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Estimation of River Discharge Outside the Regime of Uniform Flow

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Abstract

The flow discharge computation is based on assumptions about the energy grade line slope and the water level to estimate the stream flow with the hydraulic model of Manning's equation. The purpose of this paper is to improve the river discharge prediction using a modified practical Manning's equation for non-uniform but also for uniform flow conditions by using free surface slope, bed slope, friction coefficient and the flow depth. This paper presents an improvement in the integration of the average velocity in gradually varied flows. To validate the modified equation, experimentations have been carried out in the hydraulic laboratory of Ain Temouchent University (Algeria) for 23 flow discharges through two measurement sections; we have got 46 values of the average velocity, the bed slope and free surface slope. The results compare very well with the experimentations. A very good agreement has been found between measured and computed velocities with a coefficient of determination R^2 (97.91% for section A and 99.45% for Section B) with the normalized RMSE low than 2.4%. This could be a very useful contribution for the literature. No closer-form of this modified Manning equation is available in the literature.

Keywords

flow discharge, modified Manning equation, uniform and non-uniform

1 Introduction

The estimation of river flows continues to be an important subject in various issues including water resources management especially for extreme events such as floods and droughts; design of hydraulic structures; reservoir operation and river regulation [1]. The river flow propagation and the flow characteristics are critical for flood prevision and watershed modeling [2].

River discharge is very important info for several scientific and operational applications; this parameter is expressed in terms of water level variations mistreatment mathematical formulas or calibrated relationships, knowing as rating curves. A rating curve is established by concurrent measurements of rate and water levels, and a curve is fitted through the measured hydraulic variables [3]. Therefore, there's a great need for permanent continuous discharge knowledge.

For that purpose, ancient ways of quantification may be used. However, these ways offer point-based estimates of hydrological parameters [4].

The Manning equation is habitually used as a flow predictor in hydraulic models of open channel flow. However, the estimation of flow rate and discharge are subject to significantly uncertainty and error due to difficulties to define the energy gradient and roughness constant.

In natural streams, non-uniform flow may be a ubiquitous phenomenon that's not fully understood even for the case of gradually varied non-uniform flow in open channel [5].

Computations of the gradually varied flow profiles are of significant importance to hydraulic engineers [6]. In most of practical cases, a rigorous computation of the water elevation in varied sections of the channel is required, either numerically or analytically [7].

In a channel when the slope, the section, the roughness and flow rate are constant, always the uniform regime finally is established. Friction losses in this regime are fully offset by the bottom slope. The presence of a singularity (narrowing, widening, the threshold discontinuity, etc.,) not only causes a localized loss of energy, as in the pipe flow, but also a change of the free surface [8].

The water surface profile of the gradually varied flow (GVF) is function of the channel slope and other conditions.

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The GVF is formed at and around abrupt transition, sluice gate, weir, hydraulic jump, end of channel (over fall) and at the modification of channel slope [9]. Water surface profile calculations are required to get the water depth in a distance measured from an arbitrary reference. Several procedure methods take into account each profile individually, however the profiles computed by the dimensionless forms are applicable for all flows that have a similar Froude number. Thus, the dimensionless strategies greatly reduce the computational effort [10].

The average velocity in open channel flow is used to estimate shear stress, erosion and prevention against floods, the most famous equation that allows to get the average velocity is the Chezy's equation in 1768 or the very similar equation of Manning in 1890. Chezy's Equation resolves forces and hydraulic head gradients in the case of open channel flows to get the average velocity. Manning's Equation is very similar to Chezy's Equation, but it was derived empirically.

The Manning equation is used to estimate the average velocity and hence the discharge, in different flow configurations (even for the non-uniform flows), although this equation was originally developed for uniform equilibrium flows only [11] because the bed slope, the free surface slope and the energy grade line slope are even. However, in the case of non-uniform flow, there is no practical form of this equation to get the river discharge (even if we do have the bed slope and the free surface slope but without the mean velocity; we cannot get the EGL slope or the flow discharge!).

The focus of the present research is on improving the river flow prediction a new practical Manning's equation for uniform but also for non-uniform flow condition.

2 Theoretical background

2.1 Uniform and non-uniform flows

Steady flow refers to flow conditions that don't change with time. Steady flows are often either uniform or non-uniform [12]. The river flow is called uniform flow when the flow velocity at a given instant is constant within a given length of channel, otherwise, the flow is called non-uniform i.e., once the flow velocity at a time varies, this flows also are referred to as varied flows. [13]

This classification is also relying on the variation of flow rate with respect to space at a specified instant of time. Thus, the convective acceleration in uniform flow is zero. In mathematical terms, the partial derivatives of the velocity projections with in the x, y, and z direction are all zero. However, many times this restriction is relaxed by allowing a non-uniform velocity distribution at a channel section. In different words, a flow is considered uniform as long as the velocity within the direction of flow at different locations along a channel remains constant [13].

