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# The Critical Length is a Good Measure to Distinguish between Stick Balancing in the ML and AP Directions

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**Abstract**

Seven novice subjects participated in experiments of stick balancing on a linear track in the anterior-posterior (AP) and the medio-lateral (ML) directions. The goal of the experiments was to test how the depth perception in the subjects' AP direction affects balancing performance compared to balancing in the ML direction, where depth perception does not play a role. It is easier to balance longer sticks than shorter ones, therefore balancing performance is measured by the length of the shortest stick that subjects can balance. Subjects were found to be able to balance shorter sticks in the ML direction than in the AP direction: the ratio of the shortest stick lengths in the ML direction relative to the AP direction was in average 0.53. Thus, the additional challenge posed by depth perception in the AP direction is clearly observable. Additionally, repeated trials were carried out for 5 consecutive days to assess the development of balancing skill by using stabilometry analysis. The maximal balance time of the subjects significantly increased with the days of practice.

**Keywords**

dynamic balance, stabilometry parameters, stick balancing, balancing skill development

## 1 Introduction

Stick balancing is a voluntary motor skill in which theoretical predictions for the stabilization of an inverted pendulum can be evaluated experimentally [1–5]. Longer sticks are easier to balance than shorter ones. Most human subjects can balance a stick of length 1 meter, but nobody can balance a stick shorter than 20 centimeters. This observation indicates that stick balancing can be a useful device to measure and characterize balancing skill of human subjects [6–10]. The natural question arises whether the balancing skill improves for repeated stick balancing trials and how the skill development can be quantified.

One way to simplify the balancing task is to confine the movements of the hand and stick to planar motion. This can be done by balancing the stick along a linear track [2, 11–14]. The setup is similar to virtual stick balancing [6, 15–17]. Usually, in track balancing experiments human subjects balance the stick in the direction parallel to the medio-lateral direction of their bodies.

During stick balancing on the fingertip, visual perception in the medio-lateral (ML) and anterior-posterior (AP) directions are coupled. Usually, the movements in the AP direction are considered to be the critical ones and movements in the ML direction are neglected [9, 10, 18, 19]. The track balancing device makes it possible to uncouple visual perception in the ML and AP directions. In the present study, the objective was to investigate and quantify the difference between visual perception in the ML and AP directions by asking novice subjects to carry out stick balancing trials on a linear track in both directions. The goal was to determine what are the shortest stick lengths that subjects can balance in the ML and AP directions, respectively. It is assumed, that sensory uncertainties of visual perception are much larger in the subject's AP direction compared to the ML direction due to the limitation caused by depth perception in the AP direction. Thus, the following hypothesis is proposed: *humans are expected to balance shorter sticks in the ML direction than in the AP direction during stick balancing on a linear track.*

Additionally, stabilometry parameters of the measured time histories were analyzed with the goal to assess the

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learning process of novice subjects in case of balancing short sticks for 5 consecutive days of measurement. Note that the same stabilometry analysis was carried out in [14] with subjects of one day and many days of experience in track balancing in the ML direction. Subjects with many days of experience had at least 15 minutes/week of practice time for 15 weeks. However, the stick length was $l = 0.9$ m and there were no repeated trials. In [14], the stabilometry parameters that were found to be reliable indicators to distinguish between balancing performance levels are: the standard deviation of the stick angle ($\sigma_\varphi$); the standard deviation of the cart position ($\sigma_x$); the frequency power of the stick angle ($\mathrm{FP}_\varphi$); the frequency power of the cart position ($\mathrm{FP}_x$); the mean power frequency of the cart position ($\mathrm{MPF}_x$); and the frequency dispersion of the cart position ($\mathrm{FD}_x$). These indicators will serve as basis for the current investigation of balancing short sticks on a linear track.

The outline of this paper is as follows. Section 2 introduces the stick balancing device and the methods for data analysis. Then, the results regarding shortest stick lengths and stabilometry analysis are described in Section 3. Finally, Section 4 discusses the main findings and the limitations of the study.

## 2 Apparatus and methods

Stick balancing experiments were performed and the measurement results were analyzed as discussed below.

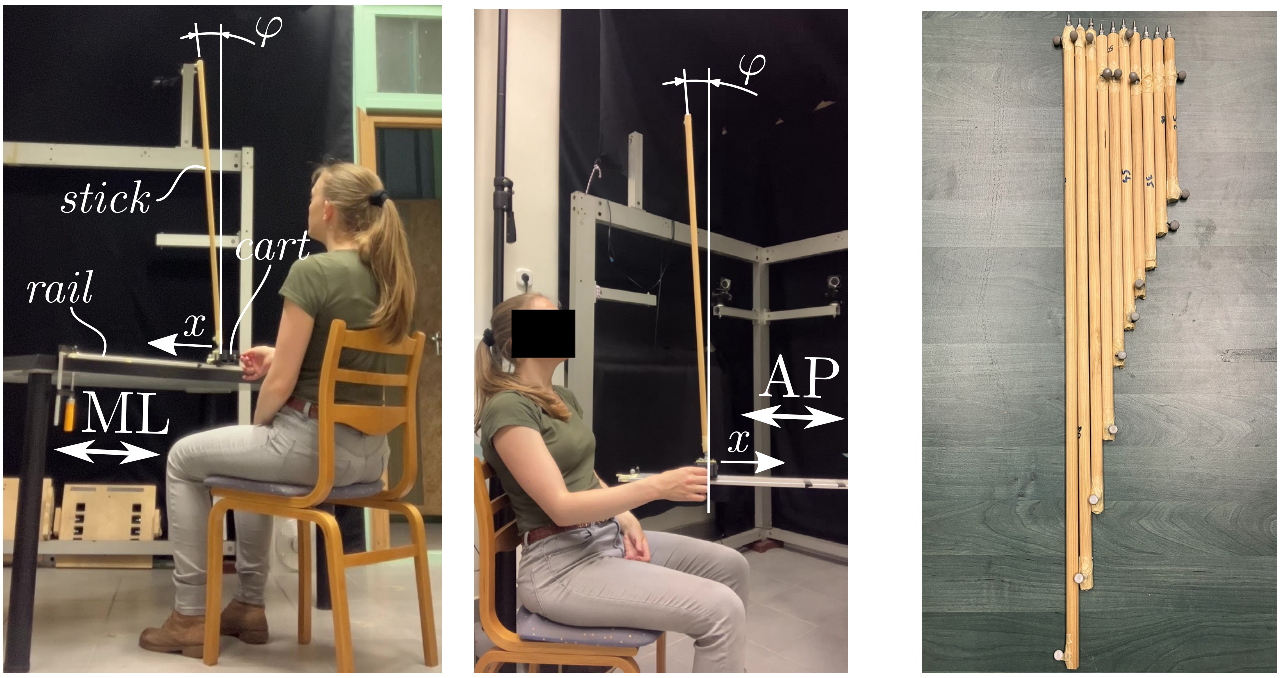
### 2.1 Stick balancing on a linear track

