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DYNAMIC MODELLING OF FRICTION CLUTCHES AND APPLICATION OF THIS MODEL IN SIMULATION OF DRIVE SYSTEMS[1]

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Abstract

Theoretical dynamic modelling of friction clutches and principles of application of this model at drive systems is presented in this paper. Provided that the friction clutch consists of two rigid shafts connected only by friction, we face with the problem simulation of drive systems. Drive systems have no unique dynamic model due to the fact that the model of the whole system on the transitional process of the engagement of the clutch is quite different from that when the driving and driven shafts are connected rigidly. So the task is to create an unique model and to present the principles of application of this model for drive systems.

Keywords: friction clutch, computer simulation, drive system, planetary system, automatic speed-change gear.

1. Introduction

As it is well known, the possibility of engaging and disengaging rotors running with different shaft speeds is one of the most important advantages of friction clutches. Disc clutches are applied as clutches or brakes frequently. If both rotors of the clutch rotate, then it works as a clutch, while the rotation of one is inhibited, then it works as a brake.

From dynamic point of view the friction clutch has two states. If the friction discs slide on each other, then the clutch transmits torque by means of sliding friction. In this case the transmitted torque is a function of the pressure acting on the discs, the relative velocity, the temperature of the surface, and last but not least the conditions of lubrication. If the friction discs rotate together, then the transmitted torque is smaller than the limit torque necessary for slip, it can be even zero.

The second order Lagrange differential equation is frequently applied for describing the dynamic model in the programmes designed for the dynamic simulation of drive systems [1], [2], [5]. If the parts are considered

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to be rigid then only the known rotating masses, driving and load torques and clutch (or brake) torques are present in the Lagrange equation. During the transition process of engagement of a clutch or brake, the transmitted torque is the sliding torque, then it can be determined directly providing that the friction coefficient is known. On the other hand, in the period of complete engagement the torque of the clutch or the brake cannot be given a value directly. Therefore in these cases we regard the dynamic model, as if the clutches and brakes did not exist in the system at all, and the two elements coupled this way become attached to the same degree of freedom. Consequently, the number of degrees of freedom of the system decreases with one in the instant when the sliding terminates. This means that the model is not valid further on, and a new dynamic model is needed to investigate the system in the following period.

The application of the new dynamic model is troublesome, especially in the case of automatic transmissions, where several brakes and clutches are built in the system. The structure of the programmes used for simulation becomes too complicated, and they usually apply specific solutions designed for specific problems only. This article proposes a model that solves this problem.

In order to specify the problem exactly, let us see a simple example.

2. Basic Problem

In the simplest case the system consists of only a prime mover, a clutch, and a driven machine. (See *Fig. 1*).

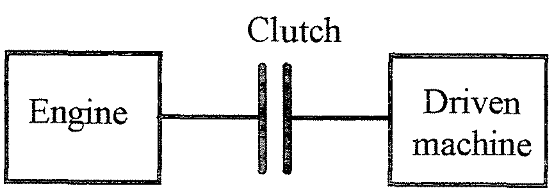

Fig. 1. Scheme of the system

The coefficient of friction is assumed to be constant during the engagement process. On this assumption in case of a specific clutch construction the sliding torque of the clutch, T_s, depends only on the contact pressure. Let us assume that the contact pressure depending on time, as well as the sliding torque, T_s, are proportional, which is similar to the ones presented in *Fig. 2* [6]. The engagement process can be analysed if the torque of the clutch and the working characteristics of the machines are known.

At $t = 0$ we apply load on the prime mover (IC engine) running with idle running speed. Due to the load, the angular speed ω_e of the driving machine will decrease. When the increasing clutch torque reaches the level

