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Multi-objective Optimization of Multi-cells Foam Mattress

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Abstract
Body pressure dispersion mattresses are useful tools for preventing pressure ulcers in patients with limited mobility who experience prolonged body weight-related compression loads at their body contact areas over time. The objective of this study is to propose and optimize a multicell finite element (FE) model of foam mattress to prevent patients from developing pressure ulcers (bed sores), by improving the contact pressure distribution on the upper mattress surface and immersion in the mattress. The NSGA-II multi-objective genetic algorithm was used to predict different configurations of cell materials to provide a more comfortable sleep. Our mattress model contains many cells (50 × 50 × 50), each of which can contain one of the nine different foam firmnesses. The NSGA-II algorithm attempts to combine the properties of soft and firm foams into a single mattress. however, the complexity and intersection of the fitness function objectives and the high number of possible chances forced the optimal solutions set to extend into the area under the result of foams that have a compressive strength between soft and firm. Based on the overall optimization results, the standard deviation ranged from 0.00325 to 0.00175 MPa and the maximum mattress immersion ranged from 50 mm to less than 20 mm. Mattresses with optimal configurations disperse body pressure smoothly to fit the patient's body shape.
Keywords
sleep comfort, sleep, quality beds, bed firmness, multi-cells mattress, foam mattress

1 Introduction

Pressure sores are the main problem common in patients with reduced mobility. They are caused by various parameters such as peak contact pressure, friction, moisture, shear forces, temperature, undernourishment, and restriction of blood flow, which limits the reconstruction of skin tissue [1]. Measures to treat pressure sores include proper distribution of body weight, reduction friction, improvement of ventilation and use of hyperelastic materials such as rubber and polymer foam [2].

The most common material used in mattresses is flexible polyurethane foam (FPF). It can be modified in many ways to adjust the different degrees of comfort and firmness by varying the composition, density and microstructure of the material [3].

The quality of the mattress is considered one of the most essential elements for sleep comfort, as the mattress is in direct contact with the human body and is the main component and useful tool for pressure dispersion and the prevention of bedsores.

There are two categories of mattresses: reactive support surfaces or low-tech mattresses, including standard foam mattresses, and active support surfaces or high-tech mattresses, which use a combination of various air cells [4–7].

The limited medical budget makes it necessary to look for more economical mattresses [8]. Cost-effectiveness analyses have shown that passive mattresses (e.g., high-performance foam) are more effective in preventing pressure ulcers than standard foam mattresses, but cost significantly less than high-tech or dynamic mattresses [9].

Experimental testing of body pressure distributions for various parameters and designs is expensive and time consuming. Simulations using finite element (FE) models were used to solve these problems and predict the various comfort parameters of the mattresses [10–13].

Many authors and researchers have devoted considerable research efforts to predict sleep comfort and contact pressure between the human body and a mattress cushion. Some of these are limited by the mechanical geometry of the model, whereas others are limited by the material models used [2, 4, 14–22].

Lee et al. [22] proposed a finite elements (FE) model to evaluate and predict the contact pressure between a foam

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mattress and the human body in the supine position to study sleep comfort.

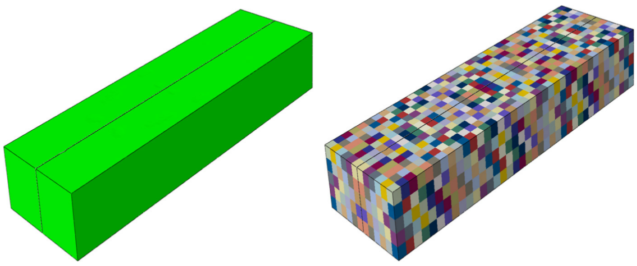
In the present work, we aim to optimize the configuration of multi-cell foam mattresses using NSGA-II genetic algorithms. Our mattress model consists of 1140 cells, each of which can accommodate one of the nine different foam firmnesses (see Fig. 1).

