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PNEUMATIC CONVEYING IN THE VERTICAL PIPELINE OF AIR LIFT

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Abstract

In this paper the authors present a procedure for determining pressure drop caused by solid material moving in the vertical pipeline of an air lift. This method can be used to determine important parameters – with regard to the expansion of the conveying gas – such as acceleration and velocity of particles, change of gas velocity along the pipeline, etc.

In this paper the authors have neglected the tightening effect on the cross-section created by the solid material. It is in a further paper, to be published later, that they shall point out what sort of mistakes this negligence causes in each physical parameter.

Keywords: pneumatic conveying, air lift, pipeline, two-phase flow, pressure drop.

1. Introduction

The paper deals with the physical parameters of pneumatic conveying performed with greater mass flow and to greater vertical distances. An example of such a pipeline conveying great mass flow to a great distance is the vertical pipe of an air lift. Diameter of the conveyor pipe is $D = (80 - 400)$ mm, length of the conveyor pipe is $H = (10 - 70)$ m, conveying capacity is in the range of $\dot{m}_s = (20 - 150)$ t/h in 'normal' cases.

All these mean that during the dimensional design phase one has to be familiar with the pressure drop of the conveyor pipeline, the change of conveying gas velocity due to gas expansion, the velocity and acceleration of the particles, etc.

In this paper the authors present a method for determining the parameters of conveying. The tightening effect of particles on the cross-section is neglected. The authors shall present a more accurate method, with regard to this tightening effect, in a later paper.

2. Determining Characteristic Parameters of Two-Phase Flow Created in the Vertical Conveyor Pipeline of Air Lift

Fig. 1 shows the schematic drawing of an air lift. According to the well-known operating principles of the air lift, gas, flowing through the distribution layer marked 1, brings the material to be transported into 'fluid state' in the tank marked 2. High-velocity conveying gas exits through the nozzle marked 3, picks up material and enters the conveying pipeline marked 4. The mixture of gas and solids, leaving the end part of the conveying pipeline, enters the separator, where material and gas is separated.

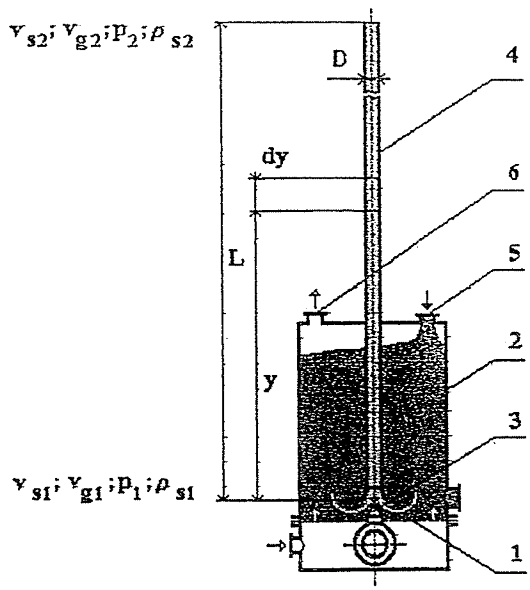

Fig. 1. Arrangement scheme of air lift

Material is constantly fed into the tank marked 2 through the flange marked 5, and the gas, having flowed through the material layer, exits through the tank flange marked 6.

2.1. Restrictions in Formulating the Equations of Two-Phase Flow

a. Velocity distribution within the pipe cross-section is considered constant.
b. The collision force affecting the particles is considered as being constant along the pipeline.
c. Change of state of the conveying gas is considered to be isotherm.
d. Effects originating from rotation of the particles are disregarded.

e. Solid material particles are considered to be spherical.
f. Tightening effect of particles on the pipeline cross-section is disregarded.

2.2. Equations Formulated for the Material Particles

Fig. 2 shows the scheme of the pipeline section, sorrounded by the control surface, as cut out from the conveying pipeline of the air lift.

