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System-theory Approach to the Operation of Heat Exchangers in District Heating Systems

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Abstract

In this study, the systematic analysis, design, and operation of district heating systems (DHS) are investigated. Particular emphasis is placed on the characterization of the heat transfer processes occurring between the primary and secondary circuits in indirect district heating configurations, with special attention given to the modeling of heat substations. The governing heat transfer equations applicable to heat substations are derived, and both the direct (fundamental) and inverse problem formulations are established. Based on the developed models, operational working points are defined in accordance with varying consumer-side heat demands. The fundamental heat balance equations are derived, input and output variables are identified, and solutions for the output variables are obtained, thereby determining the system's operating point under known boundary conditions. The analysis is limited to steady-state operation and steady-state operating points. Furthermore, an optimization framework is proposed, applicable from both the service provider's and the end-user's perspectives. The presented models are inherently adaptable to both viewpoints.

Keywords

substations, heat exchanger, DHW, input/output, capacity, optimization

1 Introduction

District heating is a significant sector of the energy industry in countries with suitable climatic conditions, playing a vital role across Europe. Millions of residential units, public institutions, and industrial facilities benefit from district heating systems (DHS) throughout the continent [1, 2]. With the increasing integration of renewable energy sources, district heating is gaining even more importance [3, 4]. These systems are uniquely suited for accommodating renewable sources, such as geothermal energy and biomass, and for large-scale applications [5–7]. DHS enable the highest efficiency in energy production, particularly through combined heat and power (CHP) generation. The environmental significance of district heating is well-documented [8, 9]. However, it is a complex technology that requires a high level of engineering knowledge and expertise for both design and operation. Currently, district heating networks (DHNs) are characterized by high supply temperatures and centralized heat generation plants [10]. The development of fourth-generation district heating (4GDH) and smart heat networks that meet the heating demands of buildings is crucial for transitioning to sustainable energy systems [11, 12]. The design and operation of heating systems and district heating substations significantly influence the required supply temperatures and achievable return temperatures. Although design temperatures in new buildings are relatively low, several studies have reported difficulties in achieving reduced return temperatures even in modern constructions [13]. Recent research has investigated the relationship between DHS and the heating requirements of buildings, in particular by optimizing the supply and return temperatures achievable in the heat supply. Operators aim [14] to reduce these temperatures, thus increasing the efficient use of renewable energy sources. The efficiency of low-temperature DHS can

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be increased by the use of automatic balancing, advanced hot water services and digital capacity monitoring systems. Mathematical models [15] of some components of district heating substations, such as heat exchangers (HE), control valves, three-way valves and frequency-controlled pumps, can help to optimize them and increase their efficiency. These models can be used to simulate real systems based on real data, facilitating the efficient operation of real-time district heating plants. The operation of conventional hot water supply systems, especially those with storage tanks and circulating loops, are receiving increasing attention for the efficiency and sustainability of district heating. The aim of Benakopoulos et al. [16] is to present the potential for low temperature hot water supply, taking into account energy balance calculations and tests in real buildings. The analysis of hot water production and simultaneity factors is important as they have a significant impact on the energy balance and return temperatures. For example [17], lower building density can result in more favorable return temperatures, while instantaneous systems can provide 8–9 °C lower return temperatures.

Achieving low flow and return temperatures is key to the efficient operation of district heating services, which requires consumer substations and secondary heating systems to operate optimally with flawless temperature control [18]. Over the decades, it has become widely recognized among district heating operators that identifying temperature faults in existing systems [19] and designing consumer equipment [20] properly can help reduce return temperatures. Temperature faults can occur in different parts of the system but are generally more significant near the consumer equipment [21, 22]. Temperature faults in DHS and optimization options are discussed in detail [23].

In recent years, a number of studies and models have been developed to optimize DHNs. Some predictive models [24] are able to predict the capacity of an urban DHS for a given period and determine the optimal operating conditions and system parameters that minimize costs and maximize energy efficiency while ensuring that all heat demand is met. To improve the operation of low temperature DHS, various optimization tools are available to help minimize heat production and distribution costs. MODEST software, for example, uses linear programming techniques to reduce capital costs, while CPLEX and GAMS systems are used to solve complex mathematical problems [25].

In summary, the future of district heating is closely linked to the increasing use of renewable energy sources, the achievement of lower operating temperatures and the drive to increase system efficiency.

In this paper, the systemic analysis, system-based design, and operation of DHS are investigated. Particular attention is given to the description of the heat transfer process between the primary and secondary systems in indirect district heating configurations, with an emphasis placed on the modeling of heat substations. The heat transfer equations occurring within the substations are derived, and both the base and inverse tasks are formulated. The investigations are conducted using input/output models with the objective of simulating the operation of heat substations. Based on the presented models, operational working points are defined and formulated to accommodate variations in consumer heat demand. The fundamental heat transfer balance equations are derived, the input and output variables are specified, and the equations are solved for the output variables, which, together with the known input variables, determine the operating point. Only stationary operation modes and stationary operating points are considered. Temporal transitions between operating points and the characterization of transient phenomena are not addressed in this study.

2 The energy balance of heating substations

The basic components of a DHS are illustrated in Fig. 1.

2.1 The structure and connection of heating substations

The system highlights the heat substation and demonstrates its connections to both the primary and secondary systems. The secondary system consists of a heating system and a domestic hot water (DHW) subsystem. In the heat substation, the components responsible for heat transfer are the heating and DHW heat exchangers.

