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CALCULATIONS OF THE ASYMMETRIC RADIATION IN DIMENSIONING FOR THERMAL COMFORT

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Abstract

Asymmetrical radiation is one of the main problems of thermal comfort in prefabricated houses in Central European countries. It is due to the fact that the radiant heat loss between the human body and the relatively poorly insulated outer walls increases. The various methods for calculating radiant heat loss and heat gain are well known. However, only the method based on the thermal equilibrium of the human body provides us with precise results. The extent of asymmetrical radiation and the possible improvements can be determined by the methods of calculation and the dimensioning diagrams based on therm.

Keywords: Thermal comfort, asymmetrical radiation, thermal equilibrium, angle factor.

1. Introduction

Dimensioning of closed spaces for thermal comfort has recently become more and more obligatory. Although the current Hungarian standards do not include dimensioning for thermal comfort, it is required by the standards of EU and other developed industrial countries [1,2].

The dimensioning method is based on the heat balance of the human body. It is more or less known in detail in Hungary. General dimensioning for thermal comfort, however, has an element – the local discomfort factors – that would be of great importance regarding Hungarian buildings. As it is well known in dimensioning for thermal comfort the local discomfort factors include two factors

- asymmetric radiation,
- draught.

Both are manifest in Hungarian buildings – especially in prefabricated houses – but the number of problems related to asymmetric radiation is significantly higher.

Taking this fact and the limits on the length of this paper into consideration we would only like to concentrate on the calculations of asymmetric radiation and briefly on the issue of domestic application. The problems of draught will be analysed in our next publication on the basis of this article.

The publication is divided into three parts:

a) theory and calculation of dimensioning for thermal comfort,
b) means of calculating asymmetric radiation,
c) possibilities and characteristics of calculations regarding Hungarian buildings.

2. Theory and Calculation of Dimensioning for Thermal Comfort

The energy released by chemical combustion in the human body is partly converted to internal body heat and partly used for useful work (in the physical sense) and muscle work.

According to FANGER's theory [3] the oxidation processes in the human body consist of two parts: external mechanical power (W) and internal body heat (H). The metabolic rate (M) is also divided into two parts

$$M = H + W, \quad \mathrm{W}. \tag{1}$$

Thus external mechanical efficiency is

$$\eta = \frac{W}{M}. \tag{2}$$

Introducing this definition into *Eq.* (1) gives

$$H = M(1 - \eta), \quad \mathrm{W} \tag{3}$$

or, expressed per body unit surface area:

$$\frac{H}{A_{Du}} = \frac{M}{A_{Du}}(1 - \eta), \quad \mathrm{W/m^2}. \tag{4}$$

The DuBois Area A_{Du} defines the body surface area of humans and is calculated as

$$A_{Du} = 0.203\, G^{0.425}\, L^{0.725}, \quad \mathrm{m}^2 \tag{5}$$

where G is the weight, kp
L is the length, m

of the person.

The human body gives down its heat produced in the body (if not used a part for mechanical work) in four ways:

- by radiation,
- by convection,
- by conduction,
- by evaporation.

In the course of calculations heat loss by convection and conduction are considered as one. The percentage of the various ways of heat loss is

- radiation 42–44%,
- convection 32–35%,
- evaporation 21–26%.

The first two can be both positive and negative (heat gain and heat loss), while evaporation is only negative, i.e. heat loss.

A heat balance will exist for the human body in a technical and health sense if heat gain equals heat loss.

The basis of the calculations is the heat balance of the human body, which can be obtained using the following factors:

$$\left[\frac{H}{A_{Du}}, I_{cl}, t_a, t_{mrt}, p_a, v, t_s, \frac{E_{sw}}{A_{Du}}\right] = 0, \tag{6}$$

where	H/A_{Du}	• internal heat production per body unit, surface area,
	I_{cl}	• thermal resistance of the clothing,
	t_a	• air temperature,
	t_{mrt}	• mean radiant temperature,
	p_a	• pressure of water vapour in ambient air,
	v	• relative air velocity,
	t_s	• mean skin temperature,
	E_{sw}/A_{Du}	• heat loss per unit body surface area by evaporation of sweat secretion.

