

# Improved Method for Determining the Feed Influence on the Tangential Cutting Force During Re-drilling, Countersinking and Boring Based on the Small Sample Theory

Volodymyr Krupa<sup>1\*</sup>, Volodymyr Kobelnyk<sup>1</sup>, Andrii Hahaliuk<sup>1</sup>, Volodymyr Dzyura<sup>2</sup>,  
Nadiia Tymoshenko<sup>3</sup>

<sup>1</sup> Department of Design of Machine Tools, Tools and Machines, Faculty of Engineering of Machines, Structures and Technologies, Ternopil Ivan Puluj National Technical University, Ruska str. 56, 46001 Ternopil, Ukraine

<sup>2</sup> Department of Automobiles, Faculty of Engineering of Machines, Structures and Technologies, Ternopil Ivan Puluj National Technical University, Ruska str. 56, 46001 Ternopil, Ukraine

<sup>3</sup> Department of Higher Mathematics, Institute of Applied Mathematics and Fundamental Sciences, Lviv Polytechnic National University, Stepana Bandery str. 12, 79000 Lviv, Ukraine

\* Corresponding author, e-mail: [krupa\\_v@tntu.edu.ua](mailto:krupa_v@tntu.edu.ua)

Received: 16 January 2023, Accepted: 06 February 2024, Published online: 02 April 2024

## Abstract

Methods of determining the tangential cutting force are analyzed in the work, including finite-element simulation models, the use of neural networks, analytical methods, and methods of natural experiments. A method for evaluating the influence of feed on the tangential component of the cutting force is proposed, which can be applied to test new tools, tool plates for a certain group of materials. The main advantage of the proposed method is the limited number of experiments and processing of the obtained data according to the theory of a small sample. In this study, the proposed method was used to determine the influence of countersinking feed on tangential cutting force. The basis of the approach is probabilistic and statistical methods and data processing criteria. An assessment of the homogeneity of the results of experimental studies was carried out, the law of distribution of the tangential cutting force, as well as the characteristics of its dispersion, were established. An approach to estimating the maximum value of the tangential component of the cutting force based on the maximum value is presented. This method is universal and can be used for drilling, countersinking, boring and other types of processing.

## Keywords

evaluation criteria, probabilistic statistical methods, impact evaluation, cutting process, maximum tangential cutting force

## 1 Introduction

One of the tasks of mechanical engineering is to increase the efficiency of the machining process, which cannot occur without determination of the force characteristics of the cutting process itself. The stresses generated during the cutting process depend on the forces occurring in the tool. This requires the improvement of its design, provision of the strength and rigidity of all tool elements. Power parameters also affect the energy consumption in the digital control of the production process, etc. While testing the new tool, or new tool plate with certain geometrical parameters, it is necessary to determine the force characteristics of the cutting process depending on the elements of the cutting mode. This also applies to the new material being processed. A significant number of various

methods are used for this purpose. One of the methods for predicting cutting forces is the application of CAE systems using the finite element method. For example, in papers [1, 2], a new finite element simulation model of radial cutting force for drills with replaceable plates, depending on the elements of the cutting mode was proposed. In paper [3], FEM modeling for the determination of the force coefficient during drilling for defining the orthogonal cutting force was used. This method is effective for ideal operating conditions. However, in real conditions, the data of this method can significantly differ from the actual values. Another method of predicting cutting forces is the use of neural networks. The focus of the paper [4] is to employ artificial neural network (ANN) and quadratic rotatable

central composite design (QRCCD) to carry out prediction and optimization for the investigation of the cutting force to minimized energy consumption. In papers [5–7], the neural network is used to predict cutting forces during milling for various materials, including Inconel 738 [7]. The method of neural networks is used for optimization of control parameters in the milling of aluminum hybrid metal matrix composites using ANN and Taguchi-grey relational analysis [8]. The method of neural networks is highly effective in predicting cutting forces, but it is quite time-consuming.

There are analytical methods for determining the cutting forces. In paper [9], mechanistic approach is used to model the cutting forces. Cutting-force coefficients are identified from measured instantaneous forces in drilling operations. Unified analytical model for determining the cutting forces using general transformation matrix is proposed on paper [10], and analytical model of the cutting force in micro-end-milling operations is proposed in paper [11]. The accuracy of analytical models strongly depends on the number of factors taken into account. With the increase in their number, the accuracy increases, but the calculations become very complicated, making the application rather limited.

