# SPECTRAL ANALYSIS OF MOIRÉ IMAGES

Ákos ANTAL* and B. ÖLVECZKY**

* Department of Precision Mechanics and Optics
Technical University of Budapest
H-1521 Budapest, Hungary
** Centre for Biological and Medical Systems
Imperial College, London

Received: Jan. 10, 1997

## Abstract

The signal-to-noise ratio of moiré images is quite low. Obtaining useful information from an image can often be problematic, since the contrast of the grid giving rise to the moiré phenomena is similar to the contrast of the moiré fringes that carry the useful information. By using optical filtering techniques it is possible to filter these images real-time. In this paper the authors give an example of an algorithm to design an appropriate spatial filter by comparing the Fourier spectra of a mathematical model of a moiré image with a real moiré image.

*Keywords:* moiré phenomena, Fourier optics, diffraction pattern, spatial filtering.

## Introduction
## The MoiréPhenomena

The word 'moiré' comes from the French language and according to the Oxford dictionary it describes 'a variegated or clouded appearance like that of silk'. The phenomena has interested physicists for a long time. Lord Rayleigh, refers to it in a paper from 1874:

'If two photograph copies containing the same number of lines to the inch be placed in contact, film to film, in such a manner that the lines are nearly parallel in the two gratings, a system of parallel bars develops itself, whose direction bisects the external angle between the directions of the original lines and whose distance increases as the angle of inclination diminishes...' (RAYLEIGH, 1874).

The effect that Rayleigh describes is the result of interference between two grids with different spatial frequencies, where the frequencies are determined by the number of periods per unit length. The phenomena is often called mechanical or geometrical interference. Interference between two wavefronts is a very precise technique and is in widespread use in measurement technology, but the domain in which it can be used is limited by the wavelength of visible light. If interferometric precision is not required, then an interference pattern produced by a longer 'wavelength' is justifiable, hence the importance of the moiré phenomena, where the wavelength is analogous to the period of the grid. Apart from Rayleigh, Foucault (FOUCAULT, 1859) and Ronchi (RONCHI, 1825) were also involved in the practical implementation of this method. The application of the moiré phenomena started in earnest in 1952 when the English physicist Sir Thomas Merton (MERTON, 1952) proposed a technique to produce high spatial frequency grids at low cost. Since then interest in moiré techniques as applied to measurement technology has grown.

The applicability of the moiré phenomena to measurement technology is based on the idea that if the two spatial frequencies of the grids producing the moiré pattern are connected to two different states of an object, then the resulting moiré pattern will be a function of the difference between the two states. Thus the method is suitable for measuring movement, deformation (TEOCARIS, 1969) and – when one grid is connected to the shape of the object, the other is a reference grid independent of the object – for topographical (TAKASAKI, 1970).

The result of the measurement is an image made up of moiré fringes, whose evaluation is the indirect and useful result of the measurement. One of the most important conditions for the successful application of the moiré technique is a high signal to noise ratio to enable efficient processing of the information inherent in the image.

In order to apply the moiré technique for measurements it is necessary to produce the moiré image with a known arrangement. It is a prerequisite to know the geometrical parameters of the arrangement in order to extract the information from the moiré image.

The great advantage of the moiré technique is that it is capable of producing good results under very primitive circumstances.

The simplicity of the underlying idea, the relative insensitivity towards outside noise, makes it useful for measurements where other techniques fail, or where their precision is not required. The instrumentation producing the moiré must comply with this concept. There are numerous moiré producing methods which give highly accurate results (DAI, 1991) (MATSUMOTO, 1973) by increasing the sophistication of the instrumentation. The practical application of these methods is not widespread, since they do not utilize the main advantage of the moiré technique: its simplicity and lucidity.

## The Basic Arrangement of Moiré Measurements

One of the simpler moiré arrangements is the so-called shadow moiré. It is in widespread use due to its simplicity and it serves us well to demonstrate the basic principles of the moiré phenomena.