These factors are well known or can be found in professional literature, thus no further explanation is necessary. Laboratory measurements have been performed to determine mean values of skin temperature (t_s) and sweat secretion (E_{sw}), as functions of the activity level, for persons in thermal comfort. The results can be shown as

$$t_s = f\left[\frac{H}{A_{Du}}\right] \tag{7}$$

and

$$E_{sw} = A_{Du} f\left[\frac{H}{A_{Du}}\right]. \tag{8}$$

Eq. (6) can take the following form

$$f\left[\frac{H}{A_{Du}}, I_{cl}, t, t_{mrt}, p_a, v\right] = 0. \tag{9}$$

These are the factors that have an impact on persons in a constant thermal comfort performing a given activity for a longer period of time. The heat balance can be established on the basis of these factors.

The heat balance equation is

$$H - E_d - E_{sw} - E_{re} - L = K = R + C, \tag{10}$$

where H • the internal heat production in the human body,
E_d • the heat loss by water vapour diffusion through the skin,
E_{sw} • the heat loss by evaporation of sweat from the surface of the skin,
E_{re} • the latent respiration heat loss,
L • the dry respiration heat loss,
K • the heat transfer from the skin to the outer surface of the clothed body (conduction through the clothing),
R • the heat loss by radiation from the outer surface of the clothed body,
C • the heat loss by convection from the outer surface of the clothed body.

Substituting the adequate terms into the equation gives the heat balance equation:

$$\begin{aligned} &\frac{M}{A_{Du}}(1-\eta) - 0.35\left[43 - 0.061\frac{M}{A_{Du}}(1-\eta) - p_a\right] - \\ &-0.42\left[\frac{M}{A_{Du}}(1-\eta) - 50\right] - 0.0023\frac{M}{A_{Du}}(44 - p_a) - 0.0014\frac{M}{A_{Du}}(34 - t_a) = \\ &= \frac{35.7 - 0.032\frac{M}{A_{Du}}(1-\eta) - t_{cl}}{0.18 I_{cl}} = \\ &= 3.4 \cdot 10^{-8} f_{cl}\left[(t_{cl}+273)^4 - (t_{mrt}+273)^4\right] + f_{cl}\alpha_c(t_{cl} - t_a) \end{aligned} \tag{11}$$

The other form of the *Eq.* (10) equation is

$$H - E_d - E_{sw} - E_{re} - L = R + C. \tag{12}$$

Thus the equation can be given as follows

$$\frac{M}{A_{Du}}(1-\eta) - 0.35\left[43 - 0.061\frac{M}{A_{Du}}(1-\eta) - p_a\right] -$$

$$-0.42\left[\frac{M}{A_{Du}}(1-\eta)-50\right]-0.0023\frac{M}{A_{Du}}(44-p_a)- \qquad (13)$$

$$-0.0014\frac{M}{A_{Du}}(34-t_a)=3.4\cdot 10^{-8}f_{cl}[(t_{cl}+273)^4-(t_{mrt}+273)^4]+f_{cl}\alpha_c(t_{cl}-t_a).$$

This equation is called the comfort equation in the professional literature. The various factors can be put into three categories:

factors connected to clothing: I_{cl} and f_{cl},
factors connected to activities: M/A_{Du}, v, η;
factors connected to the environments: v, t_a, t_{mrt} and p_a.

We would like to note that from this equation the so called comfort diagrams have been established for the various combinations of parameters, thus making practical application easier.

3. Calculations of Asymmetric Radiation on the Human Body

As it is seen above, heat exchange – heat loss and/or heat gain of the human body in a normal case are mainly influenced by the radiant fields (42–44%). Asymmetric radiation is created when the heat exchange radiation between certain parts of the human body and another surface – e.g. outer wall or window, radiator – increases. It is defined by calculating the heat exchange radiation of two surfaces. It is not so easy, however, to define the heat exchange radiation between a human body and a surface.