To get a uniform unsteady flow, the depth would have to increase (or decrease) at constant rate throughout the entire length of channel so at all ties neither the depth nor the velocity changes with respect to position, however is constantly dynamic with time. Mathematically, a flow is uniform if $\delta y/\delta x = 0$, $\delta v/\delta x = 0$, $\delta A/\delta x = 0$, etc., throughout the flow [14].

Steady uniform flow implies no modification in hydraulic conditions. For steady uniform flow, shear stress and resistance relationships can be directly written as a function of bed slope. For a uniform open channel flow, the shear forces (flow resistance) exactly balance the gravity force component [12].

Depending upon the rate of variation with respect to distance, flows may be classified as gradually varied flow or rapidly varied flow. gradually varied flow occurs when the flow depth varies at a slow rate with respect to distance, although the flow is called rapidly varied flow [13].

2.2 Gradually varied flows

Steady uniform flow is the simplest form of open channel flow to analyses because of the absence of flow acceleration, since the flow isn't changing in space or time, there's no net force acting on the flow and lots of the terms within the governing equations is set to zero. Although most open channel flow situations are approximated by steady flow, uniform flow isn't attained.

It can only be expected to occur in long, straight channels with unvarying cross-section form and roughness characteristics since only in such conditions can a balance be expected to develop between the friction force retarding the flow and the component of gravity driving the flow down the slope. Since most channels are characterized by highly variable channel geometries and boundary roughness, this balance of forces rarely happens, and flow depths and velocities vary spatially and that we got non-uniform flow conditions. However, as long because the flow non-uniformity isn't too great, it's commonly assumed that uniform flow formulae is used without significant error [15]. The accuracy in flow profile predictions depends on hydraulic computations based on one- dimensional gradually varied flow models [16].

In addition, the stream flow estimation method uses the open channel energy equation for steady non-uniform flow analysis in a nonprismatic channel, such as a natural river. The Manning equation with the EGL form still also applied in the non-uniform flow but there is no practical form in this case.

In most practical cases, the cross-section, depth, and velocity in a channel varies along the channel and therefore the uniform flow conditions are not often reached [11].

Uniform flow during which the flow depth remains constant with distance, such flows occur only in long and prismatic channel. In real-life projects, however, this flow condition exists rarely as a result of the created channels has to suit the existing topographical conditions for economic reasons. These changes within the channel geometry generate non uniform flows condition while changing from one uniform-flow condition to a different.

Because of the streamlines are more or less straight and parallel, the pressure distribution in gradually varied flow could also be assumed hydrostatic [13].

When the radius of curvature of the streamlines is large, e.g., the streamlines are nearly straight, such that the normal component of acceleration are often neglected, a non-uniform flow are going to be referred as gradually varied., the pressure can increase within the vertical direction in a gradually varied open channel flows just as it does in the same fluid for a uniform flow [14].

If the changes of depth in a varied flow as gradual so that the curvature of streamlines is not excessive, such a flow is said to be a gradually varied flow (GVF).

The demonstration of Chézy equation is made with uniform flow condition [11, 12, 17] but only Chanson [11] have recommended this equation for both uniform and gradually varied flow and we have found at the end of his demonstration that: "two similar equations were deduced (in the same conditions) but one is valid for uniform equilibrium flow and is not valid for non-uniform flows for which the second equation should be used.

Three flow resistance equations commonly used, the Darcy–Weisbach, Chezy and Manning equations. They all have a similar form.

3 Methodology

3.1 The modified Manning equation

The first stream flow-estimation method uses the Chezy's formula from 1768, which is a typical formula for calculating the average flow of a natural water stream but Manning's equation from 1891 is widely used in open channel flows according to [18–20] because it can simplify one-dimensional Saint-Venant momentum equations of partial differential form into a uniform-flow condition. Because no acceleration in the stream or changes in the water depth when assuming steady uniform flow, the friction slope can be described as a riverbed slope.

In fluvial hydraulics, uniform flow is taken as the base (reference) for all other considerations, and this despite the fact that truly uniform flow is rarely encountered in reality [17].