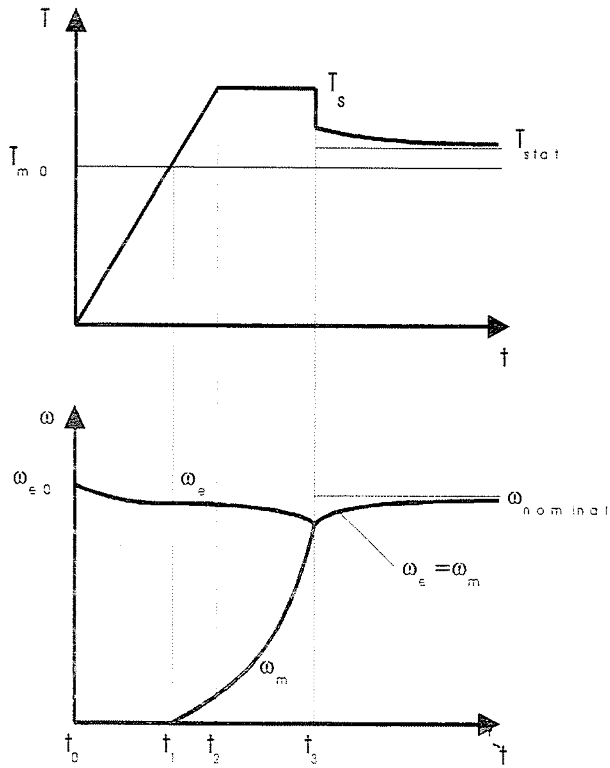

Fig. 2.

necessary to start the T_{m0} limit torque of the driven machine, the rotor of the driven machine starts to rotate. The ω_m, angular speed of the driven machine, will be equal to the angular speed of the prime mover at t_3. Having equal shaft speeds, the state of the clutch will be changed from sliding to rigid state, the clutch torque decreases suddenly, and the system accelerates until it reaches the angular velocity corresponding to the operating point.

We must define a discrete variable for deducting the equations of the system.

$$\text{State} = \begin{cases} 0, & \text{if } \omega_e = \omega_m , \\ 1, & \text{if } \omega_e \neq \omega_m . \end{cases}$$

In case of $State = 0$:

$$\dot{\omega}_e = \dot{\omega}_m = (T_e - T_m)/(\theta_e + \theta_m) . \tag{1}$$

The clutch torque can be determined with the following equation:

$$T_c = T_e - \dot{\omega}_e \theta_e = T_m + \dot{\omega}_m \theta_m , \tag{2}$$

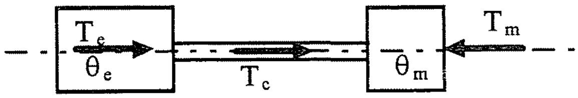

Fig. 3. Dynamic model in case of State = 0

where ω_e : angular speed of prime mover (IC engine),
ω_m : angular speed of driven machine,
T_e : torque of prime mover,
T_c : torque of clutch,
T_m : torque of driven machine,
θ_e : moment of inertia of prime mover,
θ_m : calculated moment of inertia of driven machine.

Usually the torques of the prime mover and the driven machine can be determined knowing the angular speeds:

$$T_e = T_e(\omega_e)\,, \tag{3}$$
$$T_m = T_m(\omega_m)\,. \tag{4}$$

Obviously the condition $T_c < T_s$ is valid, where T_s is the sliding torque.
In case of *State* = 1:

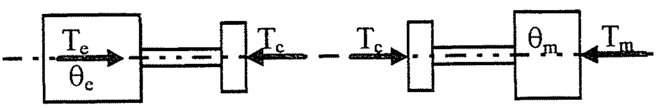

Fig. 4. Dynamic model in case of State = 1

$$\dot{\omega}_e = (T_e - T_c)/\theta_e\,, \tag{5}$$
$$\dot{\omega}_m = (T_c - T_m)/\theta_m\,. \tag{6}$$

In this case the absolute value of the clutch torque is the sliding torque.

$$\begin{aligned} T_c &= +T_s(p_n(t)), &\text{if } \omega_e &> \omega_m\,, \\ T_c &= -T_s(p_n(t))\,, &\text{if } \omega_e &< \omega_m\,, \end{aligned} \tag{7}$$

where $p_n(t)$ is the time function of the contact pressure.

This procedure is suitable for engineering considerations made during the designing of the clutch. However, if more sophisticated systems, e.g. automatic transmissions are considered, where several (2 through 4) clutches engage and disengage at the same time, after each engagement process the system needs a new model. In theory, 2^n models are required for investigation of the sliding state of n clutches simultaneously.