The measurement setup can either be used in a way that the plane of the stick's movements is parallel to the medio-lateral (ML) or the anterior-posterior (AP) direction of the subject [20]. The measurement configuration is shown in Fig. 1 (a) and Fig. 1 (b) for balancing in the ML and AP directions, respectively. The stick is pinned to the cart, which is allowed to move horizontally along a one-meter-long rail. Subjects are seated during the balancing task with their back pushed against the back of the chair.

Seven healthy individuals (aged 23–30) participated voluntarily in the measurements for 5 consecutive days, all of them were novice subjects. The goal was to test what are the shortest stick lengths that subjects can balance in the ML and AP directions, respectively. Eleven sticks of different lengths ranging from 25 to 90 cm were used for the balancing tests according to Table 1, all made of wood and had the same diameter of 16 mm (see Fig. 1 (c)). The research was carried out following the principles of the Declaration of Helsinki and subjects were allowed to withdraw from the study at any time.

### 2.2 Data analysis

The motion of the stick was captured using an OptiTrack® motion capturing system and preprocessing of the data was carried out by Motive® software. Thus, the $\{x, y, z\}$ coordinates of the markers in the global coordinate frame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(a) (b) (c)

**Fig. 1** (a) Stick balancing on a linear track in the medio-lateral direction. (b) Stick balancing on a linear track in the anterior-posterior direction. The stick is pinned to the cart via planar joint, the cart is constrained to move along the horizontal rail. (c) Sticks made for the experiments.

**Table 1** Lengths of sticks applied in the experiment $l_i$ [m] i = 1, 2, …, 11

| $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ | $l_7$ | $l_8$ | $l_9$ | $l_{10}$ | $l_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.9 | 0.8 | 0.7 | 0.6 | 0.55 | 0.5 | 0.45 | 0.4 | 0.35 | 0.3 | 0.25 |

of the motion capture system were recorded and saved. The sampling frequency was set to 120 Hz. The average tracking error of the markers was 0.2 mm resulting in a negligible error during the measurement evaluation.

Two markers were attached to the sticks, one close to the bottom and one close to the top of the stick. The markers defined a straight line which is parallel to the symmetry axis of the cylindrical stick. A marker was attached to the cart as well. Additionally, markers were used to designate the subjects' ML (parallel to their shoulder) and AP (front to back direction, normal to ML) directions. A marker was attached to the other, non-balancing hand of the subject as well so that the start and end of each trial is clearly identifiable within the recordings.

The angular deviation of the stick with respect to the vertical axis was calculated from the recorded marker positions in Matlab environment with a self-developed code. A low-pass filter with cut-off frequency of 5 Hz was applied on the measured time signals to reduce signal to noise ratio in the angle time histories.

For the evaluation of the measured time signals stabilometry parameters (SP) were used. Stabilometry was originally used as an objective tool to study body sway during quiet standing [21–23] and during different balancing exercises [24–27]. The method is usually based on the analysis of the time variant center of pressure (CoP) coordinates [28], however here stabilometry analysis was used to investigate stick balancing based on [14].

The stabilometry parameters, which were used in this study to evaluate human performance during stick balancing are the same as in [14], namely:

- standard deviation of the stick angle $\sigma_\varphi$ and cart position $\sigma_x$;
- mean power frequency of the stick angle $\mathrm{MDF}_\varphi$ and cart position $\mathrm{MDF}_x$;
- frequency dispersion of the stick angle $\mathrm{FP}_\varphi$ and cart position $\mathrm{FD}_x$;
- frequency power on the frequency range 0.1–5 Hz of the stick angle $\mathrm{FP}_\varphi$ and cart position $\mathrm{FP}_x$;
- frequency power ratio of the frequency range 0.1–1 Hz relative to 0.1–5 Hz of the stick angle $\mathrm{FPR}_\varphi$ and cart position $\mathrm{FPR}_x$.

Note that since the SPs are employed for the stick angle $\varphi$ and the cart position $x$ as well, altogether 2 × 5 = 10 SPs are considered per direction.

### 2.3 Reaction time tests

The reaction time (RT) of each subject was measured using a complex reaction time tester (CRTT) [29]. One of two lights was flashed at a random time instant and random order. Subjects had to press one of two buttons assigned to the two lights with their dominant hand as fast as possible. 10 successive random light flashes were presented for each subject with randomized time increments between flashes. This reaction time test involves a two-choice decision making, which is similar to stick balancing where subjects have to decide whether to move the stick's bottom forward/backward (AP) or left/right (ML).