The moiré pattern is observed by looking at the shadow cast by the grid lit with a point source of light through the grid itself. The shadow when seen from a given distance from the lightsource will be distorted as a function of the shape of the object, and its superposition with the original grid will yield information of the object in the form of moiré contour lines. The resolution of the method can be enhanced by increasing the frequency of the grid.

Another arrangement in frequent use is the reflection method, which can be applied if the surface can be prepared as a mirror. The result is similar to the shadow moiré with half the level difference between subsequent contour lines. It is in wide use due to high sensitivity to plane and small curvature surfaces.

For measurement purposes the projection method is the most used one. This method can only be used to measure diffuse surfaces, but the main disadvantages of the shadow moiré have here been eliminated. The arrangement consists of two main parts, a projection and a recording unit. In the projection unit the image of the grid is projected onto the surface of the object with the help of an optical system. The shadow of the grid on the object is then projected onto the grid in the recording unit, producing a moiré pattern. If we are eliminating the grid in the recording unit and record the projection of the distored shadow for two different states on the same frame then the moiré pattern will carry information about the difference between the two states, thus enabling us to measure deformation and movement.

## Spectral Analysis of Moiré Images

One of the most important tools of modern optics is the Fourier transform. The diffraction phenomena, spatial filtering and the image processing techniques form the basis that can all be explained by Fourier analysis. From a point of view of spatial filtering, the most important property of the Fourier transform is that it defines the relationship between the image and its diffraction pattern, found in the focal plane of the transforming lens. The spatial filtering of the image takes place in the diffraction pane.

Diffraction is a type of interference. Waves from different parts of the aperture interfere and cause a diffraction pattern in the far field (in infinity) of the aperture called the Fraunhofer diffraction pattern. This diffraction pattern is caused by the interference of waves propagating in the same direction. The easiest way of observing the pattern is to use a converging lens that focuses the waves with equal orientation into single points, which is equivalent with observing the interference in infinity. It is important to remember that the diffraction pattern is produced by the aperture and not by the lens; the lens only makes its observation easier. To be able to observe the diffraction pattern of an object it is essential to illuminate with a coherent lightsource, so as to ensure the constant phase relations of the interfering wavefronts.

## Relationship between the Periodic Structure and the Diffraction Pattern

A periodic function can be represented by a sum of harmonic functions. This sum is called the Fourier series of the function. In theory only functions with infinite number of periods have a Fourier series, but due to the large number of periods of an optical grid, the Fourier analysis provides a good approximation.

With the concept of spatial frequencies used in optics (number of periods per unit length) it can easily be understood that the diffraction pattern of a grid is a physical manifestation of the harmonic functions representing the grid, or in the case of a non-periodic object, the frequency spectrum of the object. Since the intensity of the Fraunhofer diffraction pattern is proportional with the Fourier transform of the aperture function, the diffraction phenomena enable us to estimate the Fourier transform of the aperture function (NUSSBAUM, 1982).

## Signal to Noise Ratio of Moiré Images

It is hard to give a general definition what is signal and what is noise in a moiré image, since they are dependent on the given application. Often the moiré fringes themselves can be considered to be noise (e.g. TV screen, typography).

Applied to measurement technology the signal is the moiré fringes carrying the information about the state or the shape of the object. The most important task is to identify the frequency domain where the moiré fringes are represented. What the frequencies outside this domain are representing needs to be subjected to further analysis.

In the following analysis we will be examining the frequency spectrum of a moiré pattern produced by two grids with square-wave transmission functions. We will then go on to examine the frequency spectrum of a real moiré image.

## The Analysis of a Moiré Pattern Produced by Optical Grids with Square-Wave Transmission Functions

The optical grid used for demonstrating the moiré phenomena can be approximated with a square-wave function:

$$\tau = \frac{1}{2}\left\{1 + sign\left[\sin\left(\frac{2^k \pi x}{K}\right)\right]\right\} ,$$

where:

$k$ is the number of periods within the examined interval,
$x$ is the variable along the examined axis,
$K$ is the length of the interval.