3. Observation of Friction Phenomena

The problem to be solved is obvious: How does the clutch change its states, and how can both states be characterised by a unified dynamic model. According to the theory of KRAGELSZKIJ [3], during the interaction of two surfaces the asperities of the harder surface penetrate into the surface layers of the softer material and they scrape along the softer surface.

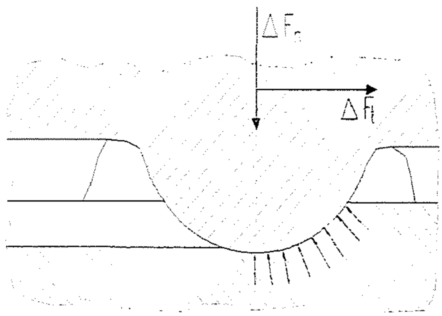

Fig. 5.

In case of a clutch the surface of the cast steel or cast iron disc is harder than the paper based friction facings. When we apply load on one side of the clutch, the harder asperities start to deform the surface of the softer material in tangential direction. The asperities will not move on the harder surface until the shear stress of the contact zones reaches a limit value.

Let us define a relative quantity that describes the shear stress level of the contact zones:

$$\xi = \frac{\tau}{\tau_{\max}} \tag{8}$$

where τ is the shear stress of the contact zones, $\tau_{\max}$ is the limit value of the shear stress. If $\xi < 1$, then the discs stick to each other, while if $\xi = 1$ then the discs slide on each other. It should be noted, that ξ describes the sliding state of the mating surfaces.

The elementary torques due to friction forces generated in the real contact zones cannot be calculated, since the position of contact zones is a random one. For sake of simplicity, let us assume, that 'τ_{average}' average shear stress is generated on each element of the friction surfaces of the driving and the driven discs. This average shear stress is equal to the quotient of the total friction force and the 'A_n' nominal contact surface.

$$\tau = \tau_{\text{average}} = \frac{F_s}{A_n} \, . \tag{9}$$

So the total friction torque is:

$$T = \int_A \tau \cdot r \cdot \mathrm{d}A = \tau_{\text{average}} \int_A r \cdot \mathrm{d}A \,. \tag{10}$$

It is obvious, using the simplified conditions, that the transmitted torque is a linear function of the average shear stress, that is:

$$\xi = \frac{\tau}{\tau_{\max}} = \frac{T}{T_{\max}} = \frac{T}{T_s} \,, \tag{11}$$

where T_s is the limit sliding torque, which can be determined with the following formula:

$$T_{\max} = T_s = \mu \cdot p_{\text{average}} \int_A r \, \mathrm{d}A \,. \tag{12}$$

4. Clutch Model

In order to determine the instantaneous values of the clutch torque, let us take into account the elasticity of the shafts, too. An elastic model can be seen on *Fig. 6*, where S is the reduced torsion spring stiffness of the shafts, gears and tangential springs, if any. L is a torque limiting device, with the same limit torque as the sliding limit torque of the friction discs. It should be noted that this limit torque is a time function. D is the reduced damping that models the internal and external friction of the elements mentioned above.

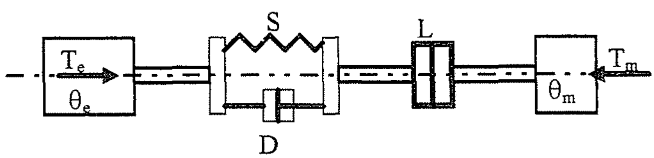

Fig. 6. Elastic clutch model

The state equations of the system [4]:

$$\dot{\omega}_e = (T_e - T_c)/\theta_e \,, \tag{13}$$

$$\dot{\omega}_m = (T_c - T_m)/\theta_m \,. \tag{14}$$

According to the model the clutch torque is composed of the load of the reduced torsion spring and the torque of the damper element. The 'L' torque limiter element limits the connecting torque to the sliding torque maximum.

$$T_{\max} = T_s\left(p_n\left(t\right), \mu\left(v_{\text{rel}}, t\right)\right) \,, \tag{15}$$

where p_n : time function of the surface pressure,
μ : friction coefficient,
v_{rel} : relative speed between contact zones.