Based on the multi-objective genetic algorithm NSGA-II, we generated a different configuration of mattress cell materials to provide a comfortable bed by dispersing the contact pressure on the surface between the human body and the mattress, and minimizing the immersion of the mattress in the direction of the gravity load.

Many studies have investigated optimization using NSGA-II in collaboration with finite element methods in many areas of engineering, physics, and industrial applications [23, 24], such as medical and bioengineering applications [25–30].

In this context, we collaborate between FEA and genetic algorithm (NSGA-II) to develop and optimize multi-cell foam mattresses.

In this study, the finite element (FE) model is based on the work of Lee et al. [22]. The generated model predicts the contact pressure between a foam mattress and the virtual human body and the displacement of the entire mattress in the direction of gravity load in the supine position (see Fig. 2) [22]. The computation was based on a nonlinear



(a) (b)

Fig. 1 Geometries of the mattress with 1140 cells, (a) mattress geometry, (b) mattress cells

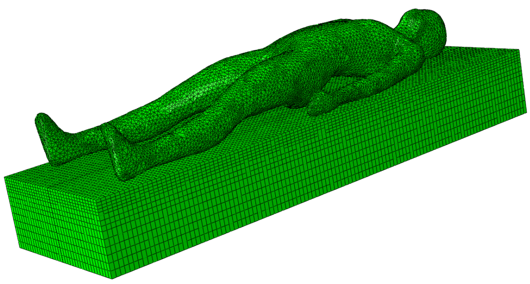


Fig. 2 FE model of the human laid on the mattress

finite element method with hyperelastic materials, such as muscle and polyurethane foam materials. A linear elastic isotropic material model was used for the skin.

2 Methods

2.1 Finite element model

A virtual 3D CAD model of the human body with a weight of 63 kg and a length of 1.67 m and a foam mattress with a length of 1900 mm, a width of 800 mm and a height of 250 mm were used as the objective function for the NSGA-II algorithm. The FE human models were placed in the supine position on the mattress, and the body gravity force was applied [22]. We used half of the symmetric FE model and reduce the width of the mattress to 300 mm, just below the human body, to minimize the computational cost (see Fig. 2).

2.2 Material properties

Our FE model consisted of three basic substances: skin, muscle, and flexible polyurethane foam. The skin was modeled as a linear elastic material. The muscle tissue was modeled using the Mooney-Rivlin model of isotropic hyperelastic materials [22].

The properties of the skin material were defined by the Young's modulus of 0.15 MPa, Poisson's ratio of 0.46, and density of 1100 kg/m^3 [22]. The Mooney–Rivlin model was used for muscle tissue because it can effectively represent the non-linear behavior of muscle. The material parameters of the Mooney-Rivlin muscle model were: $A_1 = 0.00165$ MPa, $A_2 = 0.00335$ MPa, and $v = 0.49$ [22].

The flexible polyurethane foam material was represented using a Hypefoam model [31]. The elastic strain-energy potential function U of the Hyperfoam model is used to present the relationship between stress and strain in Eq. (1):

$$U = \sum_{i-1}^{N} \frac{2\mu_i}{\alpha_i^2} \left[\hat{\lambda}_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 + \frac{1}{\beta_i} \left(\left(J^{el} \right)^{-\alpha_i \beta_i} \right) \right], \qquad (1)$$

μ_i, α_i and β_i are dependent material constants, that are determined by fitting the experimental stress-strain data curve. J^{el} is the volume ratio elastic. The λ_1, λ_2 and λ_3 are the principal stretches. N is the strain potential energy order, the coefficient β_i determines the degree of compressibility [32].

In Fig. 3 we represent the compressive strength (Nominal Stress-Strain Curve) of the nine foam materials used for mattresses' cells.