The efficiency and economic viability of heat exchangers in DHS have been studied in [26, 27]. In systems that provide DHW, a hot water storage tank is also used. Circulation in both the primary and secondary loops is maintained by circulation pumps. Regulation and set-point adjustment are managed through a computer-based process control system, which is not specifically shown in Figs. 2 and 3. The operation of bypass valves has been analyzed [28] to minimize

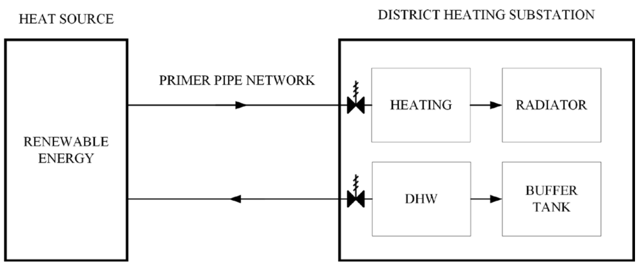

Fig. 1 Conceptual structure of a district heating system

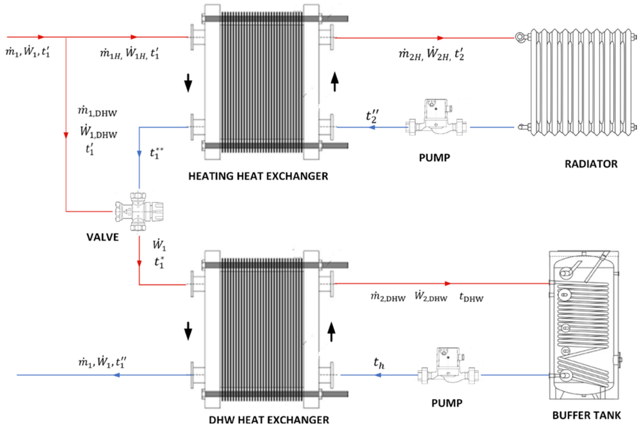

Fig. 2 Block diagram of a heat substation with sequentially connected heating and DHW heat exchangers on the primary side

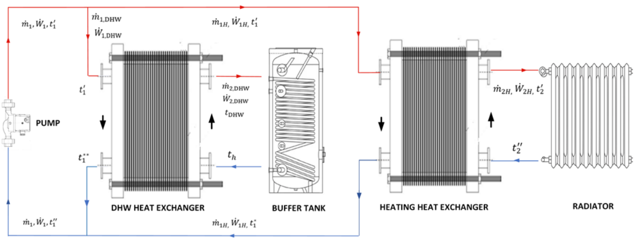

Fig. 3 Block diagram of a heat substation with parallel connections on the primary side

reduced temperature differences, and it was concluded that there is no single solution suitable for all cases.

Heat substations can be configured either in series or parallel based on the connection of the heating and DHW heat exchangers. Fig. 2 illustrates a series connection, while Fig. 3 shows a parallel connection.

2.2 The heat transfer equations of heat exchangers

The thermal sizing and performance analysis of heat exchangers rely on accurate heat transfer and energy balance equations. This section presents the fundamental formulas for heating and DHW heat exchangers in series connection without bypass operation. Key parameters such as the logarithmic mean temperature difference, mass flow ratio, and effectiveness are highlighted for system evaluation:

- Heating heat exchanger balance equations – serial connection without bypass operation:
  The heat transferred through the heating heat exchanger is depicted with the primary side in a serial connection, without bypass operation, as shown in Fig. 2:

$$\dot{Q} = kA\Delta t_k, \tag{1}$$

where:

$$\Delta t_k = \frac{t_1' - t_2' - \left(t_1^{**} - t_2''\right)}{\ln \dfrac{t_1' - t_2'}{t_1^{**} - t_2''}}. \tag{2}$$

If the nominal, rated, and maximum temperature differences for the primary and secondary fluids are known, the logarithmic mean temperature difference Δt_k can be determined, and using this, the size of the heating heat exchanger can be calculated:

$$kA = \frac{\dot{Q}}{\Delta t_k}. \tag{3}$$

The ratio of the circulating mass flows between the primary and secondary systems:

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{t_2' - t_2''}{t_1' - t_1''}, \tag{4}$$

where the larger circulating mass flow occurs in the secondary system.
Heat capacity rates is shown in Eq. (5):

$$\dot{W}_1 = \dot{m}_1 c,\ \ \dot{W}_2 = \dot{m}_2 c. \tag{5}$$

The larger heat capacity rate is in the secondary system. The effectiveness of the heating heat exchanger in the case of a counterflow heat exchanger:

$$\varphi_H\left(\dot{W}_1, \dot{W}_2\right) = \frac{t_1' - t_1^{**}}{t_1' - t_2''}. \tag{6}$$

The analytical expression of the effectiveness in Eq. (6) can be derived from the ε–NTU (effectiveness–number of transfer units) method, which takes into account the thermal conductance (kA) and the heat capacity rates of both the primary and secondary circuits. The detailed formula is:

$$\varphi_H = \frac{1 - e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_2}\right)\frac{kA}{\dot{W}_1}}}{1 - \dfrac{\dot{W}_1}{\dot{W}_2} e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_2}\right)\frac{kA}{\dot{W}_1}}}. \tag{7}$$

- DHW heat exchanger mass balance equations – series connection without bypass operation:
  Calculation of heat transferred in the DHW heat exchanger according to the notation in Fig. 2:

$$\dot{Q}_{\mathrm{DHW}} = kA\Delta t_k, \tag{8}$$

where:

$$\Delta t_k = \frac{t_1^{**} - t_{\mathrm{DHW}} - \left(t_1'' - t_h\right)}{\ln \dfrac{t_1^{*} - t_{\mathrm{DHW}}}{t_{1v} - t_h}},\ \ t_1^{*} \equiv t_1^{**}. \tag{9}$$