In addition, equations developed under these assumptions are not applied on complex cases. In the case of open channel flows, for us, the non-uniform flow (when the EGL slope is different of the bed slope or the free surface slope) is the general case and when all those slope are even (uniform flow) this is a particular case of flow (Fig. 1). The purpose of this paper is to get a practical form of the Manning equation with the non-uniform flow as the general case and when the bed slope and the free surface slope are the same, this modified Manning equation turns into the classic Manning equation for which the average flow velocity of a river channel can be expressed as:

$$V = \frac{1}{n} R_h^{2/3} S_0^{1/2} , \tag{1}$$

where V is the average flow velocity in the stream section (m/s), R_h is the hydraulic radius (m), and S_0 is the river-bed slope (m/m). If the shape of the stream section is known, the cross-sectional flow area and hydraulic radius can be expressed as a function of the flow width. Uniform stream flow can be estimated from the flow width, as:

$$Q = \frac{A(w)}{n} R_h^{2/3}(w) S_0^{1/2} . \tag{2}$$

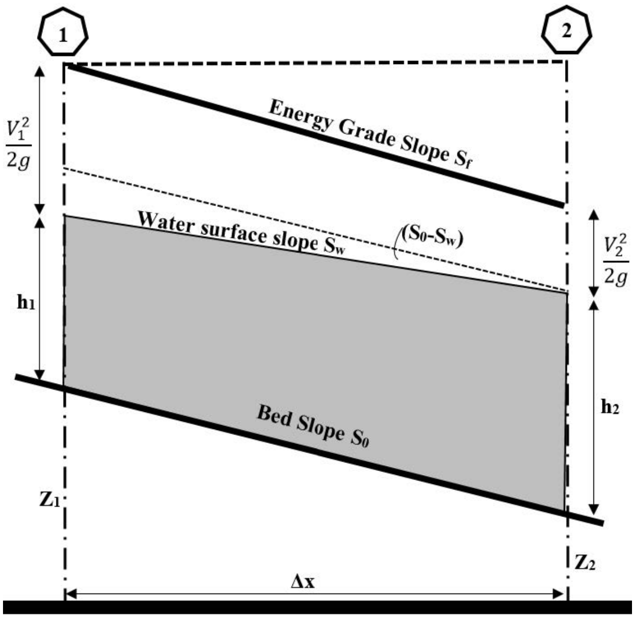

Fig. 1 Free surface flow

It is important for engineers and designers to know the value of the average velocity in any channel under all flow conditions. In the case the uniform flow, the bed slope is the same as the EGL slope, so the Manning equation is already in practical form. However, in the case of gradually varied flows, there is no such equation. In this section, we provide a practical modified Manning equation that improves the discharge estimation.

Starting with the well-known governing equation for steady, gradually varied flow (GVF):

$$\frac{dh}{dx} = \frac{S_0 - S_e}{1 - Fr^2} , \tag{3}$$

where h is the thalweg depth, x is the along-channel (thalweg) coordinate, S_0 is the bed slope, S_e is the energy grade line (EGL) slope, F_r is the Froude number.

$$Fr^2 = \frac{Q^2 B}{gA^3} = \frac{U^2 B}{gA} , \tag{4}$$

where Q is the volumetric discharge, U is the averaged velocity, B is the width, g is the gravitational acceleration, and A is the cross-sectional area.

This equation may be solved for the GVF by assuming that the water surface slope, S_w, is locally known and relating dh/dx geometrically to S_w and S_0 via

$$\frac{dh}{dx} = S_0 - S_w . \tag{5}$$

In addition, the slope of the energy grade line (EGL) may be sufficiently parameterized via the results of Manning for uniform flow as:

$$S_e = \frac{U^2}{K_s^2 R_h^{4/3}} \tag{6}$$

From Eqs. (3) and (5) we get:

$$(S_0 - S_w) = \frac{S_0 - S_e}{\left(1 - Fr^2\right)} , \tag{7}$$

$$(S_0 - S_w)\left(1 - Fr^2\right) = (S_0 - S_e) . \tag{8}$$

Adding Eqs. (4) and (6) in Eq. (8) we get:

$$(S_0 - S_w)\left(1 - \frac{U^2}{gR_h}\right) = \left(S_0 - \frac{U^2}{K_s^2 R_h^{4/3}}\right) . \tag{9}$$

Finally,

$$(S_0 - S_w) - \frac{U^2}{gR_h}(S_0 - S_w) = S_0 - \frac{U^2}{K_s^2 R_h^{4/3}} , \tag{10}$$

$$\frac{U^2}{gR_h}(S_w - S_0) + \frac{U^2}{K_s^2 R_h^{4/3}} = S_w , \tag{11}$$

$$U^2\left(\frac{1}{K_s^2 R_h^{4/3}} + \frac{(S_w - S_0)}{gR_h}\right) = S_w , \tag{12}$$

where K_s is the Strickler coefficient and R_h is the hydraulic radius ($R_h = h$). Note that this latter assumption is universally made in treatments of GVF. Solving Eq. (12) for U, we obtain

$$U^2 = \frac{S_w}{\left(\frac{1}{K_s{}^2 R_h{}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right)} . \tag{13}$$