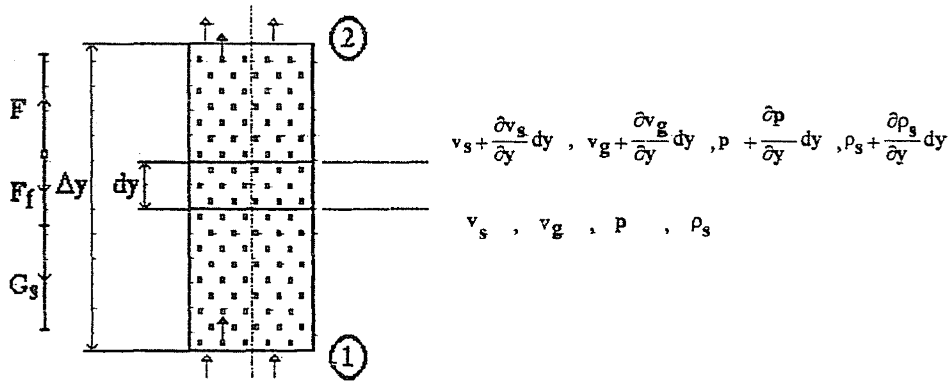

Fig. 2. The pipeline section element cut out from the vertical pipeline section

Material mass in length unit of pipeline section:

$$q_s = \frac{\dot{m}_s \, \mathrm{d}t}{\mathrm{d}y} = \frac{\dot{m}_s}{v_s}. \tag{1}$$

'ρ_s' concentration of material in volume element '$A\,\mathrm{d}y$':

$$\rho_s = \frac{q_s \, \mathrm{d}y}{A\mathrm{d}y} = \frac{\dot{m}_s}{Av_s}. \tag{2}$$

Change of material concentration:

$$\mathrm{d}\rho_s = -\frac{\dot{m}_s}{A}\frac{\mathrm{d}v_s}{v_s^2} \tag{3}$$

The continuity equation formulated for particles moving in the control surface is as follows:

$$\rho_s v_s A = (\rho_s - \mathrm{d}\rho_s)(v_s + \mathrm{d}v_s)A. \tag{4}$$

From the equation – with the secondarily small terms left out compared to the remaining terms – the following is obtained:

$$\rho_s \, \mathrm{d}v_s = v_s \, \mathrm{d}\rho_s. \tag{5}$$

The momentum equation for particles in the volume within the control surface cut out from the vertical conveying pipeline and shown in *Fig. 2* is as follows:

$$-\rho_s v_s^2 A + (\rho_s - \mathrm{d}\rho_s)(v_s + \mathrm{d}v_s)^2 A = \mathrm{d}F - \mathrm{d}F_f - \mathrm{d}G_s. \tag{6}$$

In the relation:

$\mathrm{d}F$ is the force originating from velocity difference between the gas flow and material particle in the control volume.
$\mathrm{d}F_f$ is the braking force originating from particles colliding with the wall, which is considered to affect the material particles continuously.
$\mathrm{d}G_s$ is the weight of particles in the cut-out volume element.

With regard to *Eqs.* (5) and (6), with the secondarily small terms left out compared to the remaining terms, the following is obtained:

$$\rho_s v_s A \, \mathrm{d}v_s = \mathrm{d}F - \mathrm{d}F_f - \mathrm{d}G_s. \tag{7}$$

The '$\mathrm{d}F$' force affecting the material particles:

$$\mathrm{d}F = N \frac{\rho_g - \frac{\mathrm{d}\rho_g}{2}}{2} A_o C_D \left[\left(v_g + \frac{\mathrm{d}v_g}{2} \right) - \left(v_s + \frac{\mathrm{d}v_s}{2} \right) \right]^2. \tag{8}$$

In the relation the 'N' number of pieces of particles in the control volume can be calculated as follows by the 'm_1' particle mass:

$$N = \frac{\mathrm{d}m_s}{m_1} = \left(\rho_s - \frac{\mathrm{d}\rho_s}{2} \right) \frac{A}{m_1} \mathrm{d}y. \tag{9}$$

In Relation (8) the 'C_D' drag coefficient for the particles regarded as spheres according to KASKAS [2] is as follows:

$$C_D = \frac{24}{\mathrm{Re}} + \frac{4}{\sqrt{\mathrm{Re}}} + 0.4; \quad \mathrm{Re} = \frac{w d_p}{v_g} = \frac{(v_g - v_s) d_p \rho_g}{\eta_g}. \tag{10}$$