If the nominal DHW demand is known, along with the inlet and outlet fluid temperatures in the heat exchanger, the required size of the DHW heat exchanger can be determined:

$$kA = \frac{\dot{Q}_{\text{DHW}}}{\Delta t_k}. \tag{10}$$

The efficiency of the DHW heat exchanger in a counterflow heat exchanger configuration:

$$\varphi_{\text{DHW,HE}} = \frac{t_1^* - t_1''}{t_1^* - t_h}. \tag{11}$$

In Eq. (11), the determination of $\varphi_{\text{DHW,HE}}$ involves the heat capacity rates of the fluids:

$$\varphi_{\text{DHW,HE}} = \frac{1 - e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_{2,\text{DHW,HE}}}\right)\frac{(kA)_{\text{DHW,HE}}}{\dot{W}_1}}}{1 - \frac{\dot{W}_1}{\dot{W}_{2,\text{DHW,HE}}} e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_{2,\text{DHW,HE}}}\right)\frac{(kA)_{\text{DHW,HE}}}{\dot{W}_1}}}. \tag{12}$$

2.3 Determining the outlet characteristics on the primary side of a heat exchanger, given the inlet characteristics, in a series connection

This task is referred to as the fundamental task. Fig. 4 presents the system input/output model, where the heating heat exchangers are represented as white boxes. The equations that establish the relationships between the input/output variables are included within these white boxes. Additionally, the temperature profiles are also displayed.

According to Fig. 5, the following variables are considered known: $\dot{m}_1$, t_1', $\dot{m}_2$, t_2'', t_h, $\dot{m}_{\text{DHW,HE}}$, $\dot{W}_1$, $\dot{W}_2$, $\dot{W}_{2,\text{DHW,HE}}$, $(kA)_H$ and $(kA)_{\text{DHW,HE}}$. The unknown variables at the operating point are: t_2^*, t_2', t_1'', t_{DHW}, $\dot{Q}_H$, $\dot{Q}_{\text{DHW,HE}}$.

In Section 2.4, the behavior of the output characteristics will be determined based on the known input characteristics for heating and DHW heat exchangers connected in series, within the framework of the fundamental task:

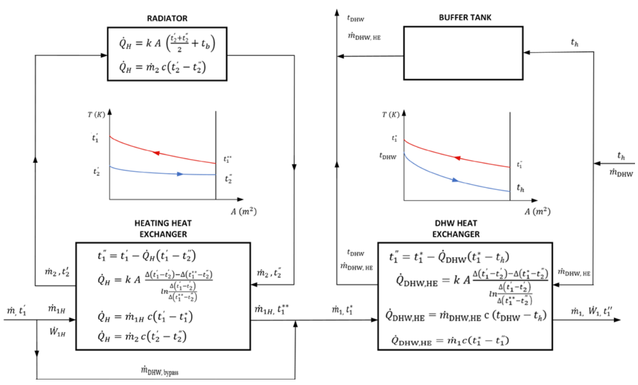

Fig. 4 White box model of a two heat exchanger heat substation on the primary side with serial connection in forward flow mode

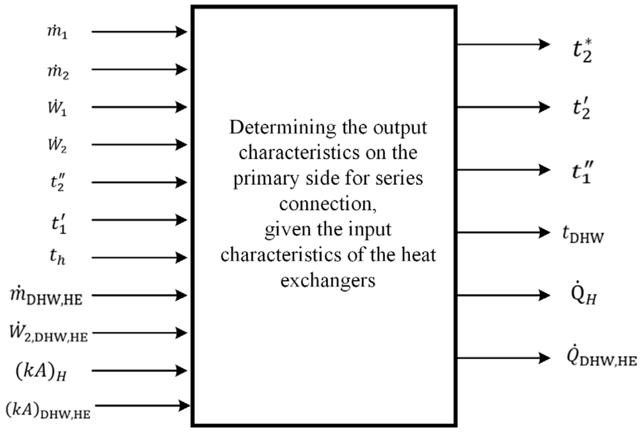

Fig. 5 Relationship between the input and output characteristics on the primary side in a serial connection for the fundamental task

- Heating heat exchanger mass balance equations:
  The Bosnjakovic φ-factor of the heating heat exchanger is calculated in Eq. (13):

$$\varphi_H = \frac{1 - e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_2}\right)\frac{(kA)_H}{\dot{W}_1}}}{1 - \frac{\dot{W}_1}{\dot{W}_2} e^{-\left(1 - \frac{\dot{W}_1}{\dot{W}_2}\right)\frac{(kA)_H}{\dot{W}_1}}} = \frac{t_1' - t_1^*}{t_1' - t_2''}. \tag{13}$$

The exit temperature of the primary hot water from the heating heat exchanger is calculated in Eq. (14):

$$t_1^* = t_1' - \varphi_H \left(t_1' - t_2''\right). \tag{14}$$

The heat transferred from the heating heat exchanger is calculated, with no bypass $t_1^* \equiv t_1^{**}$:

$$\dot{Q}_H = \dot{m}_1 c \left(t_1' - t_1^*\right) = \left(k_{H,\text{HE}} A_{H,\text{HE}}\right) \frac{\left(t_1' - t_2'\right) - \left(t_1^* - t_2''\right)}{\ln \frac{t_1' - t_2'}{t_1' - t_2''}}, \tag{15}$$

$$\dot{Q}_H = (kA)_{rad} \left(\frac{t_2' + t_2''}{2} - t_b\right). \tag{16}$$

The secondary supply hot water temperature is calculated in Eq. (17):

$$t_2' = t_2'' + \frac{\dot{Q}_H}{\dot{W}_2}. \tag{17}$$

The $(kA)_{rad}$ must be such that it satisfies this condition!