The compressive strength of flexible polyurethane foam (Nominal Stress-Strain Curve) is highly dependent on three main factors: the type of raw material (MDI or

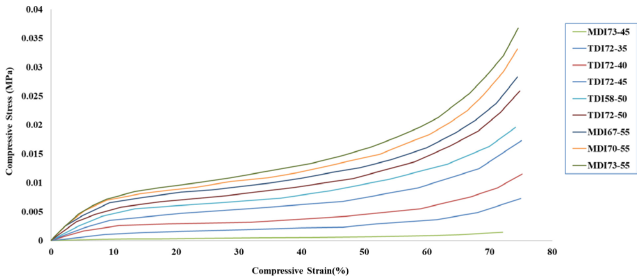


Fig. 3 Compressive strength of flexible polyurethane foam materials

TDI), density and proportion of the two main components of the foam (isocyanate and polyol) [31] (Table 1).

Fig. 4 shows the validation of the Hyperfoam material model for foam type MDI73_55 with Poisson's ratio equal to zero and the second order of the strain potential energy.

2.3 Boundary conditions, mesh and analysis step

Contact surfaces were defined between the human body and the mattress. The body surface (the outer skin surface) was defined as master surface and the top surface of the foam mattress as the slave surfaces.

The bottom surface of the mattress was fixed for translations and rotation in all the directions. The human body can only move vertically, towards the mattress, to simulate the immersion of the body in the foam of the mattress under the effect of gravity [2]. The coefficient of friction between the human model and the mattress was set to 0.4 [2, 22].

Table 1 Material data (Component proportion 45 means a proportion of between two components)

No.	Name	Isocyanate	Foam density [Kg/m^3]	Components proportion
1.	MDI73_55	MDI	73	55
2.	MDI67_55	MDI	67	55
3.	TDI72_45	TDI	72	45
4.	TDI72_40	TDI	72	40
5.	TDI72_35	TDI	72	35
6.	TDI58_50	TDI	58	50
7.	MDI72_35	MDI	72	35
8.	MDI73_45	MDI	73	45
9.	MDI70_55	MDI	70	35

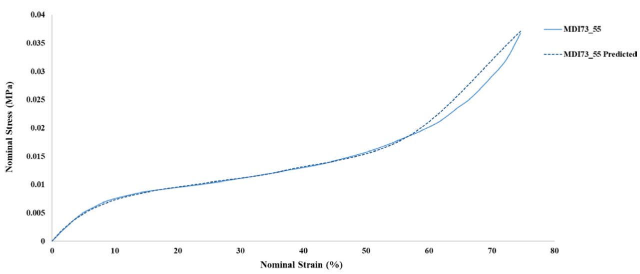


Fig. 4 Hyperfoam material model validation

The mattress mesh was graded from 10 mm from the top to 25 mm in from the bottom, and the mesh type was C3D8R. The skin mesh was generated as a solid part with a thickness of 2 mm of type C3D6 and a size of 15 mm. The mesh of the muscle tissues was generated by four-node tetrahedral solid elements, and the size of the solid elements was defined as 15 mm [22, 33].

The model was solved in a single analysis step, that is, the loading step using static analysis and all loading conditions were static. The generated model does not contain a skeleton [22].

The FE model was used to predict the standard deviation of the contact pressure to represent the dispersion of the human body weight on the mattress support surface and displacement of the mattress foam in the direction of the gravity load.

3 Optimization procedure

The NSGA-II is inspired by the well-known standard genetic algorithm GA and is considered one of the most powerful optimization algorithms. it is an improved version of the genetic algorithm and the Goldberg inspired non dominated sorting concept [34–40].

3.1 Fitness function

In this study, the fitness function of the multi-objective optimization problem was our FE model, with two objectives: the standard deviation of the contact pressure and the immersion represented by the displacement of the foam mat in the load direction.

The input of the fitness function is a chromosome as a vector with 1140 variables, represents the number of cells in our mattress model, and each variable in the vector input takes values in the range (1, 9) to represent a foam material used in a specific cell of our multi-cell mattress (see Fig. 1 (b)).

3.2 Coding of individuals

The individuals (chromosomes) are vectors of 1140 variables, each variable represents a cell in the foam mattress, and each variable takes an integer value between one and nine (1–9) to represent one of the nine compressive strengths of foam materials (Table 1). The integers is to represent the foam materials from the first one in Table 1 (MDI73_55) until the nineth foam material (MDI70_55).