When the free surface level slope and the bed slope are the same (uniform flow), this new equation turns into the Manning-Strickler's equation. Also, from Eq. (10) we get:

$$Fr^2 = \frac{(S_w - S_e)}{(S_w - S_0)} \text{ and } \frac{h_c^3 - h_n^3}{h_c^3 - h^3} = \frac{S_w}{S_0} . \tag{14}$$

3.2 The energy conservation equation

The total energy head at a point in an open channel is the sum of the potential and kinetic energy of the flowing water. The energy per unit weight is the sum of pressure head or static head, velocity head or kinetic head and potential head.

Energy equation of the gradually varied flow shows that the analytical integration of the average velocity performed on top gives the same modified Manning equation for both uniform and non-uniform flows.

There are two cases for the gradually varying flow, accelerated flow and decelerated flow. For the first case, the free surface level slope (S_w) is less than the bed slope (S_0), in the second case the free surface level slope (S_w) is higher than the bed slope (S_0).

For the two flow configurations, we have chosen two sections with a short distance between them, and we have applied the energy conservation equation.

Case 1 (Fig. 2): Accelerated flow "when $S_w \geq S_0$, $h_1 \geq h_2$"

Bernoulli's equation between Eqs. (1) and (2):

$$Z_1 + h_1 + \frac{U_1^2}{2g} = Z_2 + h_2 + \frac{U_2^2}{2g} + \Delta h_L , \tag{15}$$

$$Z_1 - Z_2 + h_1 - h_2 + \frac{U_1^2}{2g} - \frac{U_2^2}{2g} = \Delta h_L , \tag{16}$$

with $Z_1 - Z_2 = S_0 \Delta x$ and $h_1 - h_2 = (S_w - S_0)\Delta x$.

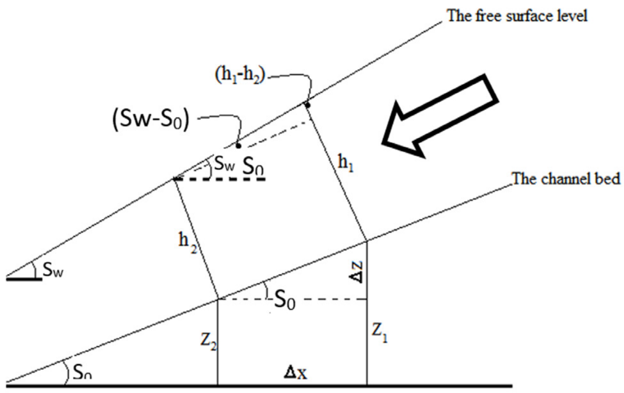

Fig. 2 Non-uniform flow configuration ($S_w \geq S_0$ and $h_1 \geq h_2$)

Also, we know that

$$U_1 = \frac{q}{h_1} \text{ and } U_2 = \frac{q}{h_2} . \tag{17}$$

With q is the flow rate per unit width (m^3/s/m), so Eq. (16) becomes:

$$S_0 \Delta x + \left(S_w - S_0\right) \Delta x + \frac{q^2}{2g}\left(\frac{1}{h^2{}_1} - \frac{1}{h^2{}_2}\right) = \Delta h_L , \tag{18}$$

$$S_0 \Delta x + \left(S_w - S_0\right) \Delta x + \frac{q^2}{2g}\left(\frac{\left(h_2 - h_1\right)\left(h_1 + h_2\right)}{h_1^2 h_2^2}\right) = \Delta h_L , \tag{19}$$

$$S_w \Delta x + \frac{q^2 \left(S_0 - S_w\right) \Delta x}{2g} \frac{\left(h_1 + h_2\right)}{h_1^2 h_2^2} = \frac{\lambda \Delta x}{D} \frac{U^2}{2g} . \tag{20}$$

With $D = 4R_h$

$$S_w + \frac{q^2 \left(S_0 - S_w\right)}{2g} \frac{\left(h_1 + h_2\right)}{h_1^2 h_2^2} = \frac{\lambda}{4R_h} \frac{U^2}{2g} , \tag{21}$$

also

$$\frac{\lambda}{8g} = \frac{1}{{K_s}^2 {R_h}^{2/6}} \tag{22}$$

and

$$q = u \cdot h , \tag{23}$$

where K_s Strickler friction coefficient.