### 2.4 Measurement protocol

The shortest stick length was assessed on the first day of the 5-day balancing training as follows. Subjects were first asked to practice stick balancing on a linear track both in the ML and AP directions for 10 minutes with the longest available stick ($l_1$ = 0.9 m) on the first day. Then, the subjects performed the reaction time test to get their RT, which served for the initial estimation of the stick length. As shown by [30], the theoretical critical length for delayed proportional – derivative (PD) feedback is linearly proportional to the square of the feedback delay, namely

$$l_{crit} = \frac{3}{4} g \tau^2, \qquad (1)$$

where $\tau$ is the feedback delay due to human reaction time and $g$ = 9.81 m/s$^2$ is the gravitational acceleration. Thus, $l_{crit}$ can be determined by substituting $\tau$ = RT in Eq. (1). Subjects started balancing either in the ML or AP direction (the order was randomized for each subject) with a stick of length taken from Table 1 that was closest to $l_{crit}$. A balancing trial was considered to be successful if the stick was balanced for 20 s at least once out of 5 trials. If the balancing trial was successful/unsuccessful, then the subjects were given the next shorter/longer stick of length available from Table 1. The measured critical length in the given direction (ML or AP) was the length of the shortest stick that subjects were able to balance successfully. Then, the measured critical length was determined in the other direction in the same way. Finally, day 1 was closed by a "2-minute" balancing session with the measured critical stick length in both ML and AP directions. The overall duration of the measurements on day 1 was about 30 minutes.

The "2-minute" balancing sessions were performed as follows. If the subjects were able to balance for 2 minutes continuously for the first attempt, then the session was terminated at 2 minutes, and it was assumed that subjects reached the maximal balance time (BT) that day. If the stick fell before 2 minutes, then subjects were asked to catch the falling stick with their other hand and start a new balancing trial straightaway. In this case, the last balancing trial was not terminated after 2 minutes from the start of the balancing session but was terminated after 2 minutes from the start of the last balancing trial. This way, the recorded balancing session may include more than one trial and/or the recording time may be longer than 2 minutes. The number of trials for ML balancing was between 1–9 and for AP balancing between 2–9 during the "2-minute" balancing sessions. The longest BT that was achieved within the "2-minute" balancing session was registered. An example for a measured time signal of S4 is shown in Fig. 2. In this case, the balancing session included 3 trials, none of them exceeded 2 minutes, and the overall "2-minute" balancing session was 173 seconds long. Balancing session is marked in green, and trials are marked in blue in Fig. 2.

On days 2–5, subjects were allowed to accommodate to the laboratory environment for a few minutes, then they were asked to perform a "2-minute" balancing session with their corresponding shortest stick length determined on day 1. The order of the balancing direction (ML or AP) was randomized for each subject on day 1, then the order was kept the same for the rest of the days.

As an extra task, on day 5, subjects were asked to perform the balancing task in the ML and AP directions first with their non-dominant eye patched (dominant-eye balancing), then with their dominant eye patched (non-dominant-eye balancing). It was investigated, whether the shortest stick length that subjects can balance remains the same or changes when balancing with one eye patched. The measured critical stick length for dominant-eye and non-dominant-eye balancing in the ML and AP directions were determined using the same procedure as on day 1 for two-eye balancing.

## 3 Results

The results and findings of the measurement evaluation are discussed below regarding shortest stick length, balancing skill development and reliability of stabilometry parameters.

### 3.1 Measured critical stick lengths for two-eye balancing

The shortest stick lengths that subjects were able to balance are shown in Fig. 3 in black. Crosses denote shortest stick lengths in ML direction ($l_{\mathrm{ML}}$) and dots denote shortest stick lengths in AP direction ($l_{\mathrm{AP}}$) as a function of subject number. It can be seen that subjects are indeed able to

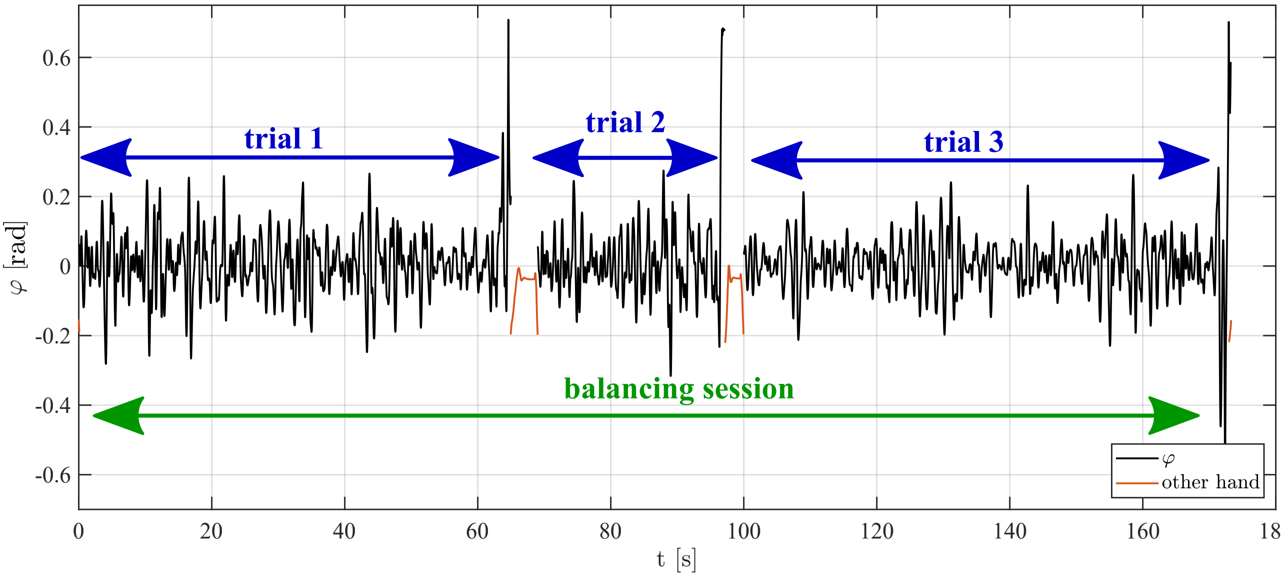


**Fig. 2** A representative trial for S4 on day 2 in the ML direction. Black denotes the stick's angular deviation measured from the vertical, while red denotes the movement of the non-balancing hand of the subject that catches the falling stick.