With regard to isothermal change of conveying gas the following can be stated:

$$\rho_g = \frac{\rho_{go}}{p_o} p. \tag{11}$$

As a result, the mass flow of the gas is:

$$\dot{m}_g = A \rho_g v_g = A \frac{\rho_{go}}{\tau p_o} p v_g. \tag{12}$$

The following gas velocity is obtained from *Eq.* (12):

$$v_g = \frac{\dot{m}_g p_o}{A \rho_{gc} p} = v_{go} \frac{p_o}{p}. \tag{13}$$

Eq. (8) takes on the following form, with regard to *Eqs.* (9) and (13), as well as neglecting also here the secondarily small terms compared to the remaining terms, with the marking $\pi_o = \frac{A}{m_1} A_o C_D$:

$$\mathrm{d}F = \frac{\pi_o}{2} \frac{\dot{m}_s}{A v_s} \rho_{go} \frac{p}{p_o} \left(v_{go} \frac{p_o}{p} - v_s \right)^2 \mathrm{d}y. \tag{14}$$

The 'F_f' braking force – following the reasoning of PÁPAI [1] – being regarded as consequence of the impacts, but with continuous effect, we interpret this effect to cause the solid particles to lose 'ξ' part of their kinetic-energy, i.e.

$$F_f = \frac{\xi}{D} \frac{m_1 v_s^2}{2} = k_i \frac{m_1 v_s^2}{D}. \tag{15}$$

In the relation 'ξ' is a factor characteristic of the conveyed material, and it is to be determined by experimental methods.

The braking force affecting the material particles in the control volume is:

$$\mathrm{d}F_f = \frac{k_i}{D} \left(\rho_s - \frac{\mathrm{d}\rho_s}{2} \right) A \,\mathrm{d}y \left(v_s + \frac{\mathrm{d}v_s}{2} \right)^2. \tag{16}$$

After the neglections, the equation evolves as follows:

$$\mathrm{d}F_f = \frac{k_i}{D} \dot{m}_s v_s \,\mathrm{d}y. \tag{17}$$

Weight of the material particles within the volume enclosed by the control surface:

$$\mathrm{d}G_s = \frac{\dot{m}_s}{v_s} g \,\mathrm{d}y. \tag{18}$$

Regarding the above, and performing the reductions, using the markings $\pi_2 = \frac{A_o}{2m_1} C_D \frac{\rho_{go}}{p_o}$ and $\pi_3 = \frac{\xi}{2D}$ we obtain the following equation:

$$\frac{\mathrm{d}v_s}{\mathrm{d}y} = \pi_2 \frac{p}{v_s} \left(v_{go} \frac{p_o}{p} - v_s \right)^2 - \pi_3 v_s - \frac{g}{v_s}. \tag{19}$$

2.3. Equations Written for the Conveying Gas

The momentum equation written for the gas in the control volume is as follows:

$$-\rho_g v_g^2 A + (\rho_g - \mathrm{d}\rho_g)(v_g + \mathrm{d}v_g)^2 A = -pA + (p - \mathrm{d}p)A - \mathrm{d}F - \mathrm{d}F_{fD} - \mathrm{d}G_s. \tag{20}$$

In this relation: 'F_{fD}' is the friction of the conveying gas on the pipeline wall. The equation written for the pipeline section element within the control surface is as follows:

$$\mathrm{d}F_{fD} = \frac{\rho_g - \frac{\mathrm{d}\rho_g}{2}}{2} \frac{\mathrm{d}y}{D} A f \left(v_g + \frac{\mathrm{d}v_g}{2} \right)^2 . \tag{21}$$

The secondarily small terms being neglected again compared to the remaining terms, and introducing the marking of $\pi_1 = \frac{Af}{2D}$, we get :

$$\mathrm{d}F_{fD} = \pi_1 \rho_g v_g^2 \,\mathrm{d}y = \frac{\pi_4}{p} \,\mathrm{d}y. \tag{22}$$

Eq. (22) can be written as follows, with regard to *Eqs.* (11) and (13), and using the marking $\pi_4 = -\pi_1 \rho_{go} p_o v_{go}^2$:

$$\mathrm{d}F_{fD} = \pi_1 \rho_{go} v_{go}^2 \frac{p_o}{p} \,\mathrm{d}y = \frac{\pi_4}{p} \,\mathrm{d}y. \tag{23}$$

Based on the above, *Eq.* (20) will be as follows, after having performed the reductions:

$$\frac{\mathrm{d}p}{\mathrm{d}y} = \frac{\pi_4 p + \pi_5 \frac{p^3}{v_s} \left(v_{go} \frac{p_o}{p} - v_s \right)^2 - \pi_7 \frac{p^2}{v_s}}{Ap^2 - 3\pi_6} . \tag{24}$$

The flow parameters of the gas solids mixture flowing in the vertical pipeline are described in *Eqs.* (19) and (24).

