the cover is not taken into consideration. This is because there are too many different materials used for the covers (almost every car seat has a different cover) that by taking it into consideration would make calculations too specific. The stress caused by a cover is similar to the stress caused by an insert. Although the cover acts on a large surface the stress it causes is much smaller, because the cover deforms more at the same compression stress than an insert or than the foam.

The last simplification is about the starting pressure of the foam inside the mould. The pressure is inside the foam from the residue gas of the polymerisation. This is an initial value of the demoulding force calculation. This means that at mould opening the foam would like to be bigger than the mould so it presses the mould apart. Since the mould does not extend a pressing force acts between the mould and the force. This can be seen when trying to demould a foam that has no undercuts. The force is larger than the weight of the foam, because the force for the pressure acting on the side of the mould times the frictional constant acts between the mould and the foam. This has to be taken into consideration at the calculations by calculating the difference of the foam's volume (at demoulding) and the volume of the mould. In this article the initial value is determined by tests. In our tests this force from this initial pressure was so small that we have neglected in the calculations.

6. Demoulding Force

For the determination of the demoulding force first let us consider a one-directional problem, when the foam shape does not change along the contour. In this case a section of the foam can be written down by a single variable function like $y = f(x)$ (see: *Fig. 7*).

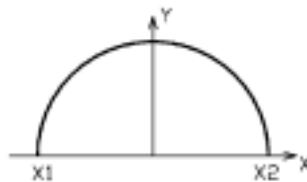


Fig. 7. Section of a simple foam

The compression stress and the size of the area where the compression stress

acts changes with the variable x . This can be formulated as:

$$F = \sum_i \sigma(x_i) \Delta A(x_i), \tag{3}$$

where

$\sigma(x_i)$ is the stress acting in the direction of x .

$\Delta A(x_i)$ is the infinitesimal area where the stress acts.

By performing the limit on (3) we get the integral for the force as:

$$\int_{x_1}^{x_2} \sigma(x) dA(x). \tag{4}$$

To convert the integral to a Riemannian integral so it can easily be calculated the $dA(x)$ is converted to $d(x)$ by the formula:

$$\frac{dA(x)}{dx} dx = A'(x) dx. \tag{5}$$

By substituting (5) integral equation (6) for the one-dimensional demoulding force problem is obtained as:

$$F = \int_{x_1}^{x_2} \sigma(x) A'(x) dx. \tag{6}$$

By taking these results into consideration the more complex problem of a real life foam can be presented.

On a general foam the surface of the undercuts can be given by a binary function $Z = f(x, y)$, see Fig. 8:

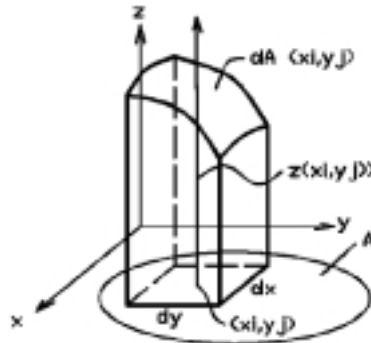


Fig. 8. An infinitesimal section of a foam

Where :

- A : is the base area where the double integral has to be calculated. On a real part it is the area of the middle section of the foam.
- dx, dy : is the elementary rectangular area. By summing these small areas and performing the limit we obtain A .
- $dA(x_i, y_j)$: is the elementary surface area of the foam.
- $z(x_i, y_j)$: is the height of the small parallelepipeds that have $dx_i * dy_j$ for the base area.

The stress $\sigma(x, y)$ and the area $dA(x, y)$ can also be considered as functions of two variables.

Since the foams are symmetrical these calculations may be done only to one side of the foam. So the middle split section of the foam may be the base area (A) of the XY plane from where the calculation of the integral can be performed.

With these parameters the deformation force can be calculated by the same way as was seen in the one dimensional problem but using the method for the double integrals:

$$F = \sum_{i,j} \sigma(x_i, y_j) \Delta A(x_i, y_j) \Rightarrow \text{by performing the limit,} \quad (7)$$

$$F = \int_A \sigma(x, y) dA(x, y) \Rightarrow \iint_{xy} \sigma(x, y) A'(x, y) dx dy. \quad (8)$$

Because our assumption says that the σ is acting in one direction only the calculation is made for $\sigma(x)$ and not for $\sigma(x, y)$. The forces then have to be multiplied by two to obtain the value for the whole foam.