- DHW heat exchanger balance equations:
  The Bosnjakovic φ-factor of the DHW heat exchanger is calculated in Eq. (18). All necessary input data is available for this calculation:

$$\varphi_{\mathrm{DHW}} = \frac{1 - e^{-\left(1-\frac{\dot{W}_{1,\mathrm{DHW,HE}}}{\dot{W}_{2,\mathrm{DHW,HE}}}\right)\frac{(kA)_{\mathrm{DHW,HE}}}{\dot{W}_1}}}{1 - \frac{\dot{W}_1}{\dot{W}_{2,\mathrm{DHW,HE}}} e^{-\left(1-\frac{\dot{W}_{1,\mathrm{DHW,HE}}}{\dot{W}_{2,\mathrm{DHW,HE}}}\right)\frac{(kA)_{\mathrm{DHW,HE}}}{\dot{W}_1}}}. \tag{18}$$

The exit temperature of the primary hot water from the DHW heat exchanger is calculated in Eq. (19):

$$t_1'' = t_1^* - \varphi_{\mathrm{DHW,HE}} \left(t_1^* - t_h\right). \tag{19}$$

The heat transferred from the DHW heat exchanger is calculated in Eq. (20):

$$\dot{Q}_{\mathrm{DHW,HE}} = \dot{m}_1 c \left(t_1^* - t_1''\right). \tag{20}$$

The secondary forward DHW temperature is calculated in Eq. (21):

$$t_{\mathrm{DHW}} = t_h + \frac{\dot{Q}_{\mathrm{DHW,HE}}}{\dot{W}_{2,\mathrm{DHW,HE}}}. \tag{21}$$

If the condition $t_{\mathrm{DHW}} \geq t_{\mathrm{DHW,design}}$ is not met, the DHW inlet temperature must be adjusted, for example, by applying priority operation.

2.4 Given the input characteristics of the heat exchangers, the output characteristics on the primary side for a purely parallel-connected heat substation are determined

Fig. 3 illustrates the parallel connection, and the task is defined in Fig. 6, where the following parameters are known: $\dot{m}_1$, t_1', $\dot{m}_2$, t_2'', t_h, $\dot{m}_{\mathrm{DHW,HE}}$, $(kA)_H$, $(kA)_{\mathrm{DHW},HE}$, $\dot{W}_{1H}$, $\dot{W}_{2H}$, $\dot{W}_{2,\mathrm{DHW,HE}}$. The unknown variables in the operating point are: t_1^*, t_1^{**}, t_1'', $\dot{Q}_H$, $\dot{Q}_{\mathrm{DHW,HE}}$. This setup suggests that a system of equations can be used to solve for the unknowns, utilizing known inputs and calculated outputs based on heat exchanger relationships and operational conditions in the system.

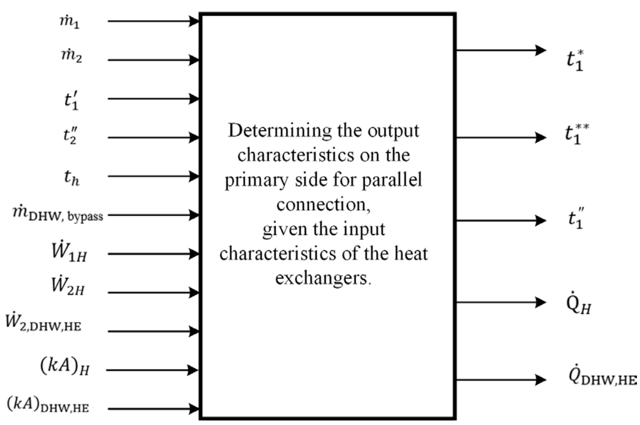

Fig. 6 Relationship between the input and output characteristics for parallel connection

In the following, we will determine the sequence of the output characteristics based on the input characteristics for the heating and DHW heat exchangers in the case of parallel connection:

- Heating heat exchanger mass balance equations:
  The Bosnjakovic φ-factor of the DHW heat exchanger is calculated. All necessary input data is available for Eq. (22):

$$\varphi_H\left(\dot{W}_{1H}, \dot{W}_{2H}, (kA)_H\right) = \frac{1 - e^{-\left(1-\frac{\dot{W}_{1H}}{\dot{W}_{2H}}\right)\frac{(kA)_H}{\dot{W}_{1H}}}}{1 - \frac{\dot{W}_{1H}}{\dot{W}_{2H}} e^{-\left(1-\frac{\dot{W}_{1H}}{\dot{W}_{2H}}\right)\frac{(kA)_H}{\dot{W}_{1H}}}} = \frac{t_1' - t_1^*}{t_1' - t_2''}, \tag{22}$$

