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# STIFFNESS OF THROTTLED HYDROSTATIC TRANSMISSIONS

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## Abstract

In hydraulic power transmission, up to a reasonable output limit, throttled hydrostatic transmissions are preferred. Although transmission can be changed here only at significant losses, they are likely to be economical because of simple construction.

Relying on the derivative characteristic of the transmission stiffness, analysis of load-dependent behaviour of various throttled hydrostatic drives is presented.

*Keywords:* hydrostatic transmission, stiffness.

## 1. Introduction

Poor efficiencies, resulting higher operational costs of hydrostatic transmissions to be altered by lossy procedures, are compensated by lower investment costs due to simple constructions. As it is seen from the practice, these systems are economical up to an output 1 kW (e.g. for driving industrial robots). This is why it is of importance to see how these drives behave under various charge conditions, how they react load changes. These questions will be answered by stiffness examinations.

## 2. Theoretical Built-up of Throttled Hydrostatic Transmission

Lossy transmission changes rely on volume flow control by throttling.

As to the essentials of functioning, a system of variable transmission consists of a pump of constant displacement $V_p$ and constant rotary speed $n_p$ and a motor of constant displacement $V_m$ under a moment $T_m$ connected to an open circuit (*Fig. 1*). For a constant pump volume flow $q_p$, volume flow $q_m$ of the motor in the circuit. and thereby the motor's rotary speed $n_m$, that is, rotary speed ratio $i_t = n_m/n_p$ in the system, are controlled by means of adjustable throttle $R_t$, while the excess volume flow $q_p - q_m$ is advisable to be returned to the reservoir across the adjustable throttle $R_r$ in the branch parallel to the energy transformers as volume flow $q_r = q_p - q_m$. Since the primary function of throttle $R_t$ is to modify volume flow $q_m$ of the motor, main function of throttle $R_r$ is to keep the total pressure difference $\Delta p_p = \Delta p_r$ of the system at a preferably constant value, while, of course, throttles $R_t$ and $R_r$ interact. In the given hydraulic network, under the described flow conditions, pressure difference and volume flow conditions are seen in the diagram exist.

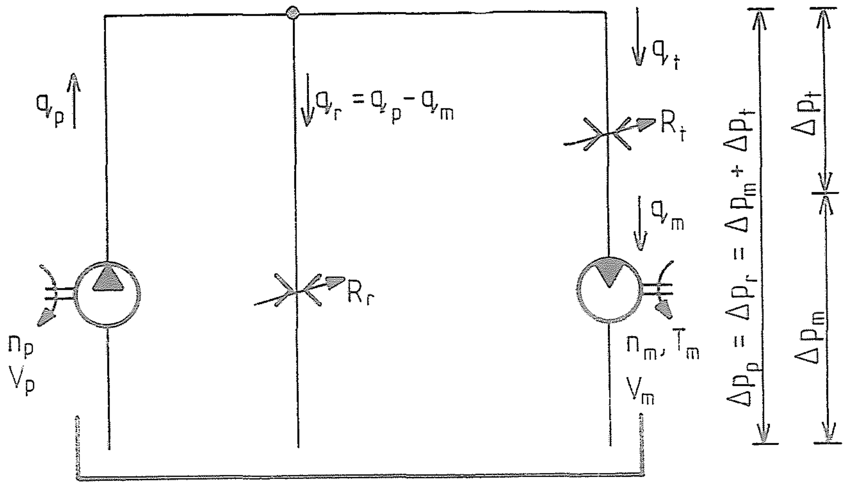


*Fig. 1.* Principle of transmission control by throttling

In practice, there are two basic arrangements for transmission control by throttling: in series (*Fig. 2*) and in parallel (*Fig. 3*). These attributes indicate the arrangement of throttle $R_t$ in relation to the motor, depending on the flow conditions. Circuit models seen in the figures have been established taking into consideration that in subsequent analyses, pumps and motors with volume losses will be involved, furthermore, that in practice, relief valves, rather than throttles $R_r$, are applied.

Remind that two alternatives of series arrangement are in use. Downstreams, for one, throttle $R_t$ precedes (meter-in), for the other it follows (meter-out) the motor. Since equations and characteristics describing the system's function are both the same control systems, in the following, series arrangement will be illustrated on meter-in.