The results associated to the initial values at points $v_s = v_{s1}$ and $p = p_1$ located at $y = 0$ in the following differential equation system

$$\begin{aligned} \frac{\mathrm{d}v_s}{\mathrm{d}y} &= f(y, p; v_s), \\ \frac{\mathrm{d}p}{\mathrm{d}y} &= f(y, v_s; p) \end{aligned} \tag{25}$$

have been solved using the Runge-Kutta type method.

In the first step, the 'p_1' starting value can only be assumed by estimation. As a means of control, the prescribed pressure (e.g. 'p_o' atmospheric pressure) should be present at the end of the pipeline. In the case the prescribed pressure is not achieved in the first step, the 'p_1' value should be constantly changed by a series of iterations until the prescribed condition is achieved at the end of the pipeline.

3. Results Obtained as Solution of the DE. System

3.1. In the Example to be Presented, the DE. System was Solved by Using the Following Data

Conveyed solid material:	Fly ash
Length of the vertical pipe:	$H = 50$ m
Pipeline diameter:	$D = 150$ mm
Mass flow of solids:	$\dot{m}_s = 25$ t/h
Mass flow of conveying gas:	$\dot{m}_g = 0.53$ kg/s
Average mass of one particle:	$m_1 = 3.88\ 10^{-9}$ kg
Average particle size:	$d_p = 150\ 10^{-6}$ m
Absolute viscosity of gas with temperature of $t = 20$ °C:	$\eta_g = 1.85\ 10^{-5}$ kg/ms
Pressure at the end of the pipeline:	$p_2 = p_o = 10^5$ Pa
Density of gas at the end of the pipeline:	$\rho_{go} = 1.2$ kg/m^3
Impact factor (experimental result):	$k_i = 0.01$
Pipeline friction factor:	$f = 0.02$

3.2. The Following Physical Quantities were Used on the Diagrams Presenting the Results of the DE. Solution

a.) The 'P_{pol}' power introduced at the $y = 0$ location, which is equal to the useful power of the compressor

$$P_{\text{pol}} = \frac{p_o}{\rho_{go}} \dot{m}_g \frac{n}{n-1} \left[\left(\frac{p_1}{p_o} \right)^{(n-1)/n} - 1 \right]. \tag{26}$$

b.) Specific energy consumption of the conveying process
The specific energy consumption 'e' is expressed by the following:

$$e = \frac{P_{\text{pol}}}{L \dot{m}_s}. \tag{27}$$

c.) Mixing ratio
The mixing ratio 'μ' is as follows:

$$\mu = \frac{\dot{m}_s}{\dot{m}_g}. \tag{28}$$

d.) The 's' slip resulting from the velocity difference of the conveyed solids and the conveying gas.

The value of the slip is defined by the following relation:

$$s = \frac{v_g - v_s}{v_g}. \tag{29}$$

3.3. The Diagrams Resulting from the Solution

On *Fig. 3* the change of pressure and velocity as a function of the pipeline length can be seen. *Fig. 4* shows these same characteristics at the starting section of the pipeline.

From the figure it can be established that the 'v_g' gas velocity and the 'v_s' material velocity gradually will increase up till the end of the pipeline due to the gas expansion, while pressure will – except for the starting section – drop in a near linear way.