The exit temperature of the primary hot water from the heating heat exchanger is calculated in Eq. (23):

$$t_1^* = t_1' - \varphi_H \left(t_1' - t_2''\right). \tag{23}$$

The heat transferred is calculated in Eq. (24):

$$\dot{Q}_H = \varphi_H \left(t_1' - t_2''\right) \dot{W}_{1,H}. \tag{24}$$

As is well known (Eq. (25)):

$$\dot{Q}_H = \dot{W}_{2f} \left(t_1' - t_2''\right). \tag{25}$$

Therefore, the forward secondary heating temperature is calculated in Eq. (26):

$$t_2' = t_2'' + \frac{\dot{Q}_H}{W_{2H}}. \tag{26}$$

- DHW heat exchanger balance equations can be defined:

$$\varphi_{\mathrm{DHW}}\left(\dot{W}_{1\mathrm{DHW}}, \dot{W}_{2\mathrm{DHW}}, (kA)_{\mathrm{DHW,HE}}\right) = \frac{1 - e^{-\left(1-\frac{\dot{W}_{1\mathrm{DHW}}}{\dot{W}_{2\mathrm{DHW}}}\right)\frac{(kA)_{\mathrm{DHW,HE}}}{\dot{W}_1}}}{1 - \frac{\dot{W}_{1\mathrm{DHW}}}{\dot{W}_{2,\mathrm{DHW,HE}}} e^{-\left(1-\frac{\dot{W}_{1\mathrm{DHW}}}{\dot{W}_{2\mathrm{DHW,HE}}}\right)\frac{(kA)_{\mathrm{DHW,HE}}}{\dot{W}_1}}}, \tag{27}$$

whereas:

$$\varphi_{\mathrm{DHW}} = \frac{t_{\mathrm{DHW}} - t_H}{t_1' - t_H} = \frac{\dot{Q}_{\mathrm{DHW}}}{\dot{W}_{2\mathrm{DHW}} \left(t_1' - t_H\right)} = \frac{\dot{W}_{1\mathrm{DHW}} \left(t_1' - t_1^{**}\right)}{\dot{W}_{2\mathrm{DHW}} \left(t_1' - t_H\right)}, \tag{28}$$

therefore:

$$t_1^{**} = t_1' - \varphi_{\mathrm{DHW}} \left(t_1' - t_H\right) \frac{\dot{W}_{2\mathrm{DHW}}}{\dot{W}_{1\mathrm{DHW}}}. \tag{29}$$

The heat transferred is calculated in Eq. (30):

$$\dot{Q}_{\text{DHW}} = \varphi_{\text{DHW}} \dot{W}_{2\text{DHW}} \left(t_1' - t_H\right). \tag{30}$$

The temperature of the produced DHW is calculated in Eq. (31):

$$t_{\text{DHW}} = t_H + \frac{\dot{Q}_{\text{DHW}}}{W_{2\text{DHW}}}. \tag{31}$$

The temperature of the returning primary hot water after mixing is calculated in Eq. (32):

$$t_1'' = \frac{\dot{W}_{1\text{DHW}}}{\dot{W}_1} t_1^{**} + \frac{\dot{W}_{1H}}{\dot{W}_1} t_1^{*}. \tag{32}$$

Finally, the secondary forward DHW temperature is calculated. If it does not meet the required temperature value, a new operating point must be set.

2.5 Determining the necessary input characteristics for series-connected heat substations on the primary side, when applying bypass control, based on the performance requirements imposed on the heat exchangers

This task is referred to as an inverse problem. According to Fig. 7, the known parameters are: $\dot{m}_1$, $\dot{m}_2$, $\dot{W}_1$, $\dot{W}_2$, t_2'', $\dot{W}_{2,\text{DHW,HE}}$, t_h, t_{DHW}, $\dot{Q}_{\text{DHW,HE}}$, $\dot{Q}_H$. At the operating point, the following output parameters are to be determined: $\dot{m}_{1H}$, $\dot{W}_{1,H}$, $\dot{m}_{\text{DHW,bypass}}$, t_1'', t_1^*, t_1'.

The process of calculating the exit characteristics is determined based on the known input characteristics for the heating and DHW heat exchangers in the case of a series connection in an inverse task:

- DHW heat exchanger balance equations:
  The capacity of the DHW heat exchanger is known:

$$\dot{Q}_{\text{DHW,HE}} = \dot{W}_1 \left(t_1^* - t_1''\right). \tag{33}$$

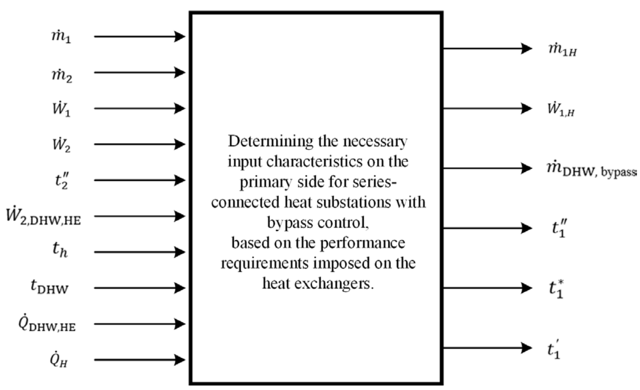

Fig. 7 Relationship of input and output characteristics in a series connection with bypass operation mode

The exit temperature of the primary hot water is calculated in Eq. (34):

$$t_1'' = t_1^* - \varphi_{\text{DHW}} \left(t_1^* - t_h\right), \tag{34}$$

which:

$$\varphi_{\text{DHW}} = \varphi_{\text{DHW}} \left(\dot{W}_1, \dot{W}_{2,\text{DHW,HE}}, (kA)_{\text{DHW,HE}}\right), \tag{35}$$

can be calculated based on the known data.