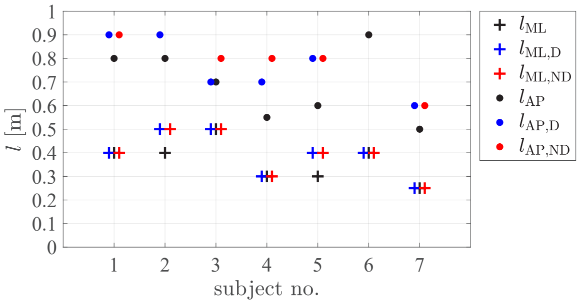


**Fig. 3** The measured critical stick lengths for two-eye (black markers), dominant-eye (D, blue markers) and non-dominant-eye (ND, red markers) balancing as a function of subject number. Crosses denote measured critical stick lengths in ML direction, dots denote shortest stick lengths in AP direction.

balance shorter sticks in the ML than in the AP direction due to the depth perception required in the AP direction.

The ratio of the shortest balanced stick lengths in the ML and AP directions as a function of subject number are shown in Table 2. The ratio is in average MEAN($l_{\mathrm{ML}}/l_{\mathrm{AP}}$) = 0.53 with standard deviation STD($l_{\mathrm{ML}}/l_{\mathrm{AP}}$) = 0.09. Since the ratio of $l_{\mathrm{ML}}$ and $l_{\mathrm{AP}}$ is around 0.5 with a relatively small standard deviation, the hypothesis proposed in Section 1 is accepted.

The mean and standard deviation of the measured reaction times (RTs) of the subjects are shown in Fig. 4 with errorbars. Using Eq. (1), the feedback delay can be indirectly estimated as

**Table 2** The ratio of the shortest balanced stick lengths $l_{\mathrm{ML}}/l_{\mathrm{AP}}$ for balancing sticks on a linear track in the ML and AP directions as a function of subject number for two-eye balancing.

| S1 | S2 | S3 | S4 | S5 | S6 | S7 | MEAN ± STD |
|---|---|---|---|---|---|---|---|
| 0.50 | 0.50 | 0.71 | 0.55 | 0.50 | 0.44 | 0.50 | 0.53 ± 0.09 |

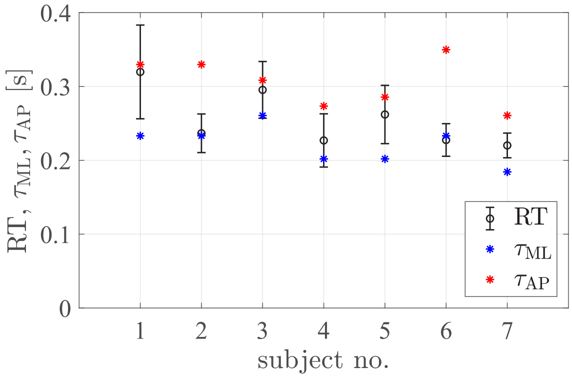


**Fig. 4** Mean and standard deviation of the measured reaction times (RTs) as a function of subject number; the feedback delay calculated based on the shortest stick lengths using Eq. (2) for the ML ($\tau_{\mathrm{ML}}$) and AP ($\tau_{\mathrm{AP}}$) directions.

$$\tau_{\mathrm{ML}} = \sqrt{\frac{4l_{\mathrm{ML}}}{3g}} \text{ and } \tau_{\mathrm{AP}} = \sqrt{\frac{4l_{\mathrm{AP}}}{3g}} \tag{2}$$

for the ML and AP directions, respectively. Therefore, the relation of the indirectly estimated feedback delays and the directly measured RTs can be investigated. The indirectly estimated feedback delays are also shown in Fig. 4 with blue and red stars for the ML and AP directions, respectively. As can be observed, $\tau_{\mathrm{ML}} \lesssim \mathrm{RT} \lesssim \tau_{\mathrm{AP}}$ for all subjects.

Linear correlation analysis between the indirectly estimated feedback delays $\tau_{\mathrm{ML}}$ and $\tau_{\mathrm{AP}}$ and the measured RTs gives coefficients $\rho(\tau_{\mathrm{ML}}, \mathrm{RT}) = 0.58$ and $\rho(\tau_{\mathrm{AP}}, \mathrm{RT}) = 0.31$, respectively. Thus, no strong linear correlation can be observed in either case. Still, the correlation for the ML direction is stronger than in the AP direction. This implies that when the balancing task is not affected by depth perception (in the ML direction), then the human control mechanism may be modelled by delayed PD feedback. However, when the role of depth perception becomes important (in the AP direction), then a more sophisticated feedback mechanism may be employed by human subjects, e.g., predictor feedback, or a combination of predictor feedback and delayed PD feedback. This result is similar to that of balancing shorter sticks on the fingertip [31].