Notice also that as Δx gets smaller $h_1 = h_2$, so we got:

$$S_w + \frac{U^2 h^2 \left(S_0 - S_w\right)}{2g} \frac{\left(2h\right)}{h^4} = \frac{U^2}{{K_s}^2 {R_h}^{4/3}} , \tag{24}$$

$$S_w + \frac{U^2 \left(S_0 - S_w\right)}{gh} = \frac{U^2}{{K_s}^2 {R_h}^{4/3}} , \tag{25}$$

$$S_w = \frac{U^2}{{K_s}^2 {R_h}^{4/3}} + \frac{U^2 \left(S_w - S_0\right)}{gh} , \tag{26}$$

$$S_w = U^2 \left(\frac{1}{{K_s}^2 {R_h}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right) , \tag{27}$$

$$U^2 = \frac{S_w}{\left(\frac{1}{{K_s}^2 {R_h}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right)} . \tag{28}$$

For the uniform flow $S_w = S_0$, we get:

$$U^2 = \frac{S_w}{\left(\frac{1}{{K_s}^2 {R_h}^{4/3}}\right)} . \tag{29}$$

$U^2 = K_s^2 R_h^{4/3} S_w$ known as Gauckler-Manning-Strickler's formula.

Case 2 (Fig. 3): Decelerated flow "when $S_w \leq S_0$, $h_1 \leq h_2$"

The difference in this case is that $h_1 \leq h_2$ so:

$$h_2 - h_1 = \left(S_0 - S_w\right) \Delta x . \tag{30}$$

Bernoulli's equation between Eqs. (1) and (2):

$$Z_1 + h_1 + \frac{U_1^2}{2g} = Z_2 + h_2 + \frac{U_2^2}{2g} + \Delta h_L , \tag{31}$$

$$Z_1 - Z_2 + h_1 - h_2 + \frac{U_1^2}{2g} - \frac{U_2^2}{2g} = \Delta h_L , \tag{32}$$

$$S_0 \Delta x + \left(S_w - S_0\right) \Delta x + \frac{q^2}{2g}\left(\frac{1}{h^2{}_1} - \frac{1}{h^2{}_2}\right) = \Delta h_L , \tag{33}$$

$$S_w + \frac{q^2 \left(S_0 - S_w\right)}{2g} \frac{\left(h_1 + h_2\right)}{h_1^2 h_2^2} = \frac{\lambda}{4R_h} \frac{U^2}{2g} . \tag{34}$$

In the same way as case 1, we get:

$$S_w = \frac{U^2}{{K_s}^2 {R_h}^{4/3}} + \frac{U^2 \left(S_w - S_0\right)}{gh} , \tag{35}$$

$$S_w = U^2 \left(\frac{1}{{K_s}^2 {R_h}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right) , \tag{36}$$

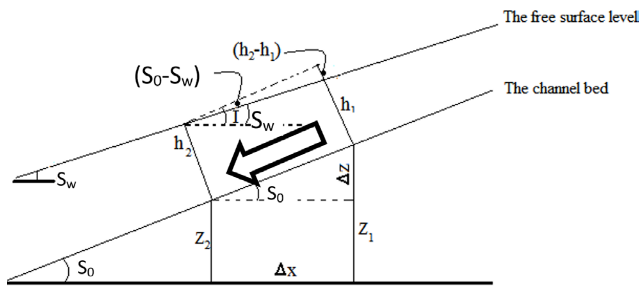

Fig. 3 Non-uniform flow configuration ($S_w \leq S_0$ and $h_1 \leq h_2$)

$$U^2 = \frac{S_w}{\left(\frac{1}{K_s{}^2 R_h{}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right)}. \tag{37}$$

Finally, the demonstration shows that for both configurations ($S_w > S_0$ and $S_w < S_0$) a single equation was derived and we can write that the average velocity is

$$U^2 = \frac{S_w}{\left(\frac{1}{K_s{}^2 R_h{}^{4/3}} + \frac{\left(S_w - S_0\right)}{gh}\right)}. \tag{38}$$

This relationship gives the average velocity based on the geometric parameters (h and R_h), topographic parameters (S_w and S_0) and mechanical parameters (K_s). Experiments were carried out to validate this new equation in the hydraulic laboratory of Ain Temouchent University.

4 Experimentations

The experiments were performed in the hydraulic laboratory of Ain Temouchent University (Algeria) in the flume sketched in Fig. 4 under different slope ranges to validate the equation, the channel is 2.5 m long, 0.086 m wide and 0.3 m deep, with transparent walls made from thick glass. A constant head pump was used for the water recirculation in the flume. Control of the discharge from the pump was achieved with an adjustable valve in the flume inlet pipe

The hydraulic conditions studied in this work were designed to investigate the change in the velocity patterns as a function of the flow depth and channel slope for a range of flow conditions. The discharge is adjusted by the flow rate valve and displayed on a rotameter. The rotameter has been calibrated using a venturi mounted at the inlet, the flow rate of the two instruments was compared, and the rotameter calibrated.