Characteristics $\Delta p = f(q)$ of the functioning of circuits according to *Figs. 2* and *3* are illustrated in *Figs. 4* and *5*, respectively. In plotting the

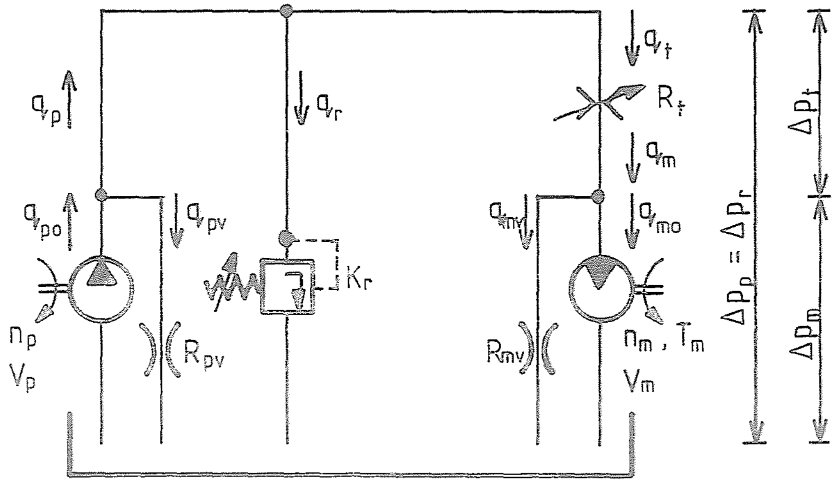


*Fig. 2.* Circuit model of series transmission control ($q_p - q_m = q_r$)

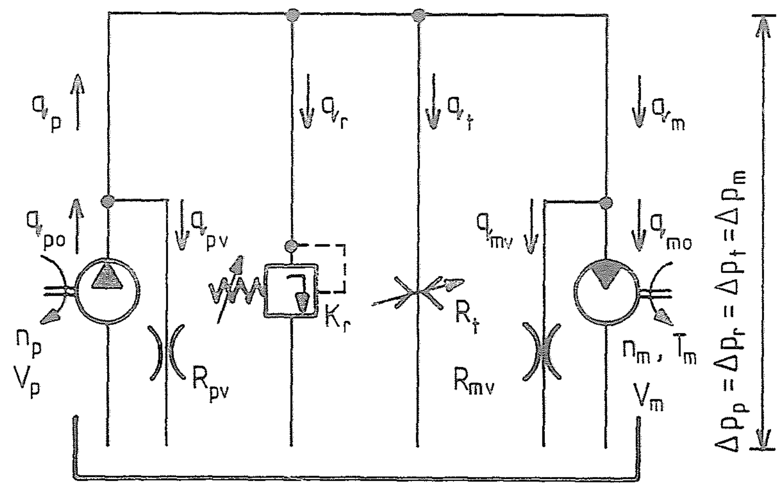


*Fig. 3.* Circuit model of parallel transmission control ($q_p - q_m = q_r + q_t$)

characteristics, the part of circuit comprising the pump with volumetric loss and the relief valve, has been considered as power source, while the part of circuit comprising the motor with volumetric loss, and its flow control valve as power consumer.