### 3.2 Measured critical stick lengths for patched-eye balancing

The shortest stick lengths that subjects were able to balance with patched eye are shown in Fig. 3. Crosses denote shortest stick lengths in the ML direction ($l_{\mathrm{ML,D}}$ for dominant-eye balancing in blue and $l_{\mathrm{ML,ND}}$ for non-dominant-eye balancing in red); and dots denote shortest stick lengths in the AP direction ($l_{\mathrm{AP,D}}$ for dominant-eye balancing in blue and $l_{\mathrm{AP,ND}}$ for non-dominant-eye balancing in red) as a function of subject number. The ratio of the shortest balanced stick lengths in the ML and AP directions as a function of subject number are shown in Table 3 for dominant-eye balancing and in Table 4 for non-dominant-eye balancing. The ratio for dominant-eye balancing is in average MEAN($l_{\mathrm{ML,D}}/l_{\mathrm{AP,D}}$) = 0.51 with standard deviation

**Table 3** The ratio of the shortest balanced stick lengths $l_{\mathrm{ML,D}}/l_{\mathrm{AP,D}}$ for balancing sticks on a linear track in the ML and AP directions as a function of subject number for dominant-eye balancing. NA refers to non-applicable.

| S1 | S2 | S3 | S4 | S5 | S6 | S7 | MEAN ± STD |
|---|---|---|---|---|---|---|---|
| 0.44 | 0.56 | 0.71 | 0.43 | 0.50 | NA | 0.42 | 0.51 ± 0.11 |

**Table 4** The ratio of the shortest balanced stick lengths $l_{\mathrm{ML,ND}}/l_{\mathrm{AP,ND}}$ for balancing sticks on a linear track in the ML and AP directions as a function of subject number for non-dominant-eye balancing. NA refers to non-applicable.

| S1 | S2 | S3 | S4 | S5 | S6 | S7 | MEAN ± STD |
|---|---|---|---|---|---|---|---|
| 0.44 | NA | 0.63 | 0.38 | 0.50 | NA | 0.42 | 0.47 ± 0.10 |

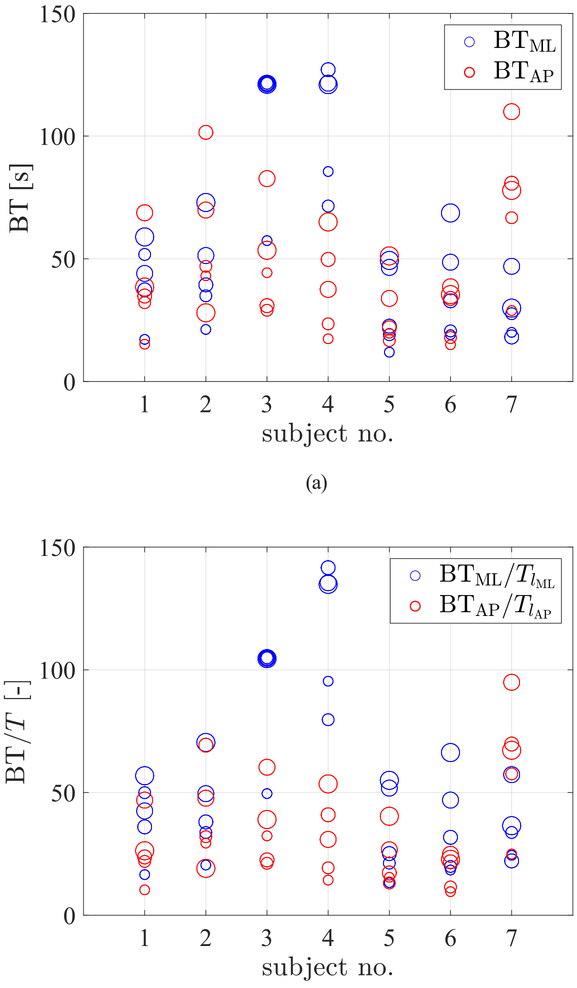
$\mathrm{STD}(l_{\mathrm{ML,D}}/l_{\mathrm{AP,D}}) = 0.11$. The ratio for non-dominant-eye balancing is in average $\mathrm{MEAN}(l_{\mathrm{ML,ND}}/l_{\mathrm{AP,ND}}) = 0.47$ with standard deviation $\mathrm{STD}(l_{\mathrm{ML,ND}}/l_{\mathrm{AP,ND}}) = 0.10$.

The measured critical stick length for balancing in the ML direction remained the same for 5 subjects for dominant-eye and non-dominant-eye balancing as well. The measured critical stick length for balancing in the ML direction increased by 10 cm for 2 subjects and the increase was the same for dominant-eye and non-dominant-eye balancing. *Thus, the measured critical stick length was the same for dominant-eye and non-dominant-eye balancing in the ML direction.*

In the AP direction, only 1 subject could balance the same stick length with their dominant eye as with two eyes, but when balancing with the non-dominant eye, the shortest stick length increased by 10 cm for this subject as well. The shortest stick lengths increased for all other subjects by at least 10 cm. S6 could not balance the longest available stick ($l_1 = 0.9$ m) in the AP direction during dominant-eye balancing (note the absence of blue dot for S6 in Fig. 3 and the NA value in Table 3). There were 3 subjects, whose shortest stick lengths differ for their dominant eye and their non-dominant eye; 2 of whom could not balance the longest available stick with their non-dominant eye (note the absence of red dots for S2 and S6 in Fig. 3 and the NA value in Table 4). *Thus, the measured critical stick length increased for patched-eye balancing in the AP direction. Additionally, the measured critical stick length was typically larger for non-dominant-eye balancing than for dominant-eye balancing in the AP direction.*

### 3.3 Maximal balance time

The maximal balance time (BT) was noted for 5 consecutive days of measurement for each subject with their corresponding shortest stick for two-eye balancing according to Fig. 3. BTs are shown in Fig. 5 (a), where BT in the ML direction is denoted by blue circles and BT in the AP direction is denoted by red circles. The achieved BT shows a generally increasing tendency as the day of the measurement increases. ANOVA revealed significant difference (significance $p < 0.05$) between BT on the first and last day of the measurement.