The entering temperature to the DHW heat exchanger can be calculated from the two equations:

$$t_1^* = t_H + \frac{\dot{Q}_{\text{DHW,HE}}}{\varphi_{\text{DHW}} \times \dot{W}_1}. \tag{36}$$

Exit temperature of the primary hot water from the DHW heat exchanger is calculated in Eq. (37):

$$t_1'' = t_1^* - \frac{\dot{Q}_{\text{DHW,HE}}}{\dot{W}_1}. \tag{37}$$

- Heating heat exchanger mass balance equations:
  The values calculated according to Eqs. (34) and (37) should be the same. Temperature of the hot water leaving the heating exchanger is calculated in Eq. (38):

$$t_1^{**} = t_1' - \frac{\dot{Q}_H}{\dot{W}_1}. \tag{38}$$

If the t_1^{**} calculated from Eq. (38) is smaller than t_1^* calculated from Eq. (36), bypass control must be applied. In the bypass control, the determination of the mass flow rates is the objective, meaning that Eqs. (39) and (40) must be satisfied:

$$\dot{m}_1 = \dot{m}_{1,\text{DHW,bypass}} + \dot{m}_{1,H}, \tag{39}$$

and:

$$\dot{W}_1 = \dot{W}_{1,\text{DHW,bypass}} + \dot{W}_{1,H}. \tag{40}$$

During the bypass operation period, the value of t_1' is assumed to be constant. Bypass control must be implemented in such a way that the mass flow distribution ensures that the t_1^{**} hot water exiting the heating heat exchanger, when mixed with the hot water coming through the bypass branch, results in the hot water required by the DHW heat exchanger at a temperature t_1^*. Equations (41) to (43) can be written:

$$\dot{W}_{1,H} t_1^{**} + \dot{W}_{1,\text{DHW,bypass}} t_1' = \dot{W}_1 t_1^*, \tag{41}$$

$$\dot{W}_{1,H} + \dot{W}_{1,\text{DHW,bypass}} = \dot{W}_1, \tag{42}$$

$$\frac{1-e^{-\left(1-\frac{\dot{W}_{1,H}}{\dot{W}_{2,H}}\right)\frac{(kA)_H}{\dot{W}_{1,H}}}}{1-\frac{\dot{W}_{1,H}}{\dot{W}_{2,H}}e^{-\left(1-\frac{\dot{W}_{1,H}}{\dot{W}_{2,H}}\right)\frac{(kA)_H}{\dot{W}_{1,H}}}}=\frac{t_1'-t_1^{**}}{t_1'-t_2''}=\varphi_H. \tag{43}$$

In Eqs. (41) to (43), the following characteristics are unknowns:

$\dot{W}_{1,H}, \dot{W}_{1,\mathrm{DHW,bypass}}, t_1^{**}$.

It is assumed that the values of $\dot{W}_1$, t_1^* and t_1' are known. The result of the bypass operation will be a change in the heat transferred by the heating heat exchanger, which will decrease. The task involves solving the system of nonlinear Eqs. (41) to (43), which are mathematically defined.

2.5.1 The solution process in the case of a serial bypass configuration

In systems where a serial bypass configuration is applied, the distribution of flow and thermal energy becomes more complex due to the interaction between the primary heating circuit and the DHW loop. To determine the thermal balance and operating point of the heating heat exchanger, a step-by-step substitution and simplification of the governing equations is required:

$$\dot{W}_{1,\mathrm{DHW,bypass}}=\dot{W}_1-\dot{W}_{1,H}. \tag{44}$$

Substitute this into Eq. (41):

$$\dot{W}_{1,H}t_1^{**}+\left(\dot{W}_1-\dot{W}_{1,H}\right)t_1'=\dot{W}_1 t_1^*, \tag{45}$$

sorted by:

$$\dot{W}_1\left(t_1'-t_1^*\right)=\dot{W}_{1,H}\left(t_1'-t_1^{**}\right). \tag{46}$$

From Eq. (46):

$$\dot{W}_{1,H}=\dot{W}_1\frac{\left(t_1'-t_1^*\right)}{\left(t_1'-t_1^{**}\right)}. \tag{47}$$

whereas:

$$\varphi_H=\frac{t_1'-t_1^{**}}{t_1'-t_2''}, \tag{48}$$

with Eq. (47):

$$\dot{W}_{1,H}=\dot{W}_1\left(t_1'-t_1^*\right)\frac{1}{\varphi_H\left(t_1'-t_1''\right)}=\dot{W}_1\frac{t_1'-t_1^*}{t_1'-t_1''}\frac{1}{\varphi_H}. \tag{49}$$

On the other hand:

$$\varphi_H=\varphi_H\left(\frac{\dot{W}_{1,H}}{\dot{W}_2}\right). \tag{50}$$

In Eq. (42), where $\dot{W}_{1,H}$ is the unknown, Eq. (49) only contains one unknown, namely $\dot{W}_{1,H}$. From Eq. (49), $\dot{W}_{1,H}$ can be determined, though only through iteration. After the calculation, the operating point of the heating heat exchanger must be recalculated, including the heat output and the exit temperatures.

2.6 Determination of optimum inlet characteristics for the required outlet characteristics of heat exchangers for primary-side heat exchangers with series connection

The objective of this task aligns with that of the fundamental task (Section 2.5), with the key difference being that the primary supply water temperature t_1' and the primary hot water flow rate $\dot{m}_1$ and $\dot{W}_1$ are derived as a result of optimization. In other words, while both tasks aim to achieve the same end goal, in this case, the optimal settings for the primary water flow and temperature are calculated through the optimization process, leading to different operational parameters compared to the fundamental task.

2.7 Determination of the inlet characteristics on the primary side for pure parallel connection, knowing the outlet characteristics of the heat exchangers

This task is called an inverse problem. According to Fig. 8, the known parameters at the operating point are $\dot{Q}_H$, t_1^*, t_2', t_2'', $\dot{W}_{2H}$, $(kA)_H$ and the following output parameters are to be determined: t_1', $\dot{W}_{1,H}$.