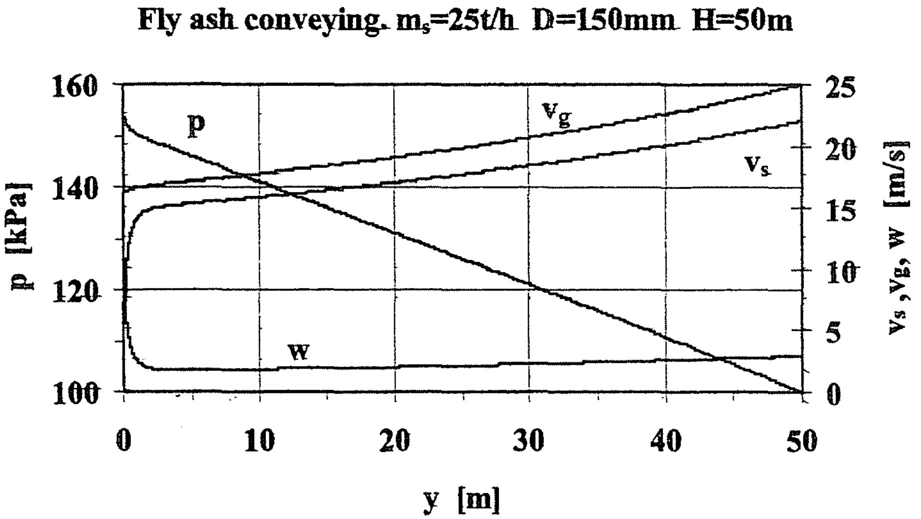

Fig. 3. Calculated pressure and velocity distribution as a function of the pipeline length

Fig. 4 shows that after having travelled a vertical distance of $y = 5$ m, material velocity will increase to $v_s = 15.35$ m/s, which value is nearly 70% of the $v_s = v_{s2} = 22$ m/s value after $y = L = 50$ m. This means that, due to the high relative velocity, the majority of the acceleration takes place at the starting section of the pipeline.

At the beginning of the conveying pipeline, i.e. at the $y = 0$ location, the value of absolute pressure is $p = p_1 = 156.47$ kPa.

Figs. 5 and *6* show the values of the slip. It can be established that the $s_{\min} = 0.104$ value is located at the $y = 5.28$ m distance from the beginning of the pipe.

Fig. 7 shows the 'a_s' acceleration and the 'v_s' material velocity as a function of time.

Minimum of the acceleration in the $t = 0.39$ s moment of time is $a_{s\,\min} = 1.59$ m/s^2. The velocity diagram has inflexion here.

Figs. 8 and *9* show the change of solids concentration along the length of the pipe.

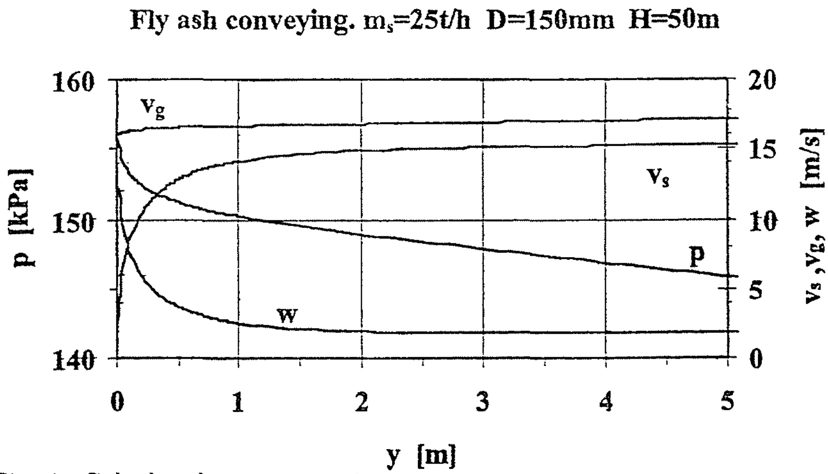

Fig. 4. Calculated pressure and velocity distribution as a function of the pipeline length at the starting section of the pipeline

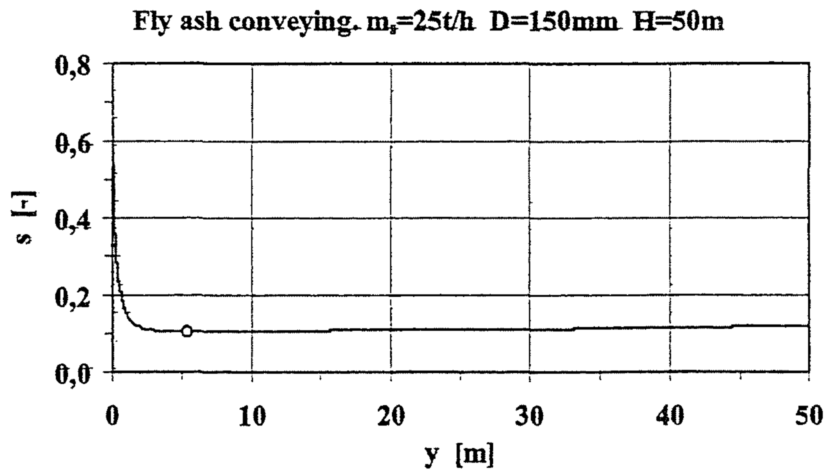

Fig. 5. Change of the slip as a function of the pipeline length. Location of the minimum indicated by circle

Fig. 10 shows the $\Delta p = p_1 - p_2 = p_1 - p_o$ pressure drop of the conveying pipeline, as well as the 'P_{pol}' change of performance in case of different 'v_{go}' end-of-pipe gas velocities.