The definition of the radiation heat exchange between two bodies (marked by 1 and 2) is established by

$$Q_{1-2}=\left[\left(\frac{T_1}{100}\right)^4-\left(\frac{T_2}{100}\right)^4\right]\frac{1}{\eta}\int_{F_1}\left[\int_{F_2}\frac{\cos\beta_1\cos\beta_2}{r^2}dF_2\right]dF_1, \qquad (14)$$

where the factor

$$\left[\int_{F_2}\frac{\cos\beta_1\cos\beta_2}{r^2}dF_2\right] \qquad (15)$$

shows the ratio of heat exchange by radiation on the surface F_2. This is called factor.

In technical practice there are various methods – mathematical equations, diagrams, tables, etc. – to determine the angle factor between two surfaces and a surface and a surface element. The method determining the radiation factor between a surface and a surface element is used for ceiling radiant heating when the top of the human head is considered as a surface element. The human body and its parts are a different case as they cannot be substituted with a surface.

The calculation of radiation for the human body was established by Fanger. The following definitions and factors are used for the calculations:

a) effective radiation area factor

$$f_{eff} = \frac{A_{eff}}{A_{Du}}, \tag{16}$$

where A_{eff} is the effective radiation area of the body

b) projected area factor

$$f_p = \frac{A_p}{A_{eff}} \tag{17}$$

as a function of the direction of radiation where A_p is the area of the human body;

c) angle factors (Φ) between the human body and any horizontal or vertical plane.

In his experiments Fanger determined the projected area factor with a technically improved photographic method previously employed, Guibert and Taylor, and Underwood. It is not necessary to explain this method in detail. He studied several persons, taking into account their weight, height, DuBois area and ponderal index (an indication of the fatness of the body). These investigations were compared and integrated with anthropologic data from American and Scandinavian studies.

On the basis of the results he used the following mathematical methods to determine the various factors and achieved the following results.

4. Effective Radiation Area Factor

Consider a person located in the centre of sphere, see *Fig.1.* The notation is also seen in the figure.

The calculation method is the same as for two surfaces in a hemisphere. Here the angle factor, however, refers to the whole sphere. The following basic equation can be given

$$A_{eff}\Phi_{P-A_2} = A_2\Phi_{A2-P}, \tag{18}$$

where A_{eff} is the effective radiation area of the subject,

Φ_{P-A_2} is the angle factor between the person and the sphere A_2,

$A_2 = 4\pi r^2{}_m$ is the area of the sphere,

Φ_{F_2-E} is the angle factor between the sphere and the person.

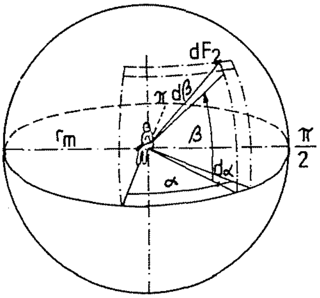

Fig. 1. Figure for the effective radiation area factor (Fanger)

As the angle factor Φ_{F_2-E} designates the fraction of the radiation leaving the person that arrives at A_2, it is obvious that Φ_{F_2-E} is unity.

Thus, the following equation can be written :

$$A_{eff} = A_2 = 4\pi r_m^2 \Phi_{A_2-P}. \tag{19}$$

The angle factor Φ_{F_2-E} cannot be directly determined, but must be found by integration. Consider a differential surface element dA_2 with the angle coordinates α and β.