The analysis of moiré patterns produced from interference of the grids with square wave transmission functions is done with the help of numerical analysis. The transmission function and the power spectrum of the grids can be seen in *Fig. 1.*

The power spectrum is proportional with the square of the Fourier transform of the transmission function.

The superposition of the two grids in *Fig. 1* yields the moiré pattern, whose transmission function and power spectrum can be seen in *Fig. 2.* We can see that the frequency domain representing the moiré fringes is easily separable from the frequencies representing the grid. We can also note that the higher harmonics of the moiré fringes are negligible. This is a property of the moiré fringes and can be explained by the harmonic transmission change of the fringes. In the power spectrum we also have the frequencies representing the sum of the fundamental frequencies and the frequencies representing the additional moiré. The additional moiré comes from interference between the higher harmonics of the first grid and the first harmonic of the second grid, as well as the sum and difference of the fundamental frequency of the second grid and the first harmonic of the first grid. Interference between higher harmonics can also be seen, but the energy of these frequencies is very small. The separation of the frequencies will start to be problematic when the difference between the two fundamental frequencies gets close to the fundamental frequencies themselves.

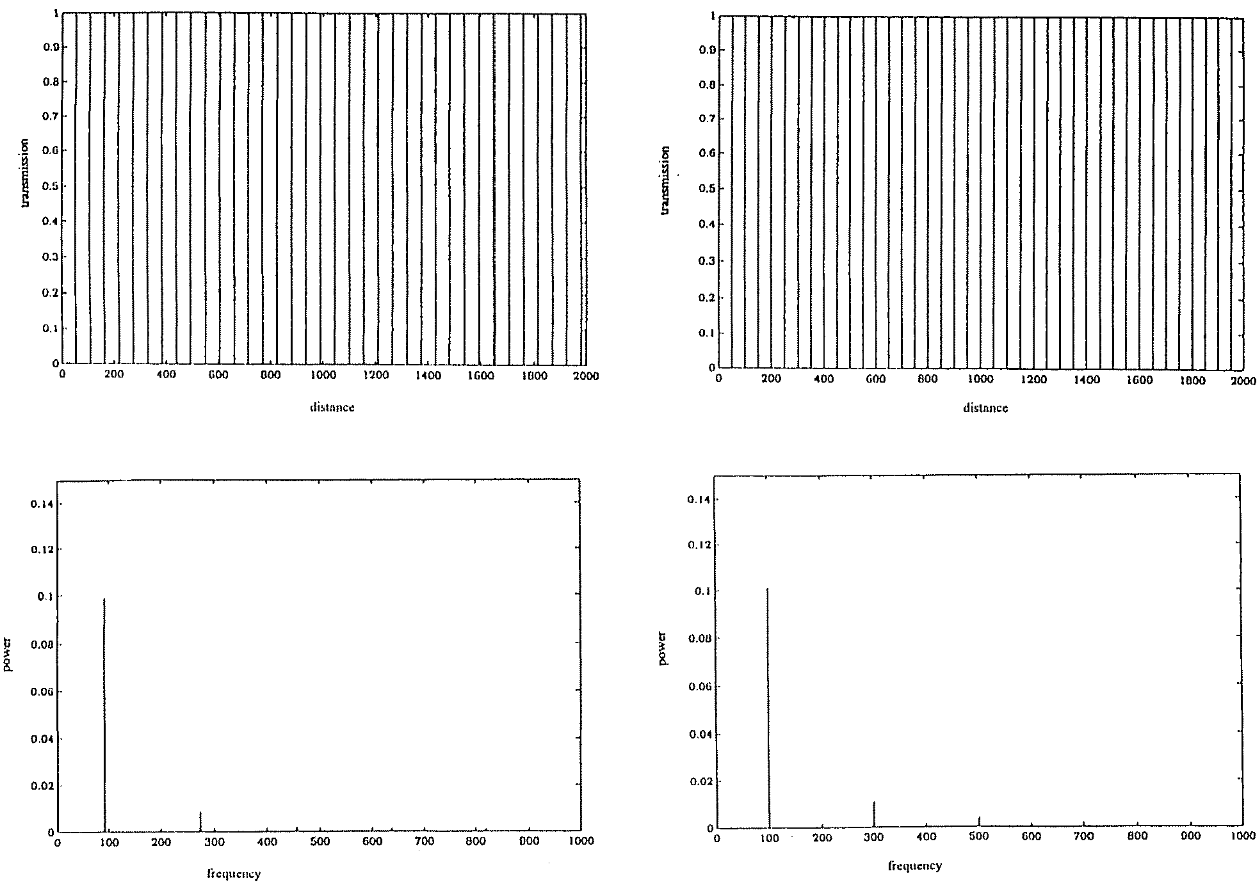


*Fig. 1.* The transmission function and power spectrum of the modelled grids used to produce the moiré phenomena