When calculating the sometimes rather difficult $F = \int_A \sigma(x, y) dA(x, y)$ integral, several methods can be used. Symbolic programming languages are available (Maple[®]) to get a closed form or numerical results of these integrals. The first three variables of Taylor formula may be used for simplifying the $\sigma(\varepsilon)$ function. The precision of the calculation does not need higher variables of the Taylor formula.

If the surface of the foam cannot be presented by one function then it has to be broken down to smaller surface parts until these parts can be written down by a function.

Another method is to use a larger surface that is tangential to the original surface that can be presented by a simple function. By calculating with this containing surface a larger force for the deformation is obtained. This pushes the results of the calculation to safer values, because the real deformation will always be smaller than the calculated ones. Of course the approximating surface should not be much larger or different from the original, otherwise the forces will be much larger than needed and would result in a heavy gripper design and an unnecessary large and expensive robot. Usually numerical values for the integral in the calculations are enough. For more precise calculation finite element analysis may be used with the suggested material model. The procedure of demoulding force calculation can be

followed on the chart of *Fig. 9*. The inputs are the foam hardness, the foam's geometry and the parameters from previous tests. The result of the process is a value of the demoulding force for the given foam part.

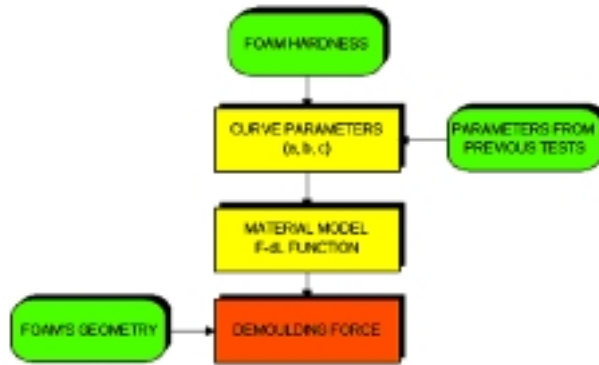


Fig. 9. The process of demoulding force calculation

If the foam is not made properly large adhesion force can occur between the mould and the foam. In this case the demoulding force can be 2–3 times larger than in a proper situation. (See: *Fig 3* in Appendix 2). Investigations of this problem can be a future task.

7. Conclusions

The results show (see Appendixes) that the force and the curves calculated by the theory give a good approximation for the real life problem.

Acknowledgement

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References

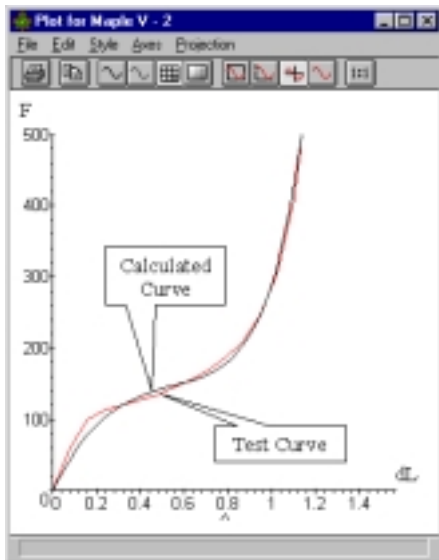
- [1] ZENTAY, P. – ZOLLER, Z. – ARZ, G. – TOTH, A. – MERKSZ, I. – MIKO, B.: Handling of Polyurethane Foams with Robots, *Gépszet '98*, Budapest, 1998. May 28–29. pp. 529–533.
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- [3] WOODS, G.: The ICI Polyurethanes Book. Second Edition. John Wiley & Sons, Chichester, 1990.

Appendix 1. Determining the Parameters from the Hardness

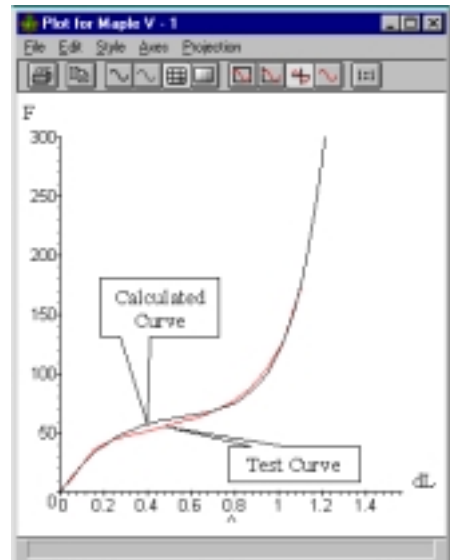
The theory has been proven and adjusted by the tests. (To obtain a strength-strain curve for the compression.)