A separate direction is the prediction of cutting forces by analytical method using several deformation areas, which is also widely used in the composite materials machining [12, 13]. The disadvantage of the above-mentioned methods is the impossibility of taking into account all factors during cutting. In some cases, the prediction model may not correspond to the actual experimental data.

However, the most widely used are methods of experimental researches (natural experiments) which make it possible to determine cutting forces more accurately. As a rule, during experimental research methods the empirical dependencies for each specific material, geometry of the cutting part, etc. are obtained in this case, cutting forces are measured using dynamometers. For example, in paper [14], the effect of cutting conditions during milling machining on cutting forces and surface roughness has been investigated. Proposes experimental models for the determination of cutting force components. The experimental method of the cutting forces determination for fiber-reinforced composites machining is used in paper [15], and similar research method is used in paper [16, 17], but additionally, the percentage distribution of each process parameter is analyzed using Analysis of Variance (ANOVA). In [18], a numerical and experimental

study was carried out to investigate the effect of tool geometry on cutting forces, temperature distribution and residual stresses on the surface of stainless steel processing. Paper [19] is devoted to the determination of cutting forces during drilling, and verification by the finite element method is performed. In particular, it is noted in this work that cutting forces are not constant but have stochastic character. However, the homogeneity of the mechanical properties of materials is noted as the cause of stochasticity. The concept of cutting forces stochasticity is developed in paper [20], and it is proved that the stochasticity of the feed rate affects the dissipation of the cutting force during turning. A similar approach for the determination of cutting forces during drilling is applied in papers [21, 22]. In paper [23], data obtained from the experiments are defined using both the Artificial Neural Network (ANN) and Response Surface Methodology (RSM). In [24], the influence 28 number of parameters on the power characteristics of the cutting process was studied by this method and their results were compared with the literature results for each of them. In work [25], a combination of methods is used for analytical determination of cutting forces taking into account the more than 50 number of parameters.

Probability-statistical methods of processing the results of real-life experiments of the cutting process have become widespread. For example, in paper [26] the problem of multicriteria optimization of the turning process using probabilistic-statistical approach is solved, in paper [27] optimization of the turning process of hardened steel AISI 52100 using stochastic programming is carried out, and in paper [28] the stochasticity of various criteria on the optimization process is taking into account. A great number of papers is devoted to the analysis of stochasticity of cutting forces occurring during metal layer cutting in turning [29] and milling [30]. Probability-statistical methods are very effective in predicting the force characteristics of the cutting process, particularly their maximum values. Their disadvantage is the need to collect large data sets for ensuring statistically reliable results.

Each tool or cutting plate can normally be designed only for machining one grade of steel or alloy, however they are usually designed for the range of materials in a certain class. Some of them can be designed for several grades (e.g. ISO P and ISO M). It is very expensive and sometimes unreasonable to test every type of plate on every material grade. Therefore, the development of efficient, statically reliable method for determining the tangential force using the minimum required number of experiments

is definitely relevant. The objective of the paper is to propose the method for determining and evaluating the tangential cutting force for a certain group of materials depending on the feed rate using the small sample theory.

### 2 The essence of the method

The proposed method is implemented in the following sequence. One or several classes of relevant alloys are selected, for example, structural quality and alloyed structural steels. From each of 2 classes, several representatives are selected. For example, in this paper, steels of grades: C22; C40; C60; C65; C70; (ISO P unalloyed) (DIN EN ISO 683) and 45Mn2; 30MnCrTi; 30CrMo; 40Cr; 40Mn2 (ISO P low-alloyed) (DIN EN ISO 3183). It is allowed to choose materials (steel) of subgroups within the same class.

We chose a tool (three cutter countersink bit made of high-speed steel with 16 mm diameter was used in this work). Blanks were made from each of the selected steels in the form of stepped cylinders with the hole with diameter equal to  $d_{eq} = D + 2$ , mm., ( $D$  – diameter workpiece) ensuring, at the same time, the cutting depth of  $t = 1$  mm.