The pressure drop of the conveying pipeline has a minimum at the $v_{go} = 19.5$ m/s value. The pressure drop then is $\Delta p = 54.45$ kPa.

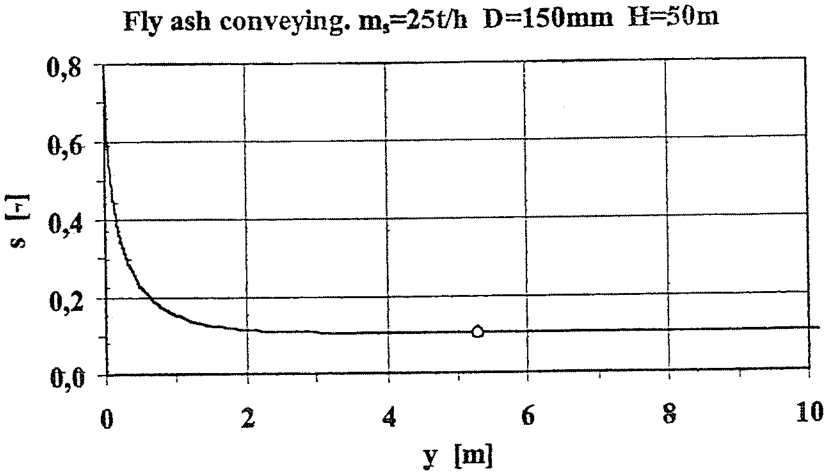

Fig. 6. Change of the slip as a function of the pipeline length at the starting section of the pipe. Location of the minimum indicated by circle

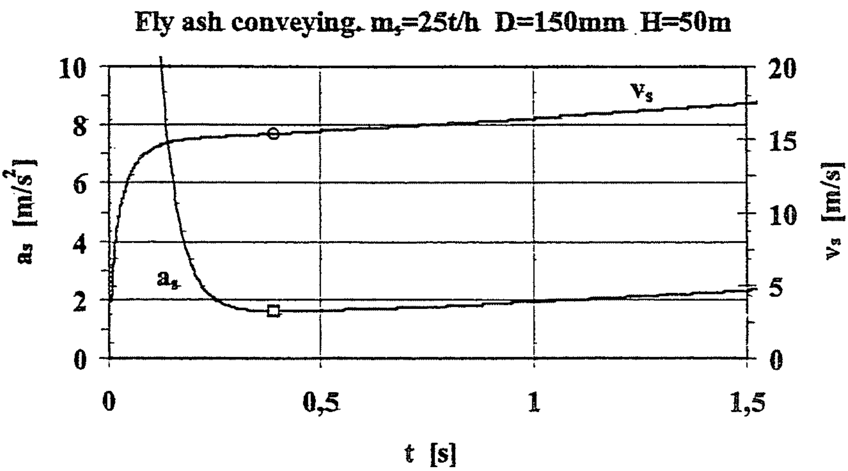

Fig. 7. Change of material acceleration and velocity as a function of time in the starting phase of the motion. Location of the inflexion indicated by circle, location of the minimum indicated by square

Fig. 11 shows the change of the 'μ' mixing ratio and the 'e' specific energy as a function of the 'v_{go}' velocity.