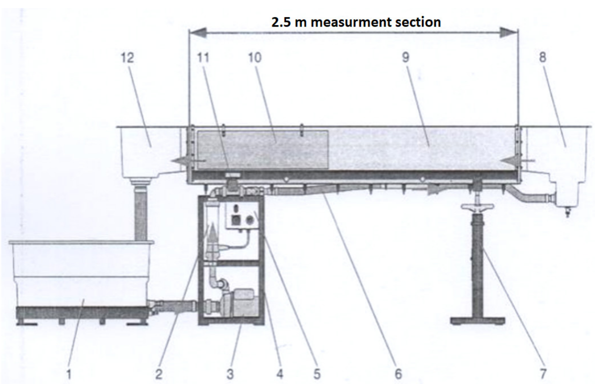

Fig. 4 Experimental flume; 1 Water tank, 2 Flow meter, 3 Pump, 4 Fixed mount, 5 Control cabinet, 6 Hose line, 7 Floating mount with inclination adjustment, 8 Inlet section, 9 Intermediate section, 10 Grid board, 11 Flow rate valve, 12 Outlet section

We have used the hand wheel (under the channel) to set the inclination adjustment to different values and we have used independent topographic instruments to get exact channel slope and free water surface slope.

Confident selection of values of the Manning roughness coefficient n usually requires considerable attention. A review of the literature on flow-resistance investigations in open channels in the case of glass or Perspex (materials of the experimental channel) shows that many authors consider a value between 0.009 and 0.012 for the Manning-Strickler friction coefficient.

Lencastre [8], Martins [21], Richards [22] and other authors consider that $n = 0.01$ for very smooth surface, for Obradovic and Lonsdale [23] the Manning-Strickler friction coefficient for glass varies between 0.009 and 0.012.

In addition, we have calculated n (and K_s) experimentally by running a uniform flow and we have found that the Manning-Strickler friction coefficient for the experimental flume compare very well with the literature. For this study we have taken $n = 0.01$ ($K_s = 100$).

4.1 The average velocity in gradually varied flow

The velocity varies in a channel section due to the friction forces on the boundaries and the presence of the free-surface.

At any point in an open channel, the flow have velocity components in all three directions. However, open-channel flow is assumed one-dimensional. Therefore, by velocity we usually refer to the velocity component in the main flow direction.

The focus of the present research is on improving the river flow prediction a modified practical Manning's equation for non-uniform but also for uniform flow condition. For the validations purposes, we have worked on an experimental channel in the hydraulic laboratory.

The major part of the experimental task was to compare the theoretical results given by the modified Manning equation and the experimental values of the average velocity measured in the laboratory.

We have first fixed the channel slope S0 using topographic instrument independently from the channel, then we have run the experiments with a fixed discharge (measured with two instruments: rotameter and venturi), after stabilization of the flow, we have measured different water depth at different sections.

Knowing the flow discharge and the cross section area (water depth and the channel width), we have computed the experimental average velocity (U_{exp}).

Then we have used the same value of water depth to compute the Hydraulic radius, the free surface level slope (S_w) and using the modified equation developed in this paper we have got the theoretical average velocity (U_{th}).

$$U_{th} = \sqrt{\frac{S_w}{\left(\frac{1}{K_s^{\,2} R_h^{\,4/3}} + \frac{(S_w - S_0)}{gh}\right)}} \tag{39}$$

4.2 The prediction model performances

For the prediction to be accurate, the model that is put into place must be carefully validated. The objective is to prove that the model generates good estimates of the variable to be predicted.

The root mean square error (RMSE) is a standard way to measure the error of a model in predicting quantitative data. It provides an indication regarding the dispersion or the variability of the prediction accuracy. It can be related to the variance of the model. Often, the RMSE value is difficult to interpret because one cannot tell whether a variance value is low or high.

To overcome this issue, it is more interesting to normalize the RMSE so that this indicator can be expressed as a percentage of the mean of the observations. It can be used to make the RMSE more relative to what is being studied.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(V_{\exp} - V_{th}\right)^2}{n}} \tag{40}$$

The results of evaluating the performance of theoretical methods shows that the RMSE is 2.4% in the Section A and 0.3% in the Section B, these results ensured the accuracy of the experimental measurements.