Using the experimental setup, the cutting process was carried out on each of the required investigated feeds  $f$ . In the investigations, we used 2N118 machine tool with a number of feeds with the denominator of geometric series, so the following feeds were taken:  $f_1 = f_{min}$ ;  $f_2 = f_1 \times \varphi$ ;  $f_3 = f_1 \times \varphi^2$ ;  $f_4 = f_1 \times \varphi^3$ ;  $f_5 = f_1 \times \varphi^4$  and  $f_6 = f_1 \times \varphi^5$ . At each pass, the value of the torque  $M_{s_i}$  was recorded. The tangential cutting force  $P_{z_{f_i}}$  in such machining operations as re-drilling, boring with multi-tooth heads, and countersinking is determined as for one cutter and multiplied by the number of teeth  $z$ . Therefore,  $P_{z_{f_i}}$  is determined by the following formula:

$$P_{z_{f_i}} = 2M_{f_i} / z \times D \tag{1}$$

where  $M_{f_i}$  is torque.

Based on the central limit theorem of probability theory, the values of forces  $P_{z_{f_1}}$ ,  $P_{z_{f_2}}$ ,  $P_{z_{f_3}}$ ,  $P_{z_{f_4}}$ ,  $P_{z_{f_5}}$ ,  $P_{z_{f_6}}$  are assumed to be random variables that follow the distribution law close to normal [31].

Using [32] and the theory of small samples [33, 34], we used the dependencies for the determination of mathematical expectations approximately equal to the mean values of forces  $P_{z_{f_i}}$ , i.e.,  $M(P_{z_{f_i}}) \approx \bar{P}_{z_{f_i}}$ , and dispersions  $D(P_{z_{f_i}})$

$$M(P_{z_{f_i}}) \approx \bar{P}_{z_{f_i}} = \frac{a+b}{2} \prod_{k=1}^{10} C_k + \sum_{k=1}^n \sum_{j=k}^n C_k \frac{\sigma}{\sqrt{2\pi}} \times \left( e^{-\frac{z_{1k}^2}{2}} - e^{-\frac{z_{2k}^2}{2}} \right) + t_k \left[ \Phi(z_{2k}) - \Phi(z_{1k}) \right] \tag{2}$$

$$D(P_{z_{f_i}}) \approx \bar{P}_{z_{f_i}}^2 = \frac{a+ab+b^2}{3} \prod_{k=1}^{10} C_k + \sum_{k=1}^n \sum_{j=k}^n C_k \frac{\sigma}{\sqrt{2\pi}} \times \left[ (\sigma \times z_{1k} + 2_{tk}) \times e^{-\frac{z_{1k}^2}{2}} - (\sigma \times z_{1k} + 2_{tk}) \times e^{-\frac{z_{2k}^2}{2}} \right] + (\sigma^2 + t_k^2) \left[ \Phi(z_{2k}) - \Phi(z_{1k}) \right] - M^2(P_{z_{f_i}}), \tag{3}$$

where  $\Phi(z_{2k})$ ,  $\Phi(z_{1k})$  – Laplace functions;  $z_{1k} = \frac{a-p_k}{\sigma}$ ;  $z_{2k} = \frac{b-p_k}{\sigma}$ ;  $a = P_{kmin}$ ;  $a = P_{kmax}$ ; ( $P_{kmin}$ ,  $P_{kmax}$  are respectively, the largest and smallest value among  $P_k$  ( $k = 1...n$ ) experimental data of random variable  $P_z$  at a certain feed value  $f$ ).

In order to determine the tangential cutting force  $P_{z_{f_i}}$ , we use the classical [35–37] dependence

$$P_z = C \times t_i^{x_p} \times f_i^{y_p} \times K \tag{4}$$

where  $C$  is coefficient,  $t_i$  is cutting depth;  $f_i$  is rotary feed;  $K$  is coefficient, which takes into account the machining conditions;  $x_p, y_p$  – are degrees that take into account the effect of cutting depth  $t$  and feed  $f$  on  $P_z$ , respectively.

Analysis of empirical dependencies for determining  $P_z$  for these types of machining presented in [35–37] etc. showed that the exponent  $x_p$  at  $t$  is equal to one or close to 1 ( $x_p \approx 1$ ).

That is, dependence Eq. (4) for a particular case at the given  $f_1 = \text{const}$  has the following form

$$P_{z_{f_i}} = K_{f_i} \times t, \tag{5}$$

where  $K_{f_i} = C \times f_i^{y_p} \times K = \text{const}$ .

Graphically, dependences  $P_{z_{f_i}} = f(t)$  are shown in Fig. 1 (here, it is accepted that  $t = 1$  mm).