If there is no sliding, then the following equation is valid:

$$T_c(t) = S\left[x_0 + \int_{t_1}^{t} (\omega_e - \omega_m)\,\mathrm{d}t\right] + D(\omega_e - \omega_m)\ , \tag{16}$$

where x_0 is the initial position of the torsion spring (at instant t_1 that precedes t), from which ξ can be expressed as a function of the other state variables

$$\xi(t) = \xi(t_1) + \frac{1}{T_s(t)}\left[S\int_{t_1}^{t} (\omega_e - \omega_m)\,\mathrm{d}t + D(\omega_e - \omega_m)\right] \quad \xi \le 1\,, \tag{17}$$

where there is no sliding in-between instants t_1 and t.

In case of sliding, $\xi = 1$ and $\dot{\xi} = 0$.

We can get $\dot{\xi}$, if we derive *Eq.* (17), when it is assumed that time period $t - t_1$ is small enough for neglecting the changes of $T_s(t)$ sliding torque during one such period.

$$\begin{cases} \dot{\xi} = \dfrac{1}{T_s(t)}\left[S(\omega_e - \omega_m) + D(\dot{\omega}_e - \dot{\omega}_m)\right] & \text{if} \quad \xi < 1\,, \\ \dot{\xi} = 0 & \text{if} \quad \xi = 1\,. \end{cases} \tag{18}$$

5. Principles of Application of the Model in Drive Systems

Clutches, speed change gears, and torque converters are frequently realised between prime movers and driven machines in drive systems. Gears transmit torque from the input shaft to the output shaft of speed change gears. In case of speed change gears with planetary gear drives an additional thing must be taken into account: the torques of the brakes and clutches. The scheme of the drive system can be seen in *Fig. 7.*

The model can be simplified if we reduce the moments of inertia to the independent shafts [4]. As an example, let us observe the sketch of a drive system in *Fig. 8.* The angular velocities of the independent shafts will be state variables. The state equations using these state variables are:

$$\dot{\omega}_i = \frac{\sum_j T_{i,j}}{\theta_i} \qquad i = 1 \ldots n\,, \tag{19}$$

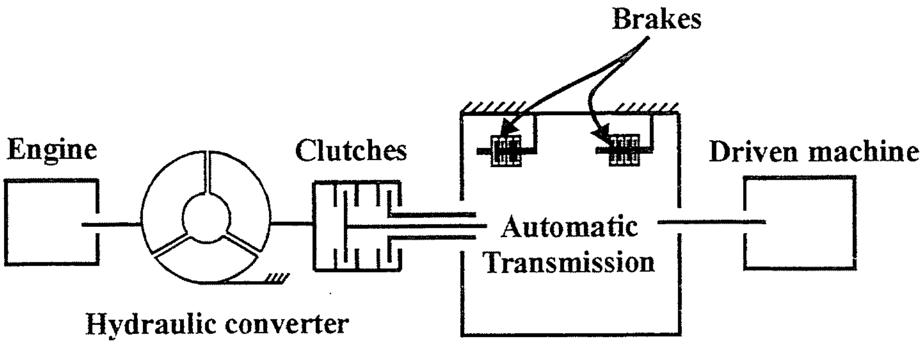

Fig. 7.

where n : number of independent shafts. The moments of inertia must be reduced on these shafts.

ω_i : the angular velocity of the shaft number i.

T_{ij} : torque reduced on shaft number i. This can be the torque of the prime mover, the driven machine, the clutch or the brake.

$$T_e = T_e(\omega_e)\,, \tag{20}$$

$$T_m = T_m(\omega_m)\,, \tag{21}$$

$$T_c = T_c\left(p_n(t)\,, \mu(v_{\mathrm{rel}}, t^o), \xi(t)\right)\,, \tag{22}$$

where $p_n(t)$: the time function of the surface pressure,

μ : friction coefficient,

v_{rel} : relative speed between contact zones.