The method of solving the curves for three points is given in MappleV R4 symbolic programming language. With this method the parameters of the function can be obtained by giving the hardness of the foam.

The verification tests were performed on a $100 \times 100 \times 100$ mm foam block on a tensile test machine in compression mode. The values in the $F - dL$ curve are represented for this specimen.



(a) Type 48 Hz foam



(b) Type 40 Hz foam

Fig. 1. Comparing the results of the tests with the calculated $F - dL$ curve on a screenshot from Mapple V

The presentation of the approximation of the compression curve for the 48 Hz (for this term see Appendix 2) foam in Mapple V R4:

restart;

$x := [0.3142, 0.6283, 0.9425];$

Value of points on the x-axis from the test curve

$x := [.3142, .6283, .9425]$

$y := [120, 154, 244];$

Value of points on the y-axis from the test curve.

$y := [120, 154, 244]$

for i from 1 to 3 do *Cycle for the equation with the*
 $\text{eq}[i] := y[i] = a^*(x[i])^b + c^* \tan(x[i]);$ *given x, y values*
 od;

$$\text{eq}_1 : = 120 = a \cdot 3142^b + .3249647319 c$$

$$\text{eq}_2 : = 154 = a \cdot 6283^b + .7265142160 c$$

$$\text{eq}_3 : = 244 = a \cdot 9425^b + 1.376446190 c$$

$\text{sol} := \text{solve}$ *Solving of the three equations for*
 $(\{\text{eq}[1], \text{eq}[2], \text{eq}[3]\}, \{a, b, c\});$ *the A, B, C parameters*

$$\text{sol} := \{b = 1.752181818, a = -767.5144927, c = 679.9156974\}$$

$B := (\text{op}(2, \text{op}(1, \text{sol})));$ *The value for the B material pa-*
rameter

$$B := 1.752181818$$

$A := (\text{op}(2, \text{op}(2, \text{sol})));$ *The value for the A material pa-*
rameter

$$A := -767.5144927$$

$C := (\text{op}(2, \text{op}(3, \text{sol})));$ *The value for the C material pa-*
rameter

$$C := 679.9156974$$

$Y := A * X^B + C * \tan(X);$ *The function with the calculated*
parameters

$$Y := -767.5144927 X^{1.752181818} + 679.9156974 \tan(X)$$

$\text{plot}(\{[[0,0],[0.0393,30],[0.0785,60],[0.1571,99],[0.2356,114],[0.3142,120],$
 $[0.3927,127],[0.4712,134],[0.5498,144],[0.6283,154],[0.7069,168],[0.7854,186],$
 $[0.8639,208],[0.9425,244],[1.0210,304],[1.0996,410],[1.131,480]], Y\},$
 $X=0..3.14/2, 0..500);$

Plotting the function and the con-
trol points.

dL is represented as the parameter of $\pi/2$ because the 100% of the compression is represented by $\pi/2$ to suit the tangent function.

The 40 Hz, 48 Hz is a factory mark of the softer (40) and the harder (48) foam.

The calculated and the measured curves are illustrated in one diagram for different hardness foams. The results show that the calculated curve follows the measured curve well. Curves for many hardness foams were determined but the calculations for the 48 Hz foam were explained in detail. The curve for the 40 Hz foam is only shown for an example.

The points used for the illustration of the test curve in *Fig. 1(a)* are given in *Table 1*.

Table 1. Sample co-ordinates of the test curve (48 Hz)

No.	X	Y	Nr.	X	Y
1	0	0	10	0.62832	154
2	0.03926	30	11	0.70686	168
3	0.07854	60	12	0.78539	186
4	0.15708	99	13	0.86394	208
5	0.23562	114	14	0.94248	244
6	0.31416	120	15	1.02102	304
7	0.39269	127	16	1.09956	410
8	0.47124	134	17	1.13097	500
9	0.54978	144			

Appendix 2

Applying the calculation for simple foam parts and verifying them with test results we used simple conical specimen for the verification of our calculation (see: Fig. 2).