4 Results and discussion

Nine different flexible polyurethane foams (FPF) were used to generate configurations of the mattress cell materials to minimize the standard deviation of contact pressure and immersion in the foam mattress under the load of a 63 kg human body in the supine position. We used half of the symmetric FE model and reduced the width of the mattress to 300 mm to minimize the computational cost.

Optimization was conducted to find a set of 90 different optimal configurations with nine different cell materials from over 4800 individuals' representative of the nondominant solution or Pareto front.

Fig. 5 shows the individuals evaluated during this lifetime and the solution space (Pareto front) representing the set of non-dominated individuals obtained after several trials over 25 generations and a population of 200 individuals.

Based on the overall optimization results, the standard deviation of the contact pressure ranged from 0.00325 to 0.00175 MPa, and the maximum mattress immersion ranged from 50 to less than 20 mm.

Fig. 6 shows the 90 solutions in the Pareto front, represented by the standard deviation of the contact pressure and displacement of the foam mattress in the direction of the gravity load.

From the 90 optimal solutions of the Pareto set, we can decide the preferred configuration for sleeping, the one that disperses the body load on the upper surface of the mattress or the one that has a low immersion of the human body.

In Fig. 7, we compare the result of the optimal configuration in the Pareto set with the result of the mattress with a single foam material (one foam compressive strength), to represent the comfort of the multi-cell mattress and make a decision about the effectiveness of this model on sleep comfort.

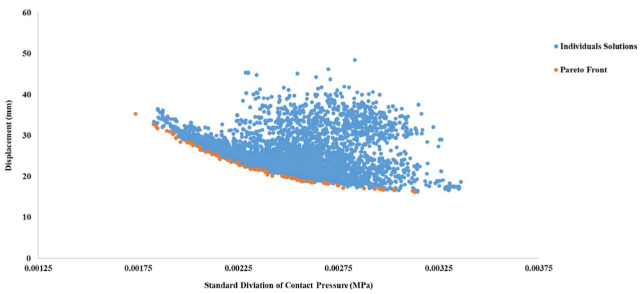


Fig. 5 Individuals' solution and Pareto front

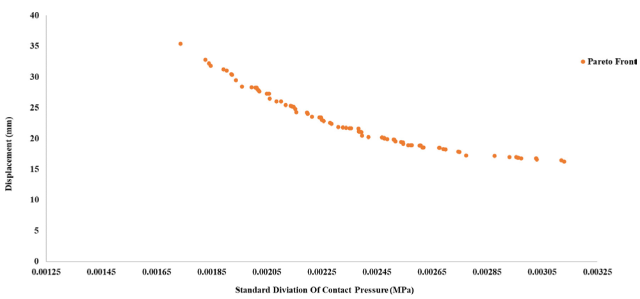


Fig. 6 Optimal Pareto solution spaces

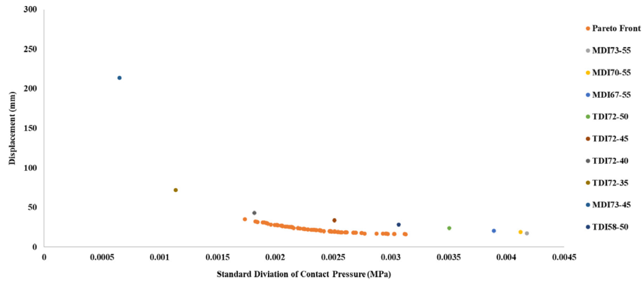


Fig. 7 Comparing the results of optimized foam mattress cell material with other foam mattress material

The result of the optimal set of individuals extends under the result of foam mattresses with a high classification (see Fig. 8).

In Fig. 9 we perform statistical analysis for the classification of materials that are highly used in mattress cells from the total number of cell materials in the 90 optimal solutions.