It follows from Fig. 1 that the angular coefficients of the inclination of the rays, illustrating the dependence

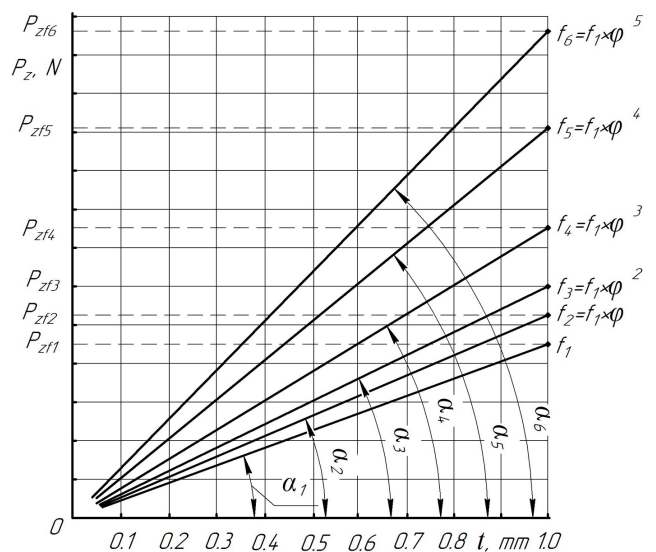


Fig. 1 Graphs of dependencies  $P_{z_{f_i}} = K_{f_i} \times t, K_{f_i} = \text{const}$

of  $P_{zi}$  on  $t$ , depend on the feed, that is:  $tq\alpha_1 = K_{f_1} = P_{z_{f_1}}$ ;  $tq\alpha_2 = K_{f_2} = P_{z_{f_2}}$ ;  $tq\alpha_3 = K_{f_3} = P_{z_{f_3}}$ ;  $tq\alpha_4 = K_{f_4} = P_{z_{f_4}}$ ;  $tq\alpha_5 = K_{f_5} = P_{z_{f_5}}$ ;  $tq\alpha_6 = K_{f_6} = P_{z_{f_6}}$ .

Taking into account that series of feeds for drilling machine can be represented by geometric progressions, the general formula for a certain case can be written in the following way:

$$K_{s_i} = C_k (f_1 \times \varphi^{i-1})^{y_p} \quad (6)$$

where  $C_k = C \times K$ .

Dependence Eq. (6) can be rewritten in the following form:

$$K_{f_i} = C_k \times f^{y_p}. \quad (7)$$

Then, substituting Eq. (7) into Eq. (5), we get

$$P_{z_{f_i}} = C_k \times f^{y_k} \times t. \quad (8)$$

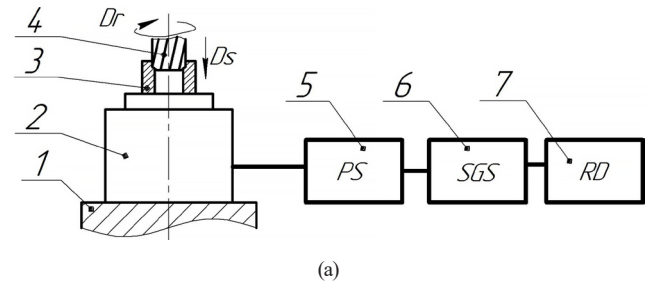
### 3 Experimental investigations

To implement this method, we will use the setup shown in Fig. 2. A two-component dynamometric device 2 is installed on the table of the vertical drilling machine. Blanks 3 with pre-drilled  $\varnothing 14$  mm hole are installed alternately in the dynamometer from different materials. The machining process is carried out by tool 4 (countersink with the following parameters: countersink diameter  $D = 16$  mm, main angle in the plan  $\varphi = 30^\circ$ , front angle  $\gamma = 10^\circ$ , number of teeth  $n = 3$ , material – S600 DIN ISO). The data from the strain gauges are transmitted through the power supply 5 and the strain gauge station 6 to the recording device 7.

Values of both axial force and torque are obtained. The value of the experimental force  $P_{z_e}$  is determined by formula Eq. (1). The results are presented in Table 1. Taking into account that the size of samples  $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = 10$  for mathematical expectation, which is approximately equal to the mean value and dispersion, respectively  $M(P_z) \approx \bar{P}_z$ , and  $D(P_z)$  using the improved method of iterations based on the theory of small samples [31] we determine the desired characteristics. And also determined the mean square deviation  $\sigma(P_z) = \sqrt{D(P_z)}$  is determined as well.