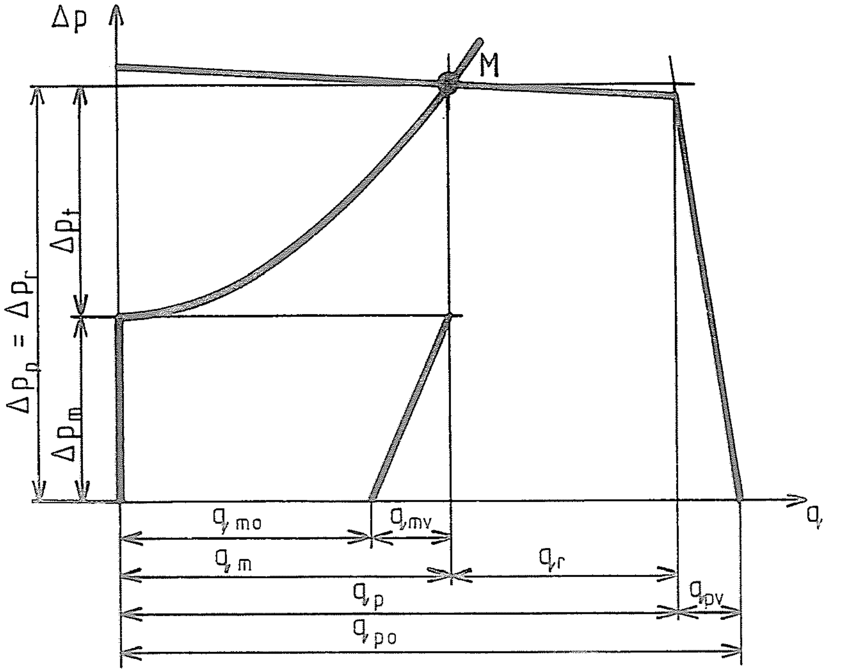


*Fig. 4.* Characteristic $\Delta p = f(q)$ of series transmission control

## 3. Stiffness of Throttled Hydrostatic Transmissions

Transmission stiffness $S_n$ can be described in general by the derivative

$$S_n = \frac{\partial T_m}{\partial n_m} \tag{1}$$

For computing the stiffness, relationship

$$n_m = f(T_m) \tag{2}$$

for the examined drive type has to be determined. For its determination, relationship

$$n_m = \frac{q_{mo}}{V_m} \tag{3}$$

may be started from, where, making use of circuit models (*Figs. 2* and *3*), and of characteristics (*Figs. 4* and *5*), for the sake of effectiveness, volume flow $q_{mo}$ has to be replaced by its value, expressed conveniently comprising also moment $T_m$.

Let us apply those above to determine stiffnesses of throttled transmissions in series, and in parallel.

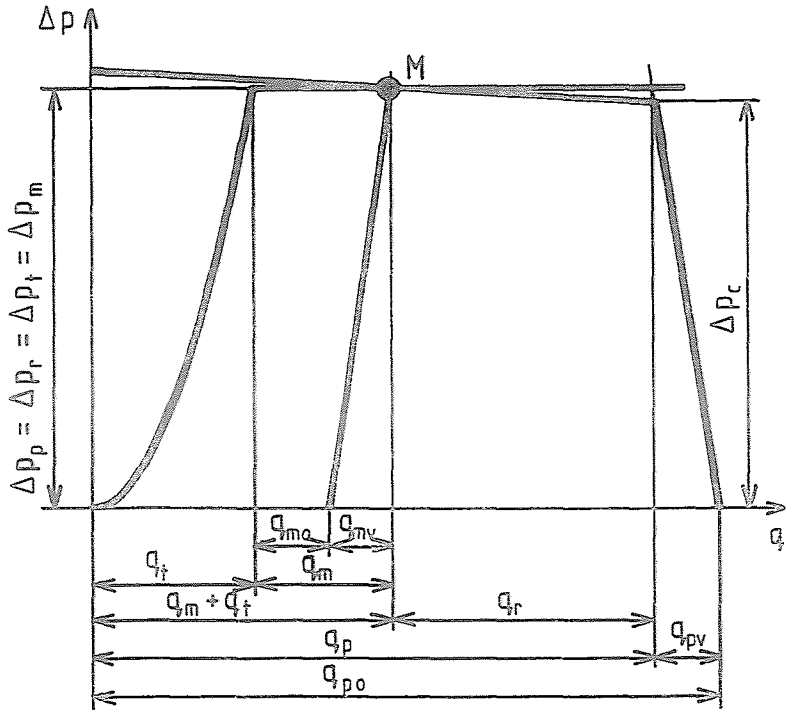


*Fig. 5.* Characteristic $\Delta p = f(q)$ of parallel transmission control

### *3.1 Stiffness of Throttled Transmissions in Series*

In transforming *Eq.* (3), the following have been reckoned with:

$$q_{mo} = q_m - q_{mv}, \tag{4}$$

$$q_m = q_t, \tag{5}$$

$$q_t = \sqrt{\frac{\Delta p_t}{R_t}}, \tag{6}$$

$$\Delta p_t = \Delta p_r - \Delta p_m, \tag{7}$$

$$q_{mv} = \frac{\Delta p_m{}'}{R_{mv}} \tag{8}$$

$$\Delta p_m = \frac{2\pi T_m}{V_m}. \tag{9}$$

Making use of *Eqs.* (3) and (4) to (9):

$$n_m = \frac{1}{V_m}\left(\sqrt{\frac{\Delta p_r V_m - 2\pi T_m}{R_t V_m}} - \frac{2\pi T_m}{R_{mv} V_m}\right). \tag{10}$$

Substituting $R_{mv} = \infty$ (taking $q_{mv} = 0$ into consideration) yields the rpm $n_{mo}$ of loss-free system:

$$n_{mo} = \frac{1}{V_m}\sqrt{\frac{\Delta p_r V_m - 2\pi T_m}{R_t V_m}} \tag{11}$$

Taking (1) into consideration, respective stiffnessess for cases with, and without losses may be expressed as:

$$S_n = -\frac{1}{\pi}\frac{1}{\frac{1}{\sqrt{R_t V_m^3 \sqrt{\Delta p_r V_m - 2\pi T_M}}} + \frac{2}{R_{mv} V_m^2}}, \tag{12}$$

$$S_{no} = -\frac{1}{\pi}\sqrt{R_t V_m^3 \sqrt{\Delta p_r V_m - 2\pi T_m}}. \tag{13}$$

For given $p_r$, $V_m$, $R_t$ and $R_{mv}$ values, *Eqs.* (10), (11), (12) and (13) yield the quality pattern according to *Fig. 6*.

### *3.2 Stiffness of Throttled Transmissions in Parallel*

In transforming *Eq.* (3), in addition to (8) and (9), still

$$q_{mo} = q_{po} - (q_{pv} + q_r + q_t + q_{mv}), \tag{14}$$

$$q_{pv} = \frac{\Delta p_m}{R_{pv}}, \tag{15}$$

$$q_r = \frac{\Delta p_m - \Delta p_c}{K_r}, \tag{16}$$

$$q_t = \sqrt{\frac{\Delta p_m}{R_t}} \tag{17}$$

have been reckoned with.

For parallelly throttled transmissions, proper use of the system does not require to have working point $M$ in the section of the power source characteristic defined by the relief valve, but – thanks to the parallel arrangement – it may also be in the section defined by the pump. In this