Fig. 9 shows the high classification foam used for mattress cells. The high classification foam used was the one that had a medium firmness between the softer (MDI-73-45 and TDI-72-35) and firmer foam (MDI-73-55) among the nine foam materials used in this optimization model.

When the firmness of the foam increased, the contact pressure increased, but the displacement decreased, when the firmness of the foam decreased, the contact pressure decreased and the displacement increased.

As a result, the NSGA-II algorithm attempts to find optimal solutions that combine the properties response of soft foam for dispersing the bodyweight on the support surface and firm foam to decrease the displacement of the

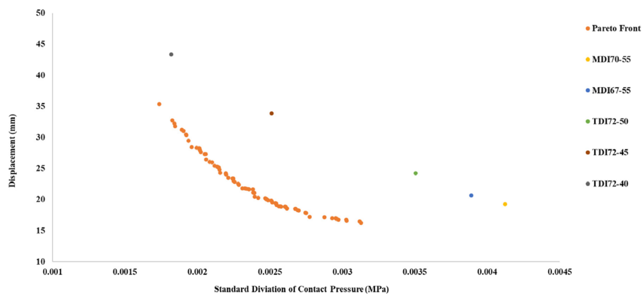


Fig. 8 The Pareto front set and the compressive strength foam high classification

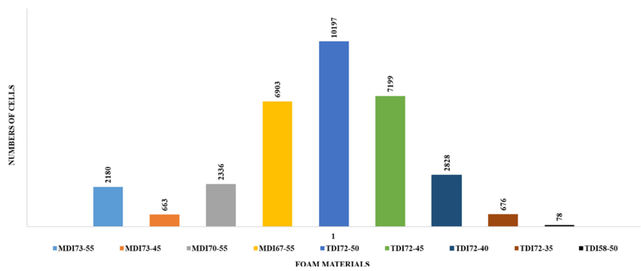


Fig. 9 Statistical analysis of compressive strength materials foam used in the optimal configuration cells for the mattress

entire mattress in the direction of gravity load. This is the reason why the Pareto set exists in the area below the medium firmness foams, and the algorithm eliminated softer foams and firmer foams.

We have two reasons for this result:

1. The first is depending on the number of possibilities can the algorithm calculate, and
2. the second depends on the model and the objectives of the model.

The number of calculations is not sufficient because the problem is very large, and the number of possibilities of configuration is very large, the number of possible iterations is equal to 1140 power 9 (114: number of cells in mattress, 9: number of foam materials), which means that any of the 1140 foam mattress cells can take one from nine different materials.

The second reason is depending on the finite element model; the computational cost of the FE model is more than 30 minutes for completion successfully, which makes the optimization take a long time to extract a good optimal set. And other reasons, depending on the objectives of the model itself (the contact pressure and displacement of the mattress), the contact pressure depends on the top surface of the foam mattress, and the displacement depends on the volume of the foam mattress. This constrains the FE model by two different objectives: one depends on the top surface of the foam mattress (contact pressure), and the other depends on the volume of the foam mattress. This problem influences the computational cost, and the NSGA-II algorithm requires more time and more iterations to find the optimal set of configuration cells.

5 Conclusion

The goal of this study was to determine the effect of multi-cell mattresses on sleeping comfort parameters. We presented an FE model of a human body lying on a multi-cellular foam mattress for multi-objective optimization using the NSGA-II algorithm.

Objectives included in this optimization, first: the dispersion of the bodyweight on the support surface, the second: the immersion of the foam mattress in the direction of gravity load.

The main objective was to improve the cell foam configuration by considering a decrease in the standard deviation of the contact pressure and immersion of the foam mattress.

The algorithm attempts to combine the properties of the softer foam which is stress dispersion in the contact, and the property of the firmer foam which is low immersion.

In future research, different FEM models of mattress foam will be proposed to reduce the computational efforts and the possible number of iterations by decreasing the total number of cells in the foam mattress to reach the optimal set faster combining between multilayering and multi-celling in one mattress model to achieve the better results for sleeping comfort.

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