### 4 Discussion and evaluation of the results

The resulting statistical series of  $P_{z_{f_i}}$  values were checked for homogeneity, i.e., the presence of anomalous values [38, 39], according to the Grebbs ( $t_k$ ), Irwin ( $\lambda_{p_i}$ ), and Romanovskyi ( $t_\beta$ ) criteria [38]. The values of these criteria were obtained from the dependencies [38, 39], respectively:



**Fig. 2** (a) Scheme of the installation and (b) photo of the machining area: 1 - machine table; 2 - dynamometer; 3 - blank; 4 - tool; 5 - power supply; 6 - strain gauge station; 7 - recording device

$$t_k = \left| P'_{z_{f_i}} - M(P_z) \right| / \sigma(P_z), \quad (9)$$

where  $P'_{z_{f_i}}$  is the studied value that stands out sharply.

$$\lambda_{p_{z_i}} = (P_{z_{f_{i+1}}} - P_{z_{f_i}}) / \sigma(P_z), \quad (10)$$

$P_{z_{f_{i+1}}}$ ,  $P_{z_{f_i}}$  are, respectively, the next and previous values of the cutting force  $P_z$ , arranged in ascending order at a certain feed rate  $f_i$

$$t_\beta = \left| P'_{z_{f_i}} - M'(P_z) \right| / \sigma'(P_z), \quad (11)$$

$M'(P_z)$ ,  $\sigma'(P_z)$  is the mathematical expectation and the mean squared deviation of the sample, calculated without the investigated value that stands out sharply

The calculated maximum values of the criteria are presented in Table 2.

**Table 1** Experimental values of force  $P_{z0}$ ,  $N$ , and its dispersion characteristics

No	Steel grade	Physical and mechanical characteristics of alloys, $\sigma_B$ , MPa/HB, MPa	Feed rates, $f$ , mm/rev					
			0.1	0.14	0.2	0.28	0.4	0.56
1	C45	650/2290	759	900	1175	1480	1885	2400
2	C40	580/1970–2250	685	850	1035	1340	1715	2185
3	C60	690/2290	855	1020	1315	1639	2095	2690
4	C70	790/2680	895	1180	1510	1940	2465	3150
5	C65	720/2425	860	1110	1380	1790	2250	2910
6	45Mn2	700/2290	794	1000	1260	1615	2065	2630
7	30MnCrTi	570/1850–2290	780	970	1220	1580	2300	2560
8	40Cr	600/2170	750	960	1195	1540	1970	2510
9	30CrMo	600/1930	690	865	1075	1405	1770	2260
10	40Mn2	670/2170	760	955	1245	1560	1995	2540
Mathematical expectation, $M(P_z) \approx \bar{P}_z(N)$			751	950	1191	1545	1993	2515
Scattering variances $D(P_z), N^2$			2470	4360	9840	12450	26000	32700
Mean square deviation, $\sigma(P_z)$			49.7	66.0	99.2	115.6	161.2	180.8

**Table 2** Calculated values of criteria  $t_k, t_b$  and  $\lambda_{P_i}$

Criteria	Feed rates, $f$ , mm/rev					
	0.1	0.14	0.2	0.28	0.4	0.56
$t_{ki}$	1.93	1.82	1.57	1.77	1.72	1.82
$t_{\beta i}$	1.58	1.87	1.57	1.78	1.72	1.83
$t_{P_{zi}}$	0.76	1.06	1.31	1.29	1.02	1.33

The tabular values of the criteria found in [38, 39] are  $t'_{k_{tabl}} = 2.43$ ;  $t'_{\beta_{tabl}} = 2.2$ ;  $\lambda_{P_{z_{tabl}}}(0.95) = 1.4$ . All the calculated values obtained according to the Grebbs  $t_{ki} < t'_{k_{tabl}}$ , Irwin  $t_{\beta i} < t'_{\beta_{tabl}}$  and Romanovskyi  $\lambda_{P_i} < \lambda_{P_{z_{tabl}}}$  criteria for the reliability level of the results are 0.95 (95%).

Analysis of the obtained data showed that there were no abnormal values (those that stood out sharply) in all statistical series  $P_{z_{f_i}}$ , presented in Table 2.