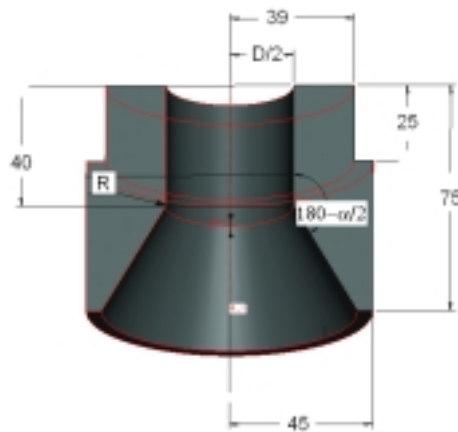


Fig. 2. The dimensions of the test mould

This simple shape was chosen because it has a large undercut and is axially symmetrical. In this case the changing of r (the radial component of the deformation) will only change with H (the variable changing with the height) and not with the radial parameter. The calculation was done in polar co-ordinates. The foam parameters given in Appendix 1 were used for the 48 Hz foam.

We have performed many demouldings from the test moulds and measured the force in the function of the length on a tensile test machine. The results of the

test for the 48 Hz foam are seen in *Fig. 3*. The value of the frictional coefficient was chosen to be 0.65 according to factory values.

restart;

The start of the session.

$b := 1.752181818$;

The value of 'b' parameter from the calculated curve (48 Hz)

$a := -767.5144927$;

The value of 'a' parameter from the calculated curve (48 Hz)

$c := 679.9156974$;

The value of 'c' parameter from the calculated curve (48 Hz)

$R0 := 40$;

The radius at the bottom of the test mould in mms.

$RH := 20$;

The smaller radius of the conical section in mms.

$H := 35$;

The height of the cone in mms.

$eq_1 := h*(R0 - RH)/H$;

The deformation x as the function of h.

$$eq_1 = \frac{4}{7}h$$

$eq_2 := eq_1/R0*(Pi/2)$;

The deformation is transferred to the period of $\pi/2$.

$$eq_2 := \frac{1}{140}h\pi$$

$eq_3 := ((c*\tan(eq_2) + a*((eq_2)^b))/10000)$;

The material model from Appendix 1 for the 48 Hz foam.

$$eq_3 := .06799156974 \tan \frac{1}{140}h\pi - .00001332539408(h\pi)^{1.752181818}$$

$F := 2*Pi*(\text{int}(eq_3, h = 0..H)) * 0.65 * RH$

The integral calculating the demoulding force.

$$F := 10.68254423\pi \approx 34 \text{ N}$$

The result of the calculation is approximately $F = 34 \text{ N}$. The maximal force on the Force-displacement curve (see: *Fig 3*) is $\approx 35 \text{ N}$. The calculated result came very close to the real life one.

If the parameters of the production are not set precisely (release agent is not applied properly, the temperature of the mould is higher/lower than the optimum, the ratio of mixture of the PU is not set according to the factory standard, etc.) the demoulding force can be much greater than the calculated one. This is because large

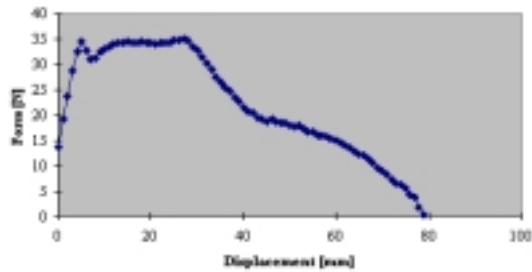


Fig. 3. The Force-displacement diagram of the 48 Hz foam from the test

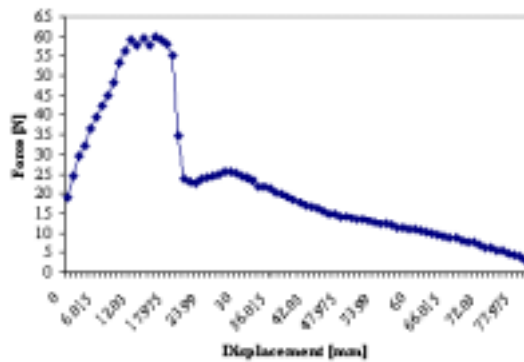


Fig. 4. Force-displacement curve when the manufacturing parameters are not set properly

adhesion force acts between the mould and the foam. This is added to the already present deformation force. To take this force into consideration a new model is needed, which requires further investigations. We reckon that by proper care in the production the large adhesion can be avoided.