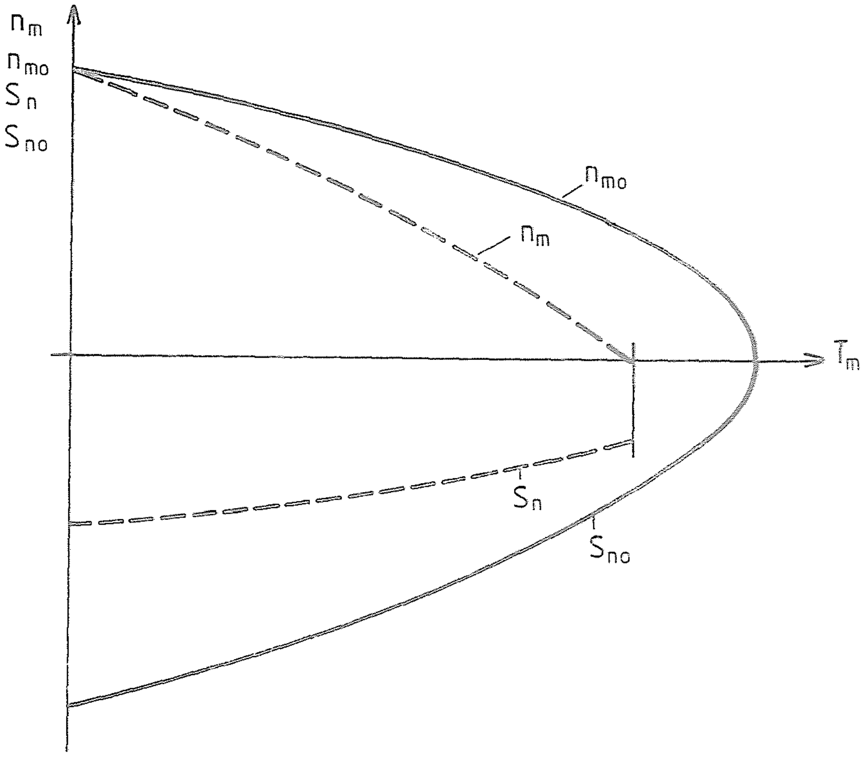


*Fig. 6.* Characteristics $n_m = f(T_m)$, $S_n = f(T_m)$ of series transmission control

case, the relief valve is closed, i.e. $q_r = 0$, hence the volume flow needless for the motor is returned into the reservoir as volume flow $q_t$. Accordingly, in analyses, cases $\Delta p_m \geq \Delta p_c$ and $\Delta p_m < \Delta p_c$ are advisably distinguished.

For the case $\Delta p_m \geq \Delta p_c$:

$$n_m = \frac{q_{po}}{V_m} + \frac{\Delta p_c}{K_r V_m} - \sqrt{\frac{2\pi T_m}{R_t V_m^3}} - \frac{2\pi T_m}{V_m^2}\left(\frac{1}{K_r} + \frac{1}{R_{pv}} + \frac{1}{R_{mv}}\right). \tag{18}$$

Replacing $R_{pv} = \infty$ and $R_{mv} = \infty$ (taking $q_{pv} = 0$ and $q_{mv} = 0$ into consideration), for a case without losses:

$$n_{mo} = \frac{q_{po}}{V_m} + \frac{\Delta p_c}{K_r V_m} - \sqrt{\frac{2\pi T_m}{R_t V_m^3}} - \frac{2\pi T_m}{K_r V_m^2}. \tag{19}$$

Taking (1) into consideration, for cases with, and without losses, respectively:

$$S_n = -\frac{1}{\pi}\frac{1}{\frac{1}{\sqrt{R_t V_m^3}\sqrt{2\pi T_m}} + \frac{2}{V_m^2}\left(\frac{1}{K_r} + \frac{1}{R_{pv}} + \frac{1}{R_{mv}}\right)}, \tag{20}$$

$$S_{no} = -\frac{1}{\pi}\frac{1}{\frac{1}{\sqrt{R_t V_m^3}\sqrt{2\pi T_m}} + \frac{2}{K_r V_m^2}}. \tag{21}$$

For the case $\Delta p_m < \Delta p_c$:

$$n_m = \frac{q_{po}}{V_m} - \sqrt{\frac{2\pi T_m}{R_t V_m^3}} - \frac{2\pi T_m}{V_m^2}\left(\frac{1}{R_{pv}} + \frac{1}{R_{mv}}\right). \tag{22}$$

Substituting $R_{pv} = \infty$ and $R_{mv} = \infty$ (taking $q_{pv} = 0$ and $q_{mv} = 0$ into consideration), for a loss-free case:

$$n_{mo} = \frac{q_{po}}{V_m} - \sqrt{\frac{2\pi T_m}{R_t V_m^3}}. \tag{23}$$

Taking (1) into consideration, for a lossy, and loss-free case, respectively:

$$S_n = -\frac{1}{\pi}\frac{1}{\frac{1}{\sqrt{R_t V_m^3}\sqrt{2\pi T_m}} + \frac{2}{V_m^2}\left(\frac{1}{R_{pv}} + \frac{1}{R_{mv}}\right)} \tag{24}$$

$$S_{no} = -\frac{1}{\pi}\sqrt{R_t V_m^3}\sqrt{2\pi T_m} \tag{25}$$

For given $q_{po}$, $\Delta p_c$, $V_m$, $K_r$, $R_t$, $R_{pv}$ and $R_{mv}$ values, *Eqs.* (18) to (25) yield the quality pattern in *Fig. 7*.