There are some other state variables, namely the ξ variables of the clutch. The state equations for these variables are:

$$\begin{cases} \dot{\xi}_k = \dfrac{1}{T_s(t)}\left[S_k(\omega_e - \omega_m) + D_k(\dot{\omega}_e - \dot{\omega}_m)\right] & \text{if} \quad \xi_k < 1\,, \\ \dot{\xi}_k = 0 & \text{if} \quad \xi_k = 1\,. \end{cases} \quad k = 1 \ldots m\,, \tag{23}$$

where m is the number of clutches,

S_k is the reduced torsion spring stiffness of the shafts, gears, and tangential springs joined to the clutch number k.

D_k is the reduced damper describing the internal and external friction of the gears and shafts.

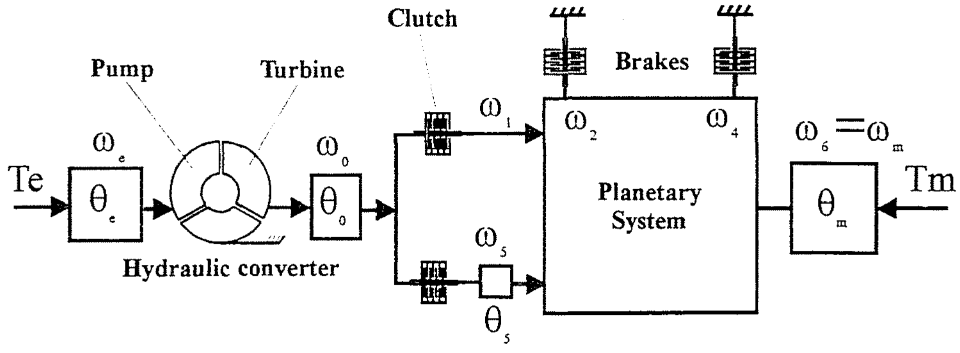

Fig. 8.

6. Presenting the Use of the Model Through an Example

The scheme of a drive system and its dynamic model can be seen in *Fig. 7* and *Fig. 8*. The key to solve the problem is to choose the independent shafts and derive the state equations of the system. It is obvious that the engine shaft and the turbine shaft are independent.

As it is well known, a O-I type planetary gear drive has two input shafts and one output shaft. Consequently, if we know the angular speeds of two shafts of the planetary gear, then the angular speed of the third shaft can be calculated [7]. In other words, the number of degrees of freedom at an O-I type planetary gear drive is two. The planetary gear drive system of the drive system can be seen on *Fig. 9* and *Fig. 10*.

The models of the planetary gear drives are presented by a triangle on *Fig. 10*. This presents an element with two degrees of freedom, for example the O-I type planetary gear drive. Since one planetary gear drive has two degrees of freedom, four planetary gear drives have eight. On the other hand, there are six joints in-between the shafts of the planetary gear drives, so the planetary gear drive system has only two degrees of freedom. Subsequently, it has only two independent shafts. In this case shafts 5 and 6 are considered to be independent.

Therefore it can be written:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \begin{bmatrix} x_{15} & x_{16} \\ x_{25} & x_{26} \\ x_{35} & x_{36} \\ x_{45} & x_{46} \end{bmatrix} \begin{bmatrix} \omega_5 & \omega_6 \end{bmatrix}, \tag{24}$$

where the x_{ij} constants can be calculated knowing the teeth number of the gears.

Shafts 1 through 4 can be considered to be internal shafts according to *Eq.* (24). The angular speeds of these shafts can be expressed as the functions of the angular speeds of the independent shafts.

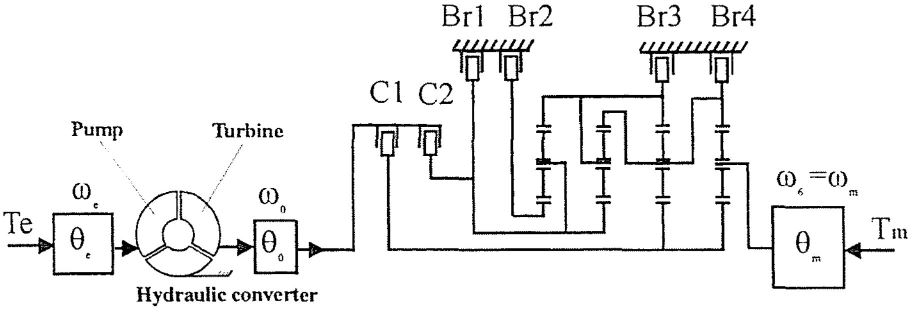

Fig. 9.