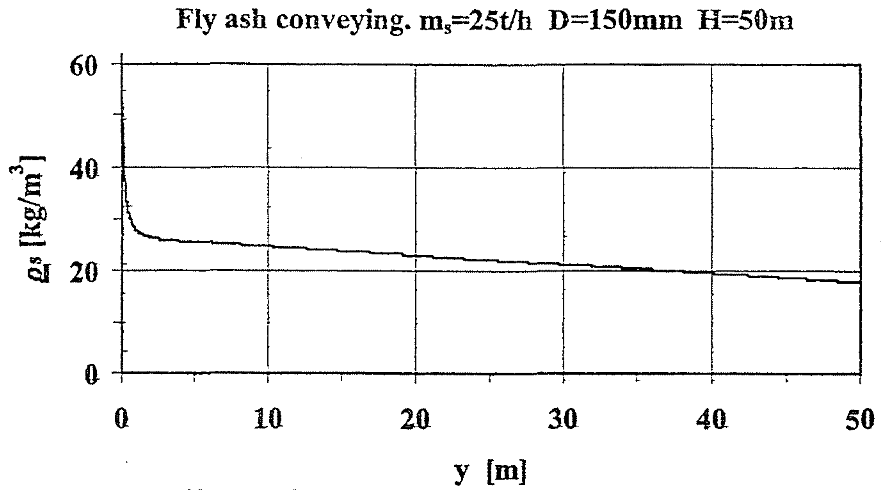

Fig. 8. Change of concentration as a function of the pipeline length

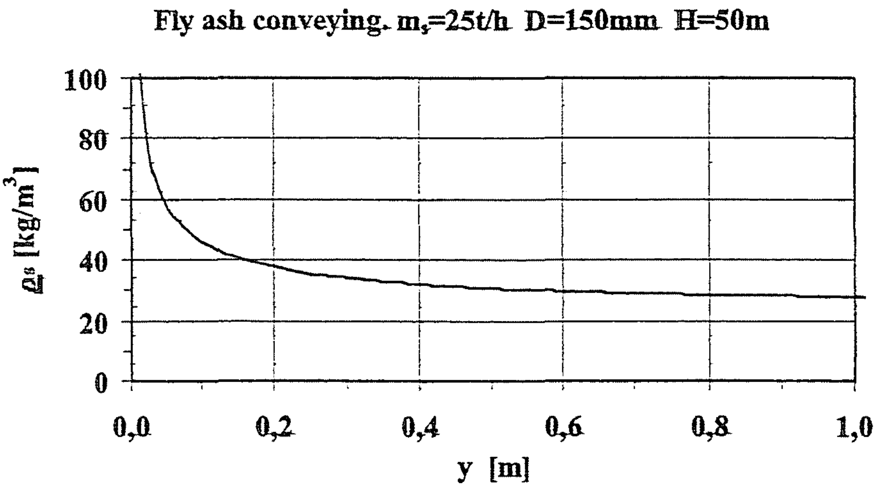

Fig. 9. Change of concentration as a function of the pipeline length at the starting-section of the pipe

4. Refining the Mathematical Model Presented in the Paper

It is in a forthcoming paper – that will take into consideration the tightening effect on the cross-section caused by the solid material – that the authors wish to point out the mistakes resulting in the individual parameter values due to neglecting the tightening effect. Accurate information on material velocity is especially important from the aspect of breaking-up and abrasion

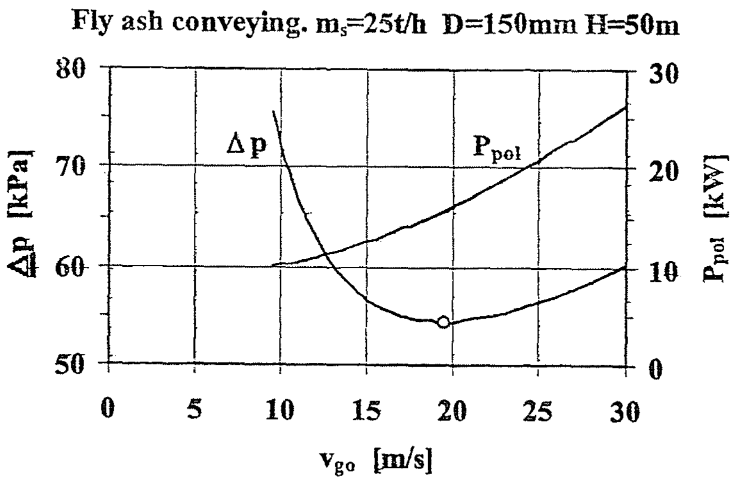

Fig. 10. Pressure drop of the conveying pipeline and polytropic power as a function of the gas velocity at the end of the pipe. Location of the minimal pressure drop indicated by circle