## 4. Conclusions

Making use of relationships $n_m = f(T_m)$, $n_{mo} = f(T_m)$, $S_n = f(T_m)$ and $S_{no} = f(T_m)$ deduced for the tested series and parallel transmissions, in knowledge of circuit parameters, quantitative analyses may be performed. Characteristics in *Figs. 6* and *7* are of help in the qualitative analysis of load-dependent behaviour of given types of transmissions, permitting to deduce conclusions of general validity, in view of the practical goal to increase the drive stiffness.

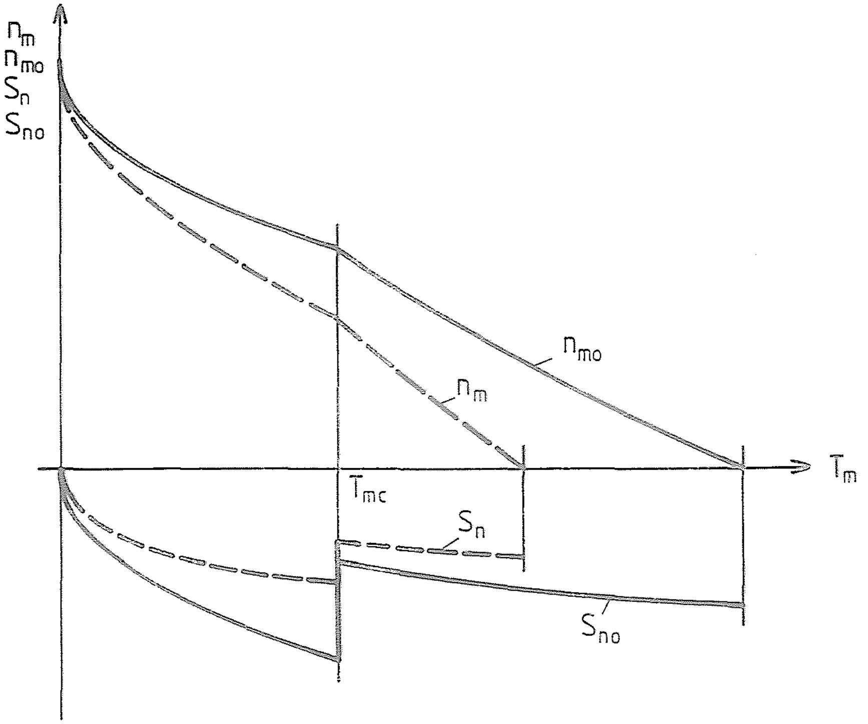


*Fig. 7.* Characteristics $n_m = f(T_m)$, $S_n = f(T_m)$ of parallel transmission control

For control valves in series with the motor (*Fig. 6*), obviously, the absolute value of stiffness $|S_n|$ is reduced with increasing load $T_m$ (for $T_m = T_{m\,\max}$, $|S_n| = 0$), so these systems are preferably applied in lower load ranges.

The opposite is true for parallel transmissions (*Fig. 7*). The absolute stiffness value $|S_n|$ is seen to decrease alongside the load $T_m$ (for $T_m = 0$, $|S_n| = 0$), so these systems are conveniently applied in higher load ranges.

## Notations

$i_t$ rotary speed ratio
$K_r$ relief valve constant
$M$ working point
$n_m$ motor rpm
$n_s$ pump rpm
$\Delta p_c$ pressure drop at the corner of the power source characteristic
$\Delta p_m$ pressure drop on the motor
$\Delta p_p$ pressure increase on the pump
$\Delta p_r$ pressure drop on the pressure control throttle or on the relief valve
$\Delta p_t$ pressure drop on the volume flow control throttle
$q_m$ metered volume flow of the motor
$q_{mo}$ theoretical volume flow of the motor
$q_{mv}$ volumetric loss of the motor
$q_p$ metered volume flow of the pump
$q_{po}$ theoretical volume flow of the pump
$q_{pv}$ volumetric loss of the pump
$q_r$ volume flow of the relief valve
$q_t$ volume flow of the flow control throttle
$R_{mv}$ resistance to volumetric loss of the motor
$R_{pv}$ resistance to volumetric loss of the pump
$R_r$ resistance of the pressure control throttle
$R_t$ resistance of the volume flow control throttle
$T_m$ moment on the motor
$V_m$ motor displacement
$V_p$ pump displacement.

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