There is only one unknown torque at each planetary gear drive, that is knowing the torque at one of the independent shafts the torques of the other shafts can be calculated. In our case the four unknown torques of the four planetary gear drives are determined by the four clutch torques. These torques $(T_{c2} - T_{b1}, T_{b2}, T_{b3}, T_{b4})$ are functions of the variables ξ_i. The torques acting on the shafts of the planetary gear drive system can be expressed with four suitably defined torques:

$$\begin{bmatrix} T_{12} \\ T_{13} \\ T_{22} \\ T_{23} \\ T_{32} \\ T_{33} \\ T_{42} \\ T_{43} \end{bmatrix} = \begin{bmatrix} t_{12} & 0 & 0 & 0 \\ t_{13} & 0 & 0 & 0 \\ 0 & t_{22} & 0 & 0 \\ 0 & t_{23} & 0 & 0 \\ 0 & 0 & t_{32} & 0 \\ 0 & 0 & t_{33} & 0 \\ 0 & 0 & 0 & t_{42} \\ 0 & 0 & 0 & t_{43} \end{bmatrix} [\, T_{11} \quad T_{21} \quad T_{31} \quad T_{41} \,] , \tag{25}$$

where t_{ij} are constants that can be calculated if the teeth numbers are known

T_{ij} is the torque acting on the shaft j of the planetary gear drive i.

In order to calculate the torques in *Eq.* (25) let us see the torque-equilibrium equations of the internal shafts. The moments of inertia of the internal shafts are neglected.

$$T_{c2} - T_{b1} - T_{12} - T_{21} = 0 , \tag{26}$$

$$T_{11} - T_{b2} = 0 , \tag{27}$$

$$T_{13} + T_{22} + T_{33} + T_{b3} = 0 , \tag{28}$$

$$T_{23} + T_{32} + T_{43} + T_{b3} = 0 , \tag{29}$$

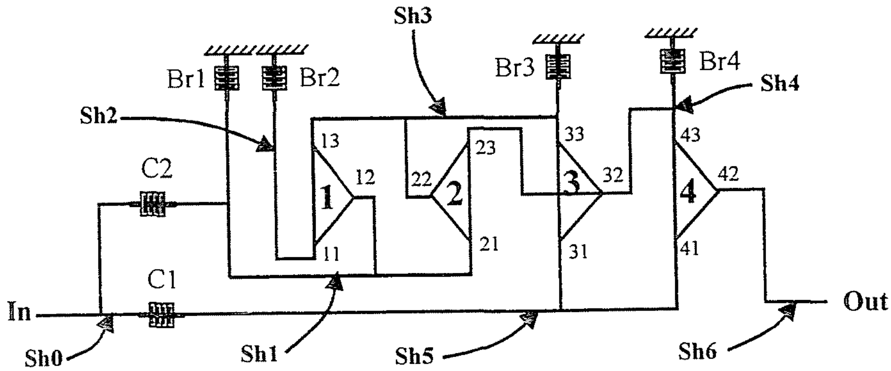

Fig. 10.

where T_{c1} and T_{c2} are the clutch torques,
T_{bi} is the brake torque.

After solving the system of linear equations, the result is:

$$\begin{bmatrix} T_{11} \\ T_{21} \\ T_{31} \\ T_{41} \end{bmatrix} =$$

$$\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & t_{12} & 0 & 0 \\ \dfrac{1}{t_{33}} & \dfrac{t_{12}-t_{13}}{t_{33}} & \dfrac{1}{t_{33}} & 0 \\ -\dfrac{t_{23}}{t_{43}}-\dfrac{t_{32}}{t_{33}t_{43}} & \dfrac{t_{13}-t_{12}-t_{23}t_{12}}{t_{43}} & -\dfrac{t_{32}}{t_{33}} & -1 \end{bmatrix} \begin{bmatrix} T_{c2}(\xi_{c2})-T_{b1}(\xi_{b1}) \\ T_{b2}(\xi_{b2}) \\ T_{b3}(\xi_{b3}) \\ T_{b4}(\xi_{b4}) \end{bmatrix}. \quad (30)$$