4.3 Results and discussions

The Manning equation is used to estimate the average velocity and hence the discharge in different flow configurations (even for the non-uniform flows), although this equation was originally developed for uniform equilibrium flows [11] because the bed slope, the free surface slope and the energy grade line slope are even. However, in the case of non-uniform flow, there is no practical form of this equation to get the river discharge (even if we do have the bed slope and the free surface slope but without the average velocity; we cannot get the energy grade line slope and hence the flow discharge).

The aim of this study was to provide a practical modified Manning equation that improves the discharge estimation in the field.

In this paper, the Analytical Integration of the equation of the average velocity in gradually varied flows allowed the development of a modified Manning equation for the average velocity which is function of the bed slope, the free surface level slope and the flow depth for uniform and gradually varied flows.

In the case of the GVF, the field computations as important as they are tough to make without the Energy grade line slope (EGL) and the mean velocity in the same time.

The experimental protocol used in this study assumes that for each flow discharge we have got two water depths in two sections with 1m distance between.

For the comparison purposes, we have plotted the computed and observed mean velocities for both Manning-Strickler and the formula developed in this paper in Fig. 5.

In the case of low bed slope in gradually varied flow and even if the Manning-Stricker equation in the first form is not very precise using the bed slope instead of the Energy grade line, but Fig. 5 shows that our new formula gives flow velocities in very good agreement with the observations for a limited number of points that verify the low slope condition.

In GVF, the water surface profile can be calculated from downstream going upstream (backwater calculations) or from upstream going downstream (front water calculations). The usefulness of our new formula is that even is such situation (GVF) we can get the flow velocity and hence the flow discharge using one formula.

In the second part of this paper, we have carried out experiments for 23 flow discharges then we have chosen two sections with different flow depth for each flow rate, we have got 46 values of the average velocity.

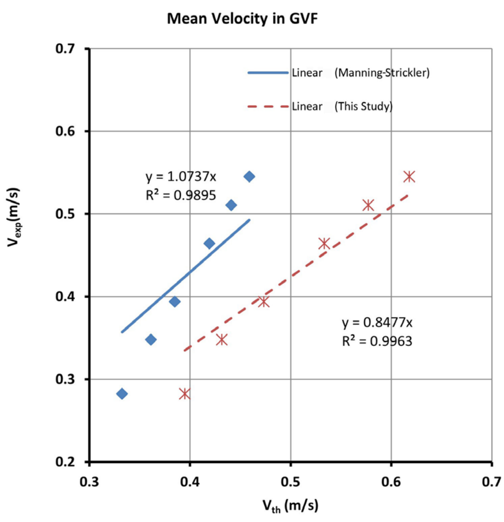

Fig. 5 Mean velocity comparison in GVF

In the Fig. 6 and 7 bellow we have plotted the theoretical average velocity computed using the Eq. (37) (since we have for each experiment the depth h, the bed slope S_0, the free surface level slope Sw and Manning-Stricker coefficient n or K_s) and the experimental average velocity computed by dividing the flow rate on the section area.

In Fig. 6, we have the scatter plot concentrated in the $y = x$ zone but even if there is a good agreement between measured and computed velocities, the trend line of $y = x$ is not very representative of the scatter plot, and we have a coefficient of determination less than the other linear models ($y = a\,x$).

For the second part, a very good agreement between the two velocities have been observed, all measurements are aligned in the axe $Y = AX$. The trend line that represents better the scatter plot is $V_{exp} = 0.94V_{th}$ with a correlation coefficient of about 99%.

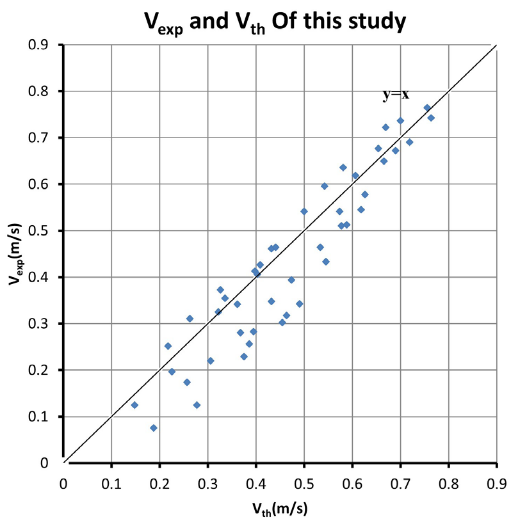

Fig. 6 Theoretical and experimental average velocity (trend line 1)