In the following, we determine the flow of the output characteristics for the heating and DHW heat exchangers in parallel for the inverse task:

- Heating heat exchanger mass balance equations:
  Heat exchanger capacity is calculated in Eq. (51):

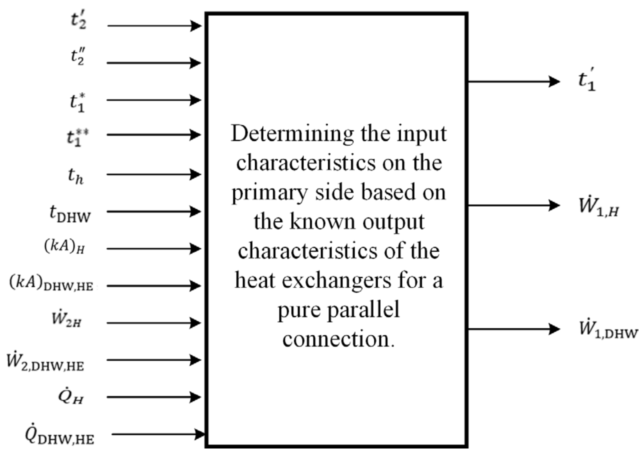

Fig. 8 Relationship between input and output characteristics in parallel connection configuration

$$\dot{Q}_H = \frac{\left(t_1' - t_2'\right) - \left(t_1^* - t_2''\right)}{\ln \dfrac{t_1' - t_2'}{t_1^* - t_2''}} \left(kA\right)_H , \quad (51)$$

From this, t_1' can be determined iteratively. Once t_1' is known, the heat transfer equation can be used to determine:

$$\dot{W}_{1H} = \frac{\dot{Q}_H}{t_1' - t_1^*} . \quad (52)$$

- DHW heat exchanger balance equations, whereas:

$$\dot{Q}_{\mathrm{DHW}} = \frac{\left(t_1' - t_{\mathrm{DHW}}\right) - \left(t_1^{**} - t_H\right)}{\ln \dfrac{t_1' - t_{\mathrm{DHW}}}{t_1^{**} - t_H}} , \quad (53)$$

from t_1' can be determined by iteration.
The value t_1' obtained from the heat exchanger calculation is compared with the value t_1' obtained from the heat exchanger calculation and the higher value is retained.
The primary heat capacity rate at the DHW heat exchanger is calculated in Eq. (54):

$$\dot{W}_{1\mathrm{DHW}} = \frac{\dot{Q}_{\mathrm{DHW}}}{t_1' - t_1^{**}} . \quad (54)$$

3 Optimal district heating substations design

3.1 Determination and sizing of the main parameters of a compact, variable mass flow, two heat exchanger, variable connection (series-parallel) district heating substation

For sizing and selecting the heating center, it is essential to know the heating and DHW heat demand that will be supplied. For heating demand, the design daily average heating capacity at a 99% reliability level must be known, while for DHW heat demand, the design daily peak demand duration diagram (with a 99% reliability level) must be understood. The sizing process aims for an economic optimum, which differs for the service provider and the investor/consumer. For the service provider, it is advantageous if the consumer uses as little mass flow as possible and returns the water as cooled as possible. Ensuring these two aspects leads to a situation where the volume flow of primary hot water circulated through the network – and consequently the circulating work – increases by the smallest possible amount, and the heat loss in the primary network either remains unchanged or decreases due to the low return water temperature.

The operator of the heating center aims to minimize the costs related to the investment in the heating center. They strive to install the smallest possible heat exchangers and hot water storage tanks, while also attempting to minimize the amount of connected primary hot water. There is an intermediate optimum when considering the sizes of the DHW heat exchanger and the hot water storage tank. If a larger hot water storage tank is chosen, the size of the DHW heat exchanger must be increased, and vice versa. Minimizing the amount of connected primary hot water results in a decrease in the return hot water temperature, but it increases the required heat exchanger surface area.

3.2 District heating substation design based on the principle of economic optimization from the investor's perspective

In district heating systems, optimizing the substation design from an economic standpoint is essential for minimizing total investment and operational costs. This section presents a method that determines the optimal configuration by combining thermodynamic sizing with economic evaluation. The procedure starts by selecting a storage volume and analyzing its impact on component sizing and cost distribution:

- The primary supply temperature t_1' is assumed to be given.
- Let's choose a storage size.
- This allows us to determine the required capacity of the DHW heat exchanger $\dot{Q}_{\mathrm{DHW,HE}}$.
- Write Eqs. (55) to (57) for the DHW heat exchanger:

$$\dot{Q}_{\mathrm{DHW,HE}} = \dot{m}_1 c \left(t_1^* - t_1''\right), \quad (55)$$

$$\dot{Q}_{\mathrm{DHW,HE}} = kA_{\mathrm{DHW,HE}} \frac{\left(t_1^* - t_{\mathrm{DHW}}\right) - \left(t_1'' - t_h\right)}{\ln \dfrac{t_1^* - t_{\mathrm{DHW}}}{t_1'' - t_h}} , \quad (56)$$

$$\varphi\left(\dot{W}_1, \dot{W}_{2,\mathrm{DHW,HE}}\right) = \frac{t_{\mathrm{DHW}} - t_h}{t_1^* - t_h} . \quad (57)$$

In Eqs. (55) to (57) mentioned, there are four unknowns: t_1^*, t_1'', $\dot{W}_1$, $A_{\mathrm{DHW,HE}}$.

Assume the value of $\dot{W}_1\left(\dot{m}_1\right)$ and treat it as an external parameter. Solve the system of equations to obtain, t_1^*, t_1'' and $A_{\mathrm{DHW,HE}}$. Afterward, determine the size of the heating heat exchanger.