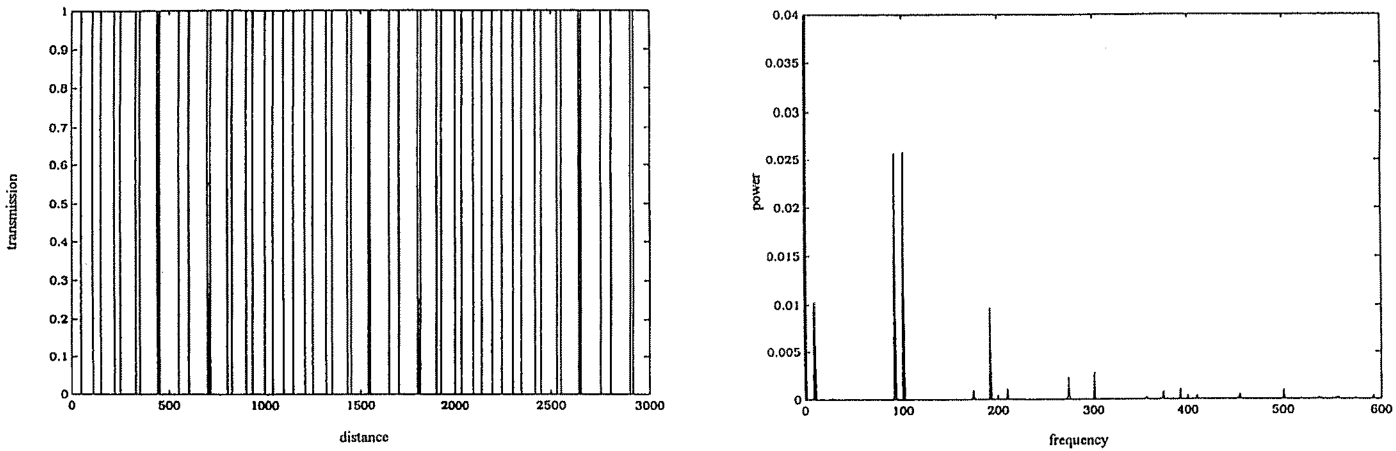


*Fig. 2.* Transmission function and power spectrum of the modelled moiré image

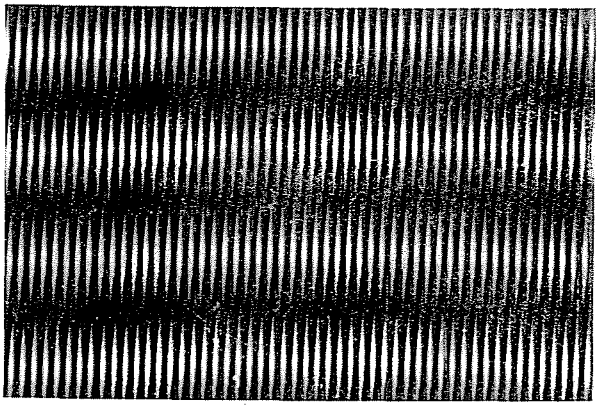

*Fig. 3.* The real moiré image

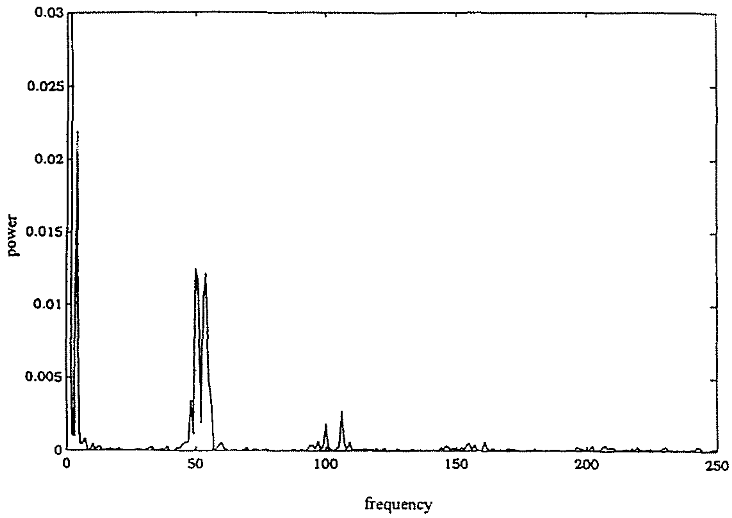


*Fig. 4.* The power spectrum of the real moiré image

## Analysis of a Real Moiré Image

*Fig. 3* shows a moiré image recorded with a digital camera. The power-spectrum calculated with the help of the discrete Fourier transform of a single row of the image can be seen in *Fig. 4*. Its character is very similar to the power spectrum of the ideal image. The frequencies representing the moiré fringes are easily separable, thus the grids can easily be separated in this case as well. We can see that in the real image we have added noise but the power of the noise is small compared to the power of the frequencies carrying the relevant information.

## Summary

The informative and very easily produced moiré images are noisy due to the very nature of their production. If the signals carrying the relevant information are separable from the noise, then it is possible to filter the images using optical or digital filtering, and thereby making further processing or evaluation of the images easier.