(a)

(b)

**Fig. 5** (a) Longest recorded balance times (BT) in the ML (blue) and AP (red) directions for the 5 consecutive days of balancing sessions. (b) Longest normalized balance times (BT/$T$) in the ML (blue) and AP (red) directions. Size of the marker is proportional to the day of measurement in both panels.

The BT itself depends highly on the length of the stick and the direction in which the balancing trial was carried out. Since the stick lengths in the ML and AP balancing trials were different for all subjects, the BT was normalized by the characteristic time of the stick, i.e., by the period of the small oscillations of the stick hanging at its downward position as BT/$T$, where the time period $T$ is:

$$T = 2\pi\sqrt{\frac{2l}{3g}}. \tag{3}$$

The ratios BT/$T$ are shown in Fig. 5 (b), where BT/$T$ in the ML direction is denoted by blue circles and BT/$T$ in the AP direction is denoted by red circles. It can be seen that most subjects are able to achieve larger BT/$T$ values when balancing in the ML direction.

### 3.4 Balancing skill development

The 2 × 5 = 10 stabilometry parameters are shown in Fig. 6 and Fig. 7 for balancing in the ML and AP directions, respectively. One-way ANOVA was applied to determine differences between first and last days of the measurement with respect to SPs with the goal to assess the learning process of the novice subjects in case of balancing short sticks. The stabilometry parameters for the subjects were of normal distribution according to the Chi squared goodness of fit test (significance $p < 0.05$). The data were homogeneous for all but SP $\mathrm{FP}_x$, therefore Welch's ANOVA was applied for SP $\mathrm{FP}_x$, and basic ANOVA for all other SPs during the statistical analysis.

ANOVA revealed no significant differences ($p < 0.05$) for any of the SPs in the ML and in the AP directions. This suggests that subjects were unable to significantly improve their balancing skill within the 5 days available for the measurement. Therefore, the 5 consecutive trials can be considered as trial repetition with the same conditions. Analysis of the coefficient of variation for the different SPs can reveal their reliability to assess balancing performance.

### 3.5 Reliability of stabilometry parameters

The coefficient of variation [32] can be obtained as the ratio of the standard deviation and the mean for each SP as:

$$c_{v,i,j} = \frac{\mathrm{STD}_k\left(\mathrm{SP}_{i,j,k}\right)}{\mathrm{MEAN}_k\left(\mathrm{SP}_{i,j,k}\right)}, \tag{4}$$

where subscript $k = 1, 2, \ldots, 5$ refers to the days of practice, subscript $i = 1, 2, \ldots, 7$ refers to subjects and subscript $j = 1, 2, \ldots, 20$ refers to different SPs introduced in Section 2.2. The mean of the coefficient of variation over the seven subjects, $c_{v,j} = \mathrm{MEAN}_i(c_{v,i,j})$, for the 2 × 5 = 10 SPs for the ML and AP directions are listed in Table 5. As can be seen, $c_{v,j} < 1$ for all $j = 1, 2, \ldots, 20$, which indicates low variability of the SPs. Furthermore, $c_{v,j} < 0.2$ for $\sigma$, MPF, FD and FPR, which indicates that the variability of these SPs is extremely low. It can also be observed that the SPs related to ML and AP directions have about the same variability.

## 4 Discussion

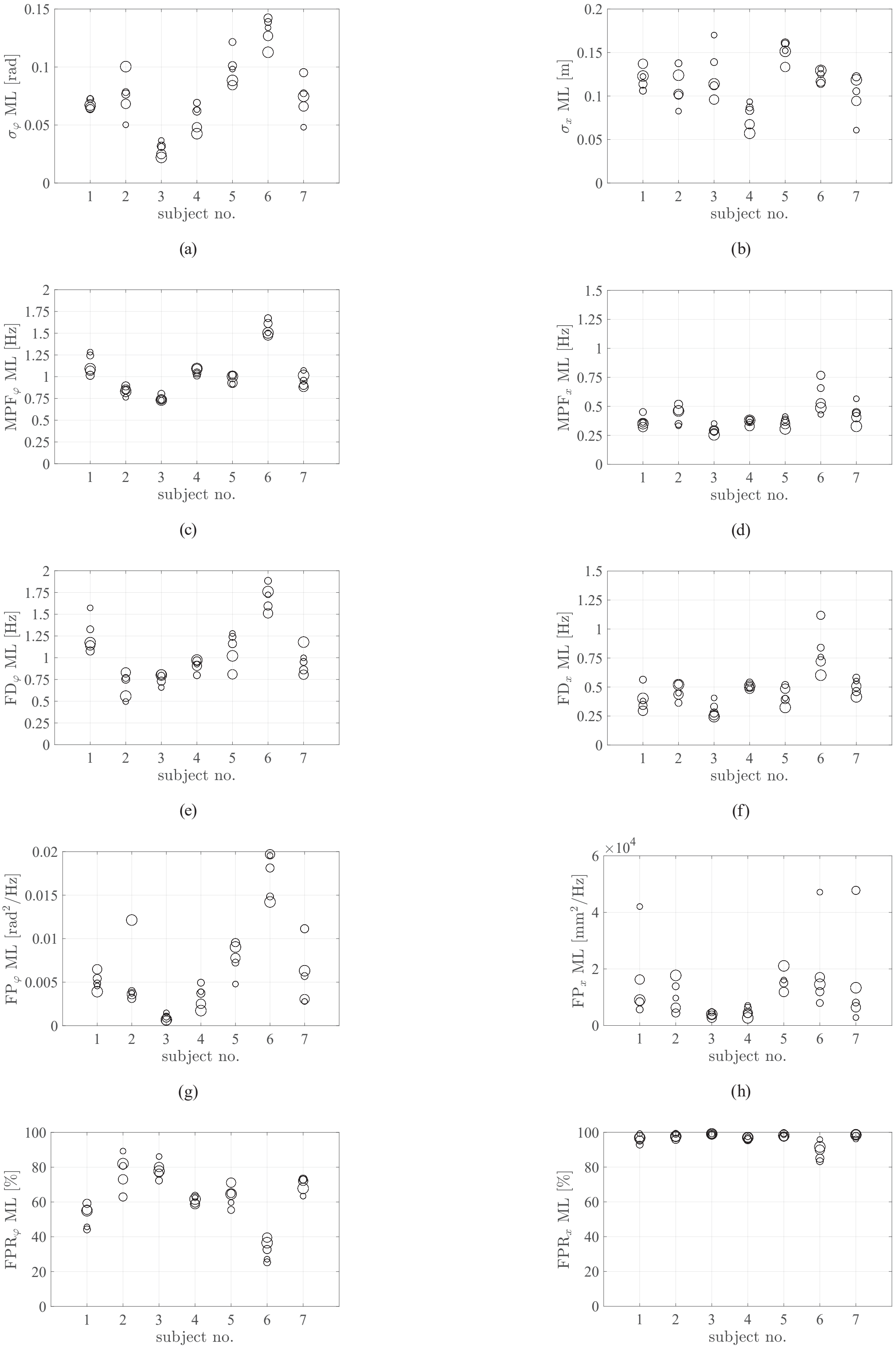
Repeated trials of stick balancing in the ML and AP direction for 5 consecutive days involving seven novice subjects revealed important and surprising findings of human motion control.