After the calculations the final equation of the radiation factor is given as

$$\Phi_{A_2-P} = \frac{1}{\pi^2 r_m^2} \int\limits_{\alpha=0}^{\pi} \int\limits_{\beta=0}^{\frac{\pi}{2}} A_P \cos \beta d\beta d\alpha \tag{20}$$

and for the A_{eff} value

$$A_{eff} = \frac{4}{\pi} \int\limits_{\alpha=0}^{\pi} \int\limits_{\beta=0}^{\frac{\pi}{2}} A_p \cos \beta d\beta d\alpha. \tag{21}$$

5. Projected Area Factors

This method uses the projected area factor f_p which is determined as

$$f_p = \frac{A_p}{A_{eff}}. \tag{22}$$

Diagrams have been established for seated and standing persons to acquire a simpler definition of f_p. The former is presented in *Fig. 2.*

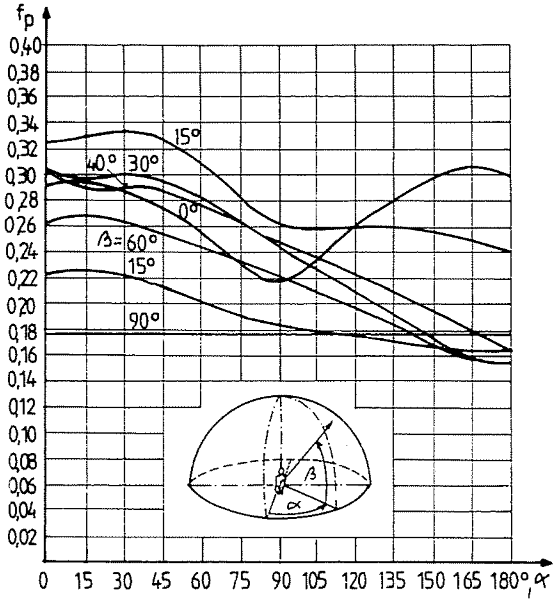

Fig. 2. Projected area factor for naked and clothed seated person (Fanger)

6. Angle Factors

Fig. 3 helps to understand the angle factor. The person is in the coordinate system x, y, z (his coordinates are O,C,O) and faces towards an area in the x, y, z plane.

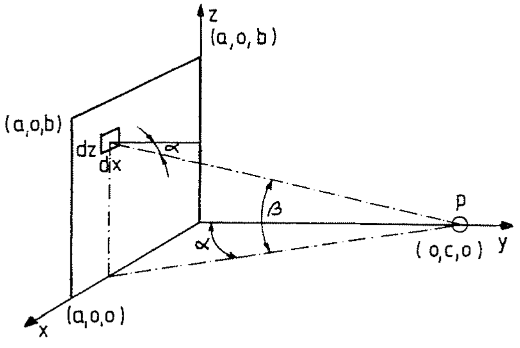

Fig. 3. Figure for the angle factor (Fanger)

In considering the area element $dA = dxdz$ and the person the following equation can be obtained

$$A_{eff} d\Phi_{P-dA} = dA\Phi_{dA-p}. \tag{23}$$

With the various equations and part calculations the angle factor Φ_{P-A} between the person and the whole area

$$\Phi_{P-a} = \frac{1}{\pi} \int_{x=0}^{a} \int_{z=0}^{b} \frac{A_p y}{(x^2 + y_2 + z^2)^{2/3}} dx\, dz =$$

$$= \frac{1}{\pi} \int_{\frac{x}{y}=0}^{\frac{a}{c}} \int_{\frac{z}{y}=0}^{\frac{b}{c}} \frac{A_p}{\left[1 + \left(\frac{x}{y}\right)^2 + \left(\frac{z}{y}\right)^2\right]^{2/3}} d\left(\frac{x}{y}\right) d\left(\frac{z}{y}\right) \tag{24}$$

In calculating the angle factors for the surfaces for a person in a closed space or room, six cases can be drawn up as it is seen from *Fig. 4*:

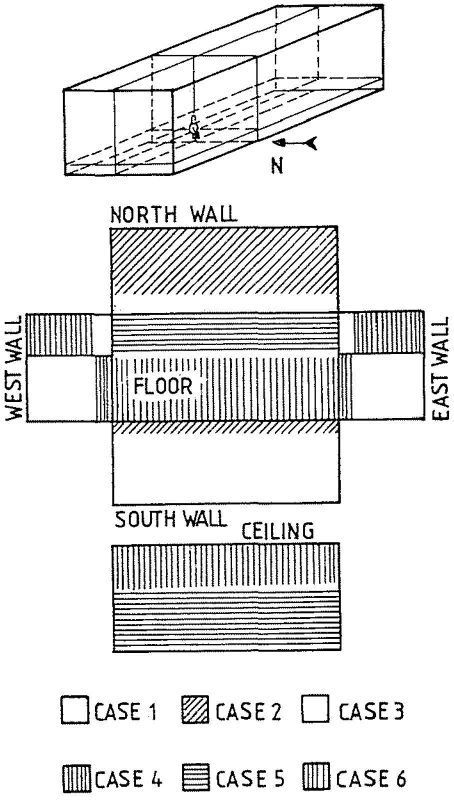

Fig. 4. Distribution of the angle factor for a person in a closed space (Fanger)

a) Vertical area in front of the person and above his centre, or behind him and below his centre (marked by dots in *Fig. 4*),

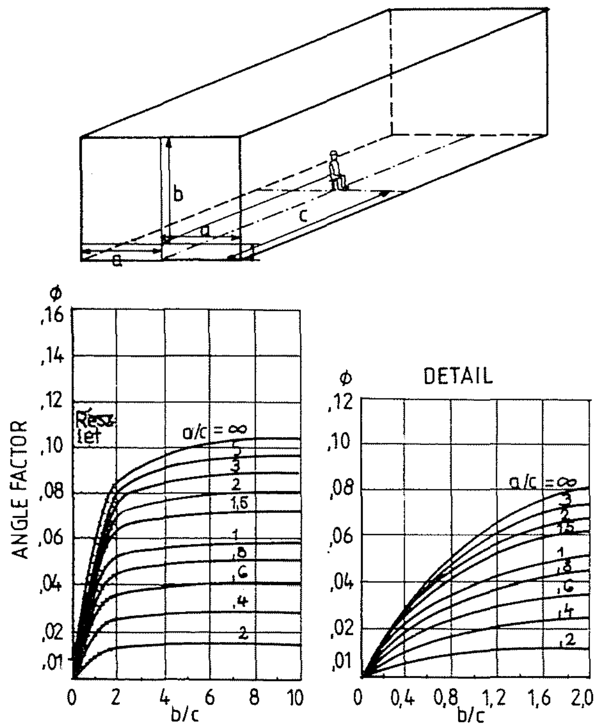

Fig. 5. Angle factors of a seated person in a closed space (Fanger)

b) Vertical area in front of the person and below his centre, or behind him and above his centre (marked by diagonal lines to the left in *Fig. 4*),

c) Vertical area on the side wall, above and forward of his centre, or below and behind his centre (marked by diagonal lines to the right in *Fig. 4*),

d) Vertical area on the side wall, below and forward of his centre, or above and behind his centre (marked by vertical lines in *Fig. 4*),

e) Horizontal area above his head (in the ceiling) and forward of his centre, or on the floor and behind his centre,

f) Horizontal area on the floor and forward of his centre, or above his head (in the ceiling) and behind his centre.

In all six cases, the normal of the area passes through the centre i.e. the area is perpendicular to him. The angle factor was shown for the first case and is seen in *Eq.* (24). The value of Φ can be similarly determined for the other five cases.

In cases where the location of the person in the room is known, but not his orientation (α), the value of Φ for $0 < \alpha < 2\pi$ can be determined