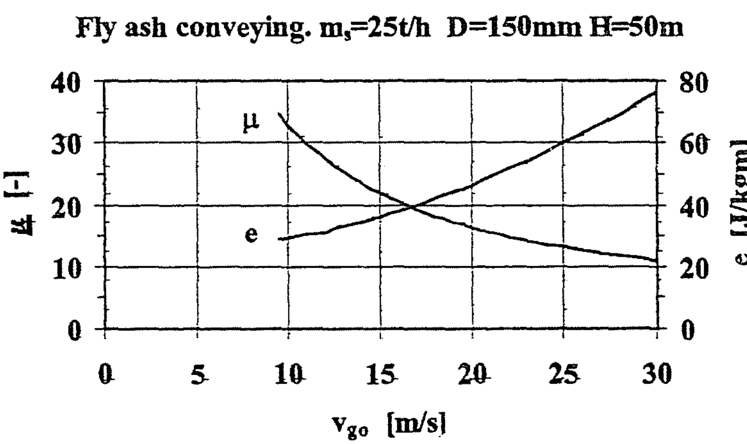

Fig. 11. Mixing ratio and specific energy consumption as a function of the gas velocity at the end of the pipe

of particles. From an energetic aspect it is equally important to know what mistakes are caused in the pressure drop of the pipeline by observing or neglecting the tightening effect on the solid material cross-section by the material.

Nomenclature

A	[m^2]	Cross-section of conveying pipeline
A_o	[m^2]	Cross-section, perpendicular to flow direction, of one solid particle
a_s	[m/s^2]	Acceleration of solid particle
C_D	[–]	Drag coefficient
d_p	[m]	Average diameter of solid particle assumed as sphere
e	[J/kgm]	Specific energy consumption
F	[N]	Forward-driving force affecting the particle
F_f	[N]	Braking force affecting the particle
F_{fD}	[N]	Friction force arising on the wall of the pipe
f	[–]	Friction factor
G_s	[N]	Force of weight
g	[m/s^2]	Acceleration of gravity
$k_i = \frac{\xi}{2}$	[–]	Impact factor
L	[m]	Length of the conveying pipeline
m_s	[kg]	Mass of solid material
$\dot{m}_g$	[kg/s]	Mass flow of the conveying gas
$\dot{m}_s$	[kg/s]	Mass flow of solid material
m_1	[kg]	Mass of one particle
N	[–]	Number of pieces of the solid material particles
n	[–]	Polytropic exponent
P_{pbol}	[W]	Polytropic power
p	[Pa]	Pressure
$p_2 = p_o$	[Pa]	Pressure at the $y = L$ location. Atmospheric pressure
p_1	[Pa]	Pressure at the $y = 0$ location
$\Delta p = p_2 - p_1$	[Pa]	Pressure drop of the conveying pipeline
Re	[–]	Reynolds number
$s = \frac{v_g - v_s}{v_g}$	[–]	Slip
t	[s]	Time
v_g	[m/s]	Velocity of the conveying gas
v_s	[m/s]	Velocity of the particle
$v_{g_2} = v_{go}$	[m/s]	Velocity of the conveying gas at the $y = L$ location
$w = v_g - v_s$	[m/s]	Relative velocity
η_g	[kg/ms]	Absolute viscosity of the gas
μ	[–]	Mixing ratio
ν_g	[m^2/s]	Kinematic viscosity of the gas

ξ	[–]	Factor characteristic of the conveyed material, established by experiment
$\pi_o = \dfrac{A}{m_1} A_o C_D$	[m^4/kg]	Constant
$\pi_1 = \dfrac{Af}{2D}$	[m]	Constant
$\pi_2 = \dfrac{\pi_o}{2} \dfrac{\rho_{go}}{A p_o}$	[s^2/kg]	Constant
$\pi_3 = \dfrac{\xi}{2D}$	[1/m]	Constant
$\pi_4 = -\pi_1 \rho_{go} p_o v_{go}^2$	[kg^2/ms^4]	Constant
$\pi_5 = -\dfrac{\pi_o}{2} \dfrac{\dot{m}_s}{A} \dfrac{\rho_{go}}{p_o}$	[s]	Constant
$\pi_6 = v_{go}^2 \rho_{go} A p_o$	[kg^2/s^4]	Constant
$\pi_7 = \dot{m}_s g$	[kgm/s^3]	Constant
ρ_g	[kg/m^3]	Density of the gas
ρ_{go}	[kg/m^3]	Density of the gas at the $y = L$ location Density of gas with atmospheric pressure
ρ_s	[kg/m^3]	Concentration of solid material

Subscripts

D	Wall of pipe
g	Gas
s	Solid material
1	One particle
i	Impact
f	Friction
$o, 1, 2$	Characteristic parameters along the length of the pipe

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