Subjects were able to balance shorter sticks in the ML than in the AP direction, the ratio of the shortest stick lengths in the two directions was in average 0.53 with low standard deviation. This means that the difficulty caused by the depth perception in the AP direction can be measured by the critical length. Using a delayed PD feedback model without any sensory uncertainties, the relation between the critical length and the feedback delay is given by Eq. (1). In this model, the critical length is smaller by the factor 0.53 if the feedback delay is shorter by the factor of 0.73. Hence, depth perception can be considered as an additional sensory process, which increases the overall feedback delay by a factor of 1/0.73 = 1.37. This relation can also be observed in [14] for estimation of feedback delay in case of track balancing in the ML direction and estimation of feedback delay for stick balancing on the fingertip [6, 10].

When balancing with eye-patch, subjects were still able to balance shorter sticks in the ML than in the AP direction. The ratios of the shortest stick lengths were in average 0.51 for dominant-eye balancing, and 0.47 for non-dominant-eye balancing. These ratios are close to that of the two-eye balancing (0.53). This means that covering one eye affects the balancing performance to about the same extent in the ML and AP directions.

The head of the subjects was not restrained for either of the directions. Therefore, it is possible that subjects were able to partially compensate the movements of the stick when balancing in the AP direction by looking a little bit sideways at the stick and thus seeing movements partially in the ML direction. The partial compensation from ML movement likely decreases the difference between ML and AP balancing performance. Nevertheless, in our experiments the balancing performance in the ML and AP direction was clearly different, which is manifested in different measured critical lengths in the ML and AP directions.

Based on the stabilometry analysis of stick balancing experiments, it was shown that subjects were not able to significantly improve their balancing skills within the available time for the measurement. This means that practice time ~2 minutes/day with the shortest stick was not enough to significantly increase expertise. This observation implies that the consecutive trials can be considered as repeated trials with similar conditions. Hence, the coefficient of variation of the SPs can be used to assess the reliability of the SPs. It was found that the variations of $\sigma$, MPF, FD and FPR are extremely low (with coefficient of variation being less than 0.2), while the variation of FP is low (with coefficient of variation being less than 0.7). This means that all the SPs used in this paper can be considered as reliable parameters to describe balancing abilities.



(a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

**Fig. 6** Stabilometry parameters determined from the measured time signals of $\varphi$ and $x$ as a function of subject number in the medio-lateral direction. Markers show stabilometry values for measurements and the size of the marker is proportional to the number of days the time signal was recorded.