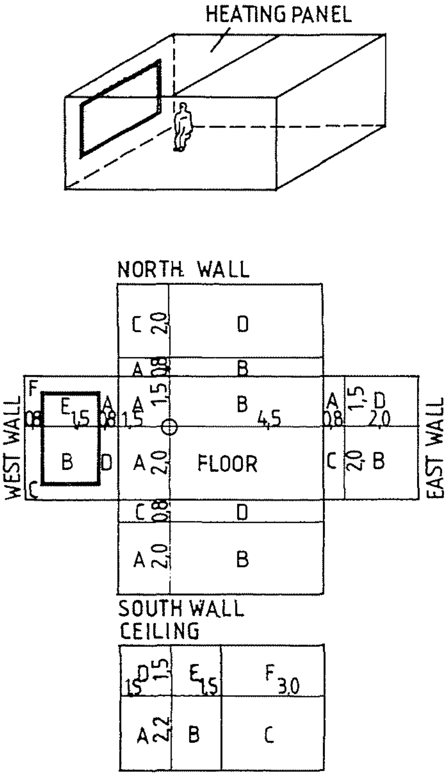

Fig. 6. Angle factor between a seated person and vertical planes on his side, in front of him and below his centre, or behind him and above his centre (Fanger)

for a vertical area with the following equation (25)

$$\Phi_{P-A} = \frac{1}{2\pi^2} \int\limits_{\frac{x}{y}=0}^{\frac{a}{c}} \int\limits_{\frac{z}{y}=0}^{\frac{b}{c}} \int\limits_{x=0}^{2\pi} \frac{F}{\left[1 + \left(\frac{x}{y}\right)^2 + \left(\frac{z}{y}\right)^2\right]^{2/3}} d\left(\frac{x}{y}\right) d\left(\frac{z}{y}\right) d\alpha \qquad (25)$$

Φ_{P-A} represents the radiation factor when the person is not perpendicular to the area but rotates around a vertical axis.

In all cases diagrams for seated and standing persons have been established. We would like to include one here (*Fig. 5*): angle factor between a seated person and vertical planes, in front of him and above his centre, or behind him and below his centre.

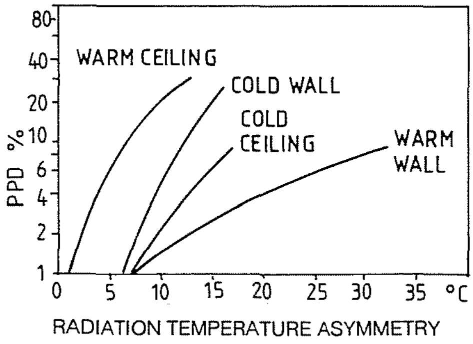

Fig. 7. Radiation temperature asymmetry in relation to PPD values

7. The Justification and Forms of Practical Application in Hungary

As we have mentioned in the introduction, local discomfort factors, especially asymmetric radiation cause numerous problems mainly in prefabricated houses in Hungary. As a result of very low heat transfer coefficient (U) - 1 to 1.2 W/m^2K – in prefabricated houses, the internal surface temperature is 15 – 16 °C when the external dimensioning temperature is –15 °C. If the mean skin temperature of a normally clothed human body (26 – 27 °C) is taken into account, then the difference between the surface temperatures is 11 – 12 °C.

This is a 30 W heat loss if the surface of the human body is 1.8 m^2 and 50% of this value and the angle factor $\Phi = 0.4$ were used in the calculations. On the other hand there is heat loss by radiation (roughly 15 W) between the other half of the body and the internal surfaces of 20 °C. This justifies the complaints of persons in rooms of otherwise adequate temperature. The exact heat loss by radiation between the areas of body and the outer walls can be determined by the diagrams in *Fig. 5* and the basic definition [6]

$$Q_s = AC\Phi \left[\left(\frac{T_1}{100} \right)^4 - \left(\frac{T_2}{100} \right)^4 \right]. \tag{26}$$

It is also possible to use the diagram in *Fig. 7* that is also found in the new European standard [7]. In it the PPD values are shown for four cases

- cold wall,
- cold ceiling,
- warm wall,
- warm ceiling

in relation to the radiation asymmetry. The PPD value and the related dimensioning diagram of PMV-PPD are well known [3,6], therefore – due to the limits on the length of the article – we do not discuss them in detail.

We would like to note that Hungarian researchers participated in establishing the values of *Fig. 7* [8].

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