**5 Testing the hypothesis of the normality of the distribution of values of the tangential cutting force  $P_z$  during countersinking according to the criterion of the distribution matching  $W$**

$W$  criterion is a powerful and more effective criterion for evaluating the validity of the assumption of normality of the distribution than Kolmogorov and Pearson  $\chi^2$  criteria, especially in the case when the amount of data is limited and can be applied to normal, log-normal, and exponential distribution laws [40].

The method of testing the hypothesis of the normality of the distribution of force values  $P_z$  according to  $W$  criterion is as follows:

- Using the data from Table 1,  $P_{z_{f_i}}$  values were placed in ascending order in the rank series.
- The value of coefficient  $S^2$  was determined from dependence [37]

$$S^2 = \sum_{i=1}^{10} \bar{P}_{zi}^2 - \frac{\left(\sum_{i=1}^{10} P_{zi}\right)^2}{10} \quad (12)$$

where  $\bar{P}_{zi}$  is the empirical mean value of the tangential component of the cutting force.

- Let us determine criterion  $W$  from dependence

$$W_p = b^2 / S^2 \quad (13)$$

where  $b$  is the coefficient determined from dependence [40]

$$b = \sum_{i=1}^k a_{n-i+1} \times (P_{z_{n-i+1}} - P_{z_i})$$

where  $a_{n-i+1}$  is constant coefficient [40],  $k = n/2 = 10/2 = 5$ ,  $P_{z_{n-i+1}}, P_{z_i}$  are the next and previous values of force  $P_z$  in the ranked series.

- Value  $W_p$  obtained by Eq. (13) was compared with the table value of this criterion  $W_{kp}$  [40]. The given table gives the minimum value  $W_{kp}$  that would be obtained at probabilities of 1, 2, 5, 10 and 50% at different values  $n$ , if the experimental data were really subjected to the normal distribution law.

For the sample of  $n = 10$ , and probability 50% according to [40]  $W_{kp} = 0.938$ . The calculated data are presented in Table 3.

**Table 3** Calculated  $W_p$  and tabular values  $W_{kp}$  of criterion  $W$

Criterion	Feed rates $f$ , mm/rev					
	0.1	0.14	0.2	0.28	0.4	0.56
Value	0.943	0.944	0.973	0.953	0.967	0.953
$W_p$	0.938					
$W_{kp}$	0.938					

Analysis of the data from Table 3 shows that for all feed rates  $W_p > W_{kp}$ . This gives us the right to assert that the hypothesis of the normal distribution of random values  $P_{zfi}$ , obtained at all investigated feed rates, has been confirmed.

**6 Determination of dispersion characteristics of random normally distributed values  $P_{zfi}$**

In the case of rejecting certain data as statistically insignificant, it is necessary to redefine the characteristics of the dispersion of random variables  $P_{zfi}$ . In addition, the correlation coefficient  $K_{var}$  and maximum values of the tangential components of cutting forces  $P_{zmax}$  were determined.

After processing the data by the method of the least squares, we obtained dependencies for determining the average value of  $\bar{P}_z$

$$\bar{P}_z = 3776 \times f^{0.705} \times t \tag{14}$$

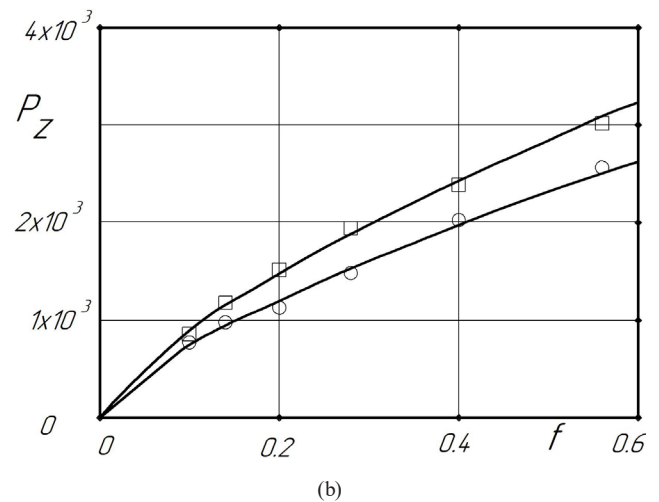
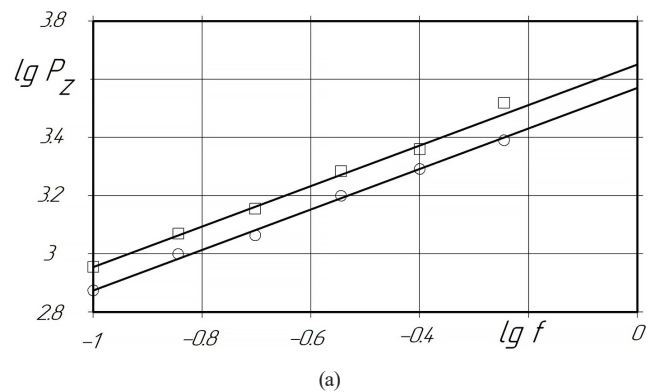
However, the calculated value should not be the average one, but its maximum value. The maximum value obtained experimentally will be determined from dependence