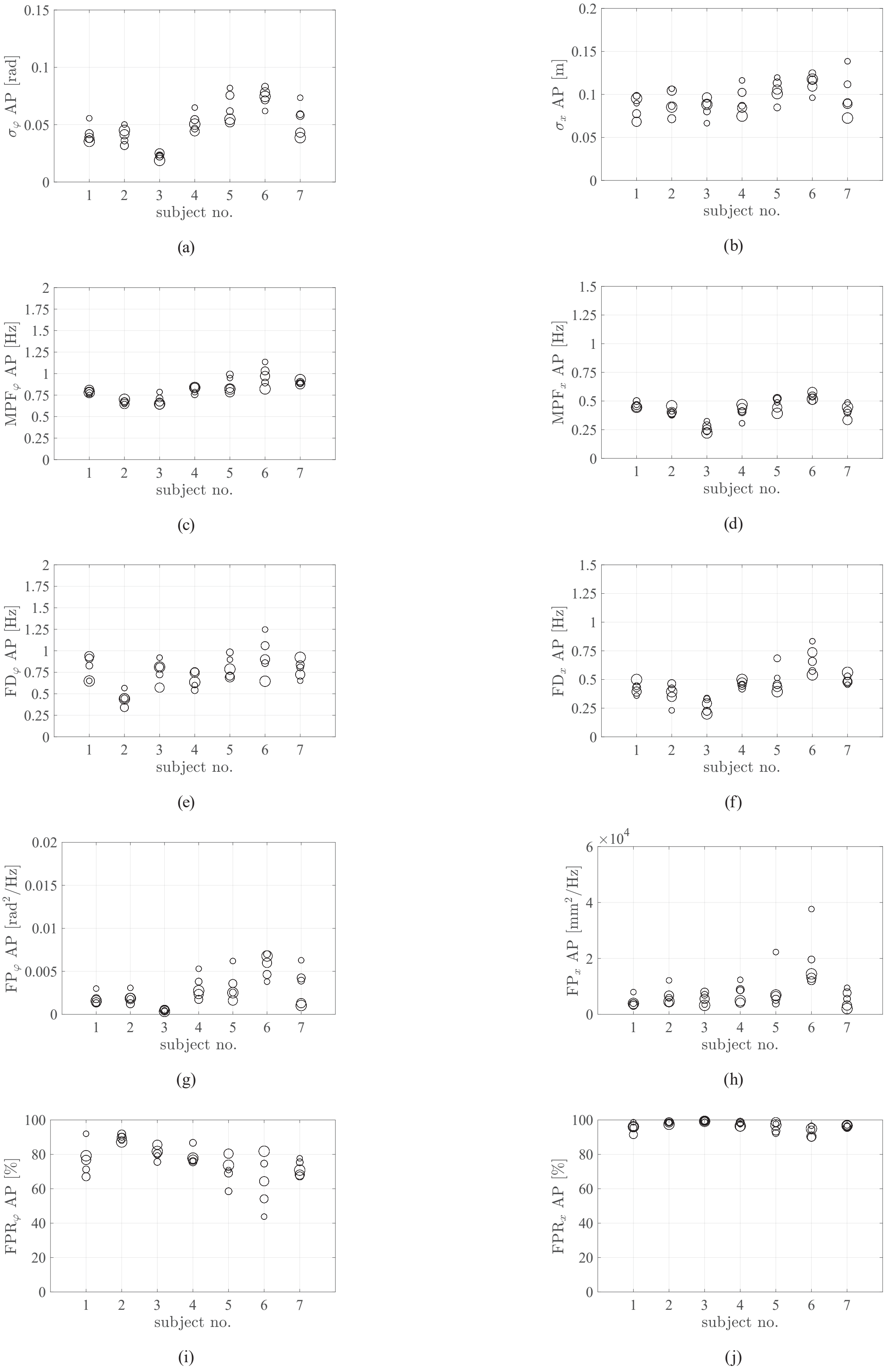


**Fig. 7** Stabilometry parameters determined from the measured time signals of $\varphi$ and $x$ as a function of subject number in the anterior-posterior direction. Markers show stabilometry values for measurements and the size of the marker is proportional to the number of days the time signal was recorded.

**Table 5** The mean of the coefficient of variation over the seven subjects, $c_{v,j} = \mathrm{MEAN}_i(c_{v,i,j})$ for the 2 × 5 = 10 SPs for the ML and AP directions.

| | | $\sigma$ | MPF | FD | FP | FPR |
|---|---|---|---|---|---|---|
| ML | $\varphi$ | 0.17 | 0.06 | 0.14 | 0.38 | 0.10 |
| | $x$ | 0.16 | 0.15 | 0.17 | 0.71 | 0.02 |
| AP | $\varphi$ | 0.17 | 0.06 | 0.17 | 0.41 | 0.10 |
| | $x$ | 0.16 | 0.11 | 0.16 | 0.52 | 0.02 |

**Table 6** Average ratio of the standard deviation of the cart position in the ML direction relative to the AP direction $\sigma_{x,\mathrm{ML}}/\sigma_{x,\mathrm{AP}}$ for each subject for the 5 days of measurement.

| S1 | S2 | S3 | S4 | S5 | S6 | S7 | MEAN ± STD |
|---|---|---|---|---|---|---|---|
| 1.44 | 1.24 | 1.57 | 0.84 | 1.47 | 1.10 | 1.09 | 1.25 ± 0.41 |

Nevertheless, the maximal balance time increased significantly between first and last day of the measurement. The BT can also be considered a stabilometry parameter, which is more sensitive to highly unstable balancing tasks such as stick balancing. Note that the other 2 × 5 stabilometry parameters were originally developed for human quiet standing, where the BT can be considered "infinite", and thus other stabilometry parameters were needed to describe balancing skill. The increase of the BT in the current measurements means that subjects in fact were able to improve balancing performance in the sense that they could recover from critical situations better when the stick was about to fall. The recovery from critical situations corresponds to the concept of barrier-function-based safety control [33, 34] in the sense that additional control actions are initiated in order to prevent the stick from falling, but the amplitude of the stick is not of interest provided it stays within the limits of fall.

The larger sensory uncertainties in the AP direction could be a major cause of subjects being able to balance about 2 times longer sticks in the AP direction. However, there might be other reasonings, such as the operability of the human arm, which would depend on the arm stiffness. Gomi and Kawato [35] showed larger stiffness for the arm reaching in the AP direction, compared to that in the ML direction. The ratio of the standard deviation of the cart position in the ML direction relative to the AP direction could reveal if the operability of the arm is direction-dependent for this task. The average ratios $\sigma_{x,\mathrm{ML}}/\sigma_{x,\mathrm{AP}}$ for the 5 days of measurement were determined for each subject and are shown in Table 6. The ratio $\sigma_{x,\mathrm{ML}}/\sigma_{x,\mathrm{AP}}$ is in average 1.25 with standard deviation 0.41, meaning that there is higher mobility (and thus likely operability) in the ML direction by in average 25%, than in the AP direction. Therefore, direction-dependent operability might also be a contributing factor to the shortest stick lengths ratios shown in Tables 2, 3 and 4.

## Declaration of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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