Since shafts 5 and 6 were chosen as independent ones, the moments of inertia must be reduced on these shafts. Without going into the details, the reduction can be carried out the following way:

$$\theta_{5r} = \theta_5 + \sum_{i=1}^{4} x_{i5}^2 \theta_i , \quad (31)$$

$$\theta_{6r} = \theta_m + \sum_{i=1}^{4} x_{i6}^2 \theta_i , \quad (32)$$

where i is the moment of inertia of the shaft No. i,
m is the moment of inertia of the driven machine.

The state equations are:

$$\dot{\omega}_e = \frac{T_e(\omega_e) - T_p(\omega_e)}{\theta_e}, \tag{33}$$

$$\dot{\omega}_0 = \frac{T_t(\omega_e, \omega_0) - T_{c1} - T_{c2}}{\theta_0}, \tag{34}$$

$$\dot{\omega}_5 = \frac{T_{c1} - T_{31} - T_{41}}{\theta_{5r}}, \tag{35}$$

$$\dot{\omega}_6 = \frac{-T_{42} - T_m(\omega_6)}{\theta_{6r}}, \tag{36}$$

where T_p and T_t are the torques of the pump and the turbine of the torque converter.

$$\begin{cases} \dot{\xi}_{c1} = \dfrac{1}{T_{s,c1}(t)}\left[S_{c1}(\omega_0 - \omega_5) + D_{c1}(\dot{\omega}_0 - \dot{\omega}_5)\right] & \text{if} \quad \xi_{c1} < 1, \\ \dot{\xi}_{c1} = 0 & \text{if} \quad \xi_{c1} = 1, \end{cases} \tag{37}$$

$$\begin{cases} \dot{\xi}_{c2} = \dfrac{1}{T_{s,c2}(t)}\left[S_{c2}(\omega_0 - \omega_1) + D_{c2}(\dot{\omega}_0 - \dot{\omega}_1)\right] & \text{if} \quad \xi_{c2} < 1, \\ \dot{\xi}_{c2} = 0 & \text{if} \quad \xi_{c2} = 1, \end{cases} \tag{38}$$

$$\begin{cases} \dot{\xi}_{bi} = \dfrac{1}{T_{s,bi}(t)}\left[S_{bi}(\omega_i) + D_{bi}(\omega_i')\right] & \text{if} \quad \xi_{bi} < 1, \\ \dot{\xi}_{bi} = 0 & \text{if} \quad \xi_{bi} = 1, \end{cases} \quad i = 1 \ldots 4, \tag{39}$$

where T_{c1}, T_{c2}, T_{bi} are function of $\xi_{c1}, \xi_{c2}, \xi_{bi}$ (*Eq.* (30)).
$T_{s,x}$ is the sliding torque of the clutch or brake No. x.

7. Conclusions

Provided that the friction clutch is a set of two rigid rotors connected by friction, we have to face a big problem in simulation of drive systems that, the drive systems having no unique dynamic model, the calculations consequently are very complicated because of the systematical changing of different models. On the process of the engagement the friction surfaces slide on each other, the current model is based on the model shown on the *Fig. 4*, the contact torque is the sliding limit torque. When the revolutions of the driving and driven shafts are equal, the friction surfaces only have to transmit the necessary torque, which is less than the sliding limit torque. In this case although the *Eqs.* (3) and (4) are valid, the T_k contact torque in the equations is unknown and it cannot be calculated directly. Therefore to construct a unique model we need to define a new state variation, which determines the current value of the contact torque. This is nothing else but the average shear stress of the contacted friction surfaces, which is about a linear function of the torque twisting the shafts. So the unique model must include a reduced torsion spring that models the twist of the shafts, a torque limiting device that limits the twist of the shafts to the sliding limit torque of the friction discs, and in addition a reduced damping that models the internal and external friction of the elements mentioned above.

A demonstration was also carried out to complete the principles of application of the model in driving systems. The author believes that more researches and discussions will follow in order to develop the model and make more applications of the model on more fields, but not only in the automatic planetary speed-change gears.

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