$$P_{zmax} = \bar{P}_z + 3\sigma(P_z) \tag{15}$$

After constructing the graph and processing the data using the method of the least squares and logarithmic coordinates, we obtained:

$$P_{zmax} = 4620 \times f^{0.711} \times t \tag{15}$$

The graphs of dependences Eq. (14) and Eq. (15) in logarithmic coordinates are presented in Fig. 3 (a), and in Cartesian – in Fig. 3 (b) Sample characteristics, as well as



**Fig. 3** Graphs of dependencies  $\bar{P}_z$  and  $P_{zmax}$  on the feed rate, respectively, in (a) logarithmic and (b) Cartesian coordinates: maximum  $P_{zmax}$  marked by a dotted line with squares; average  $\bar{P}_z$  circles marked by a dotted line with circles

some other calculated data are presented in Table 4. Thus, the resulting graph of the maximum cutting force  $P_{zmax}$  is the desired dependence while machining a certain class of materials (in this case, ISO P class). The computer program for applying this method, which makes it possible to obtain graphical and/or analytical dependencies of the tangential component of the cutting force on the feed rate by entering data has been developed. This method is universal and can be used for drilling, countersinking, boring, etc.

**Table 4** Sample characteristics of the dispersion of the random variable  $P_z(N)$ , obtained during the processing of experimental data

Dispersion characteristics of the random variable $P_z$	Feed rates, $f$ , mm/rev					
	0.1	0.14	0.2	0.28	0.4	0.56
Mathematical expectation, $M(P_z) \approx \bar{P}_z(N)$	751	950	1191	1545	1993	2515
Dispersion $D(P_z), N^2$	2470	4360	9840	12450	26000	48700
Mean square deviation, $\sigma(P_z)$ ,	49.7	66.0	99.2	115.6	161.2	220.68
Coefficient of variation $K_{var}, \%$	6.62	6.95	8.33	7.48	8.08	8.77
Maximum value, $P_{zmax} = \bar{P}_z + 3\sigma(N)$	900.1	1148.0	1488.6	1891.8	2426.6	3177.04
$P_{zmax} - M(P_z)$	149.1	198	297.6	346.8	433.6	662.04
$P_{zmax} / M(P_z)$	1.199	1.208	1.250	1.224	1.218	1.266

## 7 Conclusions

The conclusions are the following:

1. The proposed method of determining the interdependence between the feed rate and the tangential component of the cutting force based on a small number of experiments using the theory of small sample makes it possible to save resources and obtain statistically reliable results
2. As the feed rate increases, the tangential component of the cutting force increases, and as the feed rate increases, the difference between the maximum and average cutting force increases. With the increase in feed rate, the coefficient of variation increases from 6.62 to 8.87% (at the feed rate of 0.56 mm/rev), indicating the increase in process variability. However, since  $K_{var} < 10$ , the process variability remains weak. It is

recommended to combine the classes and subclasses of the investigated materials in such a way that  $K_{var} < 10$ . If  $K_{var} > 10$ , then it is necessary to obtain separate dependencies for this class or subclass.

3. As the result of the experimental investigations, there was no trend in the influence of the feed rate on ratio  $P_{zmax}/M(P_z)$ , which was the lowest 119.9% (at  $f = 0.1$  mm/rev) and the highest 126.6% (at  $f = 0.56$  mm/rev).

## 8 Investigation perspective

This method can be applied to other types of machining while testing both new tool sharpening and plates of new design. It makes it possible to obtain and analyze small amounts of data at low cost using mathematical apparatus, obtaining statistically reliable results.

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