The capacity of the heating heat exchanger is:

$$\dot{Q}_H = kA_H \frac{(t_1' - t_2') - (t_1^* - t_2'')}{\ln \dfrac{t_1' - t_2'}{t_1^* - t_h}}. \quad (58)$$

In this case, t_1^* is already known. From Eq. (54), the size of the heating heat exchanger is:

$$A_H = \frac{\dot{Q}_H \ln \dfrac{t_1' - t_2'}{t_1^* - t_2''}}{k\left((t_1' - t_2') - (t_1^* - t_2'')\right)}. \quad (59)$$

Based on the known heat exchanger areas and storage volume, the annual investment and operating costs of the heating plant are determined. By varying the storage volume – while keeping the value of $\dot{m}_1$ constant – the optimal plant configuration resulting in minimal total cost is identified. For each $\dot{m}_1$ value, the corresponding optimal heating plant size is determined.

4 Results

Figs. 9 and 10 illustrates the key characteristics of the heat exchanger's operation and the mutual influence of important parameters as a function of the kA/W_1 ratio. The main characteristics include the heat exchanger efficiency (φ_H) and the outlet temperatures on both sides (t_1^* on the primary side and t_2' on the secondary side). These variables directly indicate the effectiveness of heat transfer within the system. The most important parameters affecting these characteristics are:

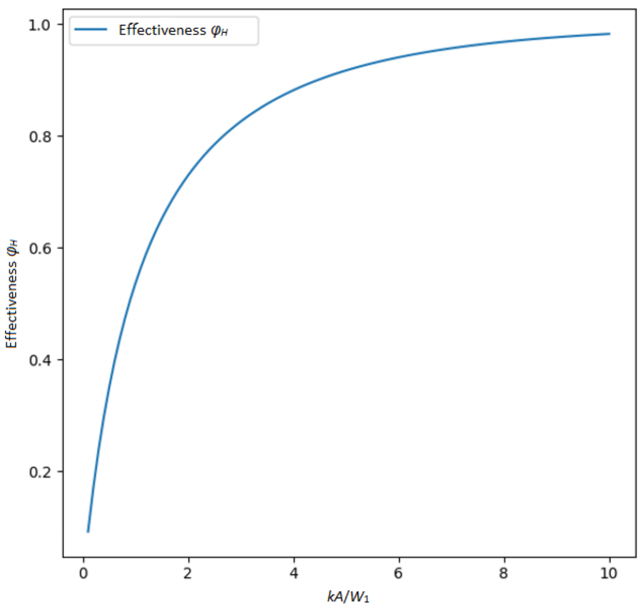

Fig. 9 Effect of the kA/W_1 ratio on heat exchanger efficiency of the heat exchangers

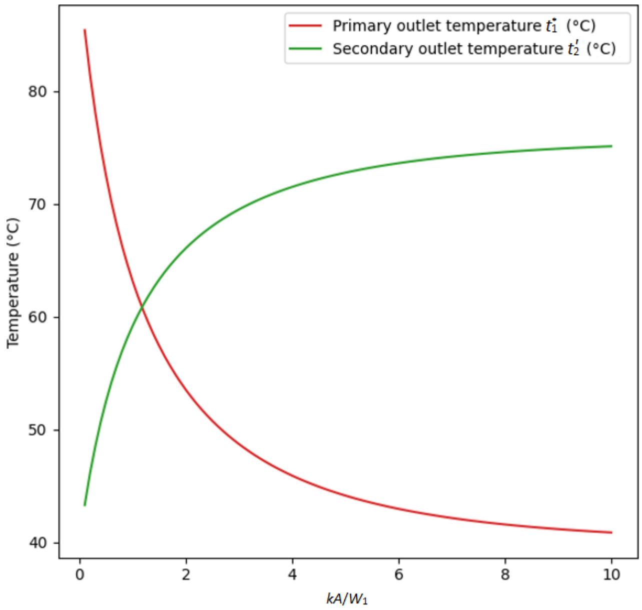

Fig. 10 Effect of the kA/W_1 ratio on the outlet temperatures of the heat exchangers

- kA: the product of the heat transfer coefficient and the heat exchanger surface area, representing the capacity of the heat exchanger. A higher kA value generally leads to more efficient heat transfer.
- W_1: the heat capacity rate on the primary side, which influences the rate of temperature change. A larger heat capacity rate tends to slow down temperature variations.
- The ratio kA/W_1: this ratio combines the effects of kA and W_1, showing how the heat exchanger's capacity relates to the thermal capacity of the primary fluid.

Fig. 9 shows that the heat exchanger efficiency increases with rising kA/W_1 values. This indicates that enhancing the heat transfer surface and capability or reducing the heat capacity rate on the primary side, leads to improved performance. Fig. 10 further illustrates this improvement through the outlet temperature trends: the primary side outlet temperature decreases, while the secondary side outlet temperature increases, signaling a more effective energy transfer. Understanding this relationship is essential for heat exchanger design, as selecting an appropriate kA value is critical for achieving optimal operation – especially under given heat capacity conditions. The interplay of these parameters supports the proper sizing of the heat exchanger to maximize efficiency while considering the system's physical characteristics.

5 Conclusions

The design and operation of DHS is a highly complex task. Due to variable weather conditions, it is often necessary to establish new operating points almost hourly to meet changing consumer heating demands. One of the most critical components of DHS control and process management is the substation. In this study, the fundamental balance equations for both the space heating and DHW heat exchangers were derived based on typical configurations. The operating points of the secondary and primary systems, as well as their interactions, were determined separately. The analyses were conducted using input/output models, enabling the parametrization and optimization of the relevant variables. An optimization approach was presented, applicable from both the service provider's and the consumer's perspectives.

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