Our analyses have shown that the power spectrum of a mathematically modelled moiré image is almost identical with the real power spectrum, and that a preliminary investigation is often helpful when designing a filtering algorithm.

## References

1. DAI, Y. Z. – CHIANG, F. P. (1991): Contouring by Moiré Interferometry, *Experimental Mechanics*, March 1991, pp. 76–81.
2. FOUCAULT, L. (1859): Memoire sur la construction des telescopes en verre argente, *Annls. Obs.*, Vol. 5, 1859, p. 197.
3. MATSUMOTO, K. – TAKASHIMA, M. (1973): Improvement on Moiré Technique for In-Plane Deformation Measurements, *Applied Optics*, Vol. 12, April 1973, pp. 858–864.
4. MERTON, T. (1952): Nouvelles méthodes de fabrication des réseaux, *Journal of Physics – Radium*, Vol. 13, 1952, p. 49.
5. NUSSBAUM, A. – PHILLIPS, R. A. (1982): Modern optika mérnököknek és kutatóknak (Contemporary Optics for Scientists and Engineers), Müszaki Könyvkiadó, Budapest, 1982 (in Hungarian).
6. RAYLEIGH, J. W. (1874): *Philosophical Magazine*, Vol. 47, 1874, p. 197.
7. RONCHI, V. (1925): La prova di sistemi ottici, *Attual. Scient.*, No. 9, 1925.
8. TAKASAKI, H. (1970): Moiré Topography, *Applied Optics*, Vol. 9. June 1970, pp. 1467–1472.
9. TEOCARIS, P. S. (1969): Moiré Fringes in Strain Analysis, Pergamon Press, 1969.