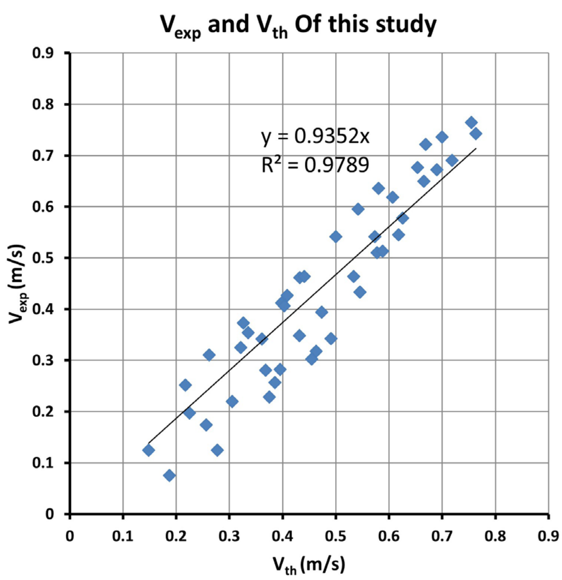

Fig. 7 Theoretical and experimental average velocity (trend line 2)

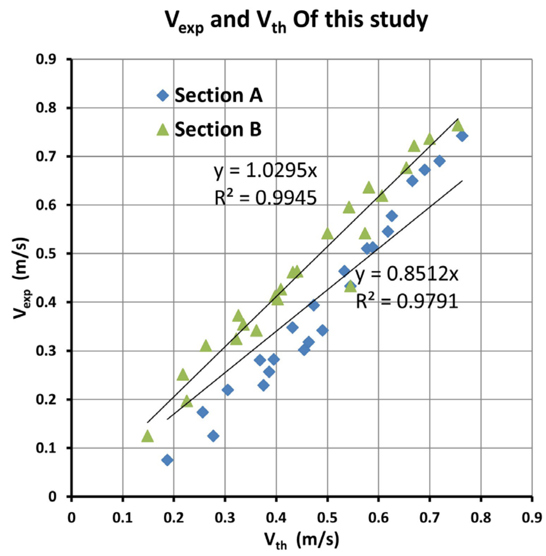

Fig. 8 Theoretical and experimental average velocity for each flow section

Second, we have plotted the velocities for every section alone, we have 23 values for the first Section A and 23 values for the second section with "1 m" between the two sections, notice that these sections are used along the experiments and kept in the same position for all the flow configurations and the slopes are calculated using the topographic instruments.

In Fig. 8, we have plotted the two sections velocities separately. The trend line sketched for the 23 values of the velocities at the first section A have given a very good correlation coefficient of about 99.72% with $V_{exp} = 1.029V_{th}$ which is an important result in this research. In other hand, at the second section B the correlation coefficient is about 98.94% with $V_{exp} = 0.85V_{th}$.

Even if many precautions have been taken for more accuracy in the experimentations, but we thought that the uncertainties are due to the instruments and channel dimensions.

No similar form exists for this modified Manning equation in the literature, and this could potentially be a very useful contribution, allowing the estimation of river discharge.

Finally, the remote sensing could use this formula to estimate the flow discharge basing on the database of rivers bed slope and roughness. The new formula limitations could be the large-scale roughness and the high bed slope, and this will be investigated in further research.

5 Conclusions

Monitoring and assessment of hydrological parameters are the key elements for the sustainable development of water resources of any country. The various components of the hydrological cycle are highly changing in space and time. Quantification of hydrological parameters using traditional methods provides limited, point based, information, which is not sufficient for assessing spatio-temporal variations in these parameters.

The average velocity in open channel flows is a very important parameter; it is used to estimate shear stress, erosion, and prevention against floods. Three flow resistance equations commonly used: the Darcy–Weisbach, Chezy and Manning equations. They all have a similar form.

The solution of the gradually varied flows equation allows the tracing of the flow depth along a channel length.

The purpose of this paper was to get a practical form of the Manning equation with the non-uniform flow as the general case and when the bed slope and the free surface slope are the same, this modified Manning equation turns into the classic Manning equation

In this paper, the Analytical Integration of the equation of the average velocity in gradually varied flows allowed the development of an analytical modified Manning equation for the average velocity in both uniform and gradually varied flows. It can be written as Eq. (13).

The theoretical results show that when the free surface slope and the bed slope are even, the equation developed in this paper turns into the classic Manning equation.

The experiments were performed in the hydraulic laboratory of Ain Temouchent University (Algeria) under different slope ranges to validate the equation, the channel is 2.5 m long, 0.086 m wide and 0.3 m deep, with transparent walls made from thick glass.

The results obtained by this analytical equation compare very well with the results of the experimentations. A very good agreement has been found between measured and computed velocities with a coefficient of determination R^2 (97.91% for section A and 99.45% for section B) with the RMSE low than 1%.

This could be a very useful contribution to the literature, allowing the estimation of river discharge.

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