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THE MATHEMATICAL MODEL OF PLUG FLOW CONVEYING

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Abstract

In the paper the movement characteristics of a special plug moving in the pneumatic pipe are described considering forces influencing the plug that acceleration, velocity, and the path made by the plug.

The state of motion in horizontal and vertical pipe sections, respectively, in a bend built into the pipe section will be determined. The bend examined in this paper is laying in the vertical plane and is connecting a horizontal pipe section with a vertical one.

Equations describing the movement characteristics are approximative because authors simplify the physical process, neglect different things. In the knowledge of the movement characteristics the data of the delivering machine can be determined.

Keywords: Pneumatic conveying, bends, plug flow conveying.

1. The Movement Equation of the Plug Moving in the Straight Pipe Section

Describing the mathematical-physical model of the plug the limitations are made as follows:

– The linear measure 'L' of the plug is not changing. If the cross-section of the pipe is constant, then the 'ε' proportion of the voids volume, the 'ϱ_h bulk density, and the 'c_a' material velocity are also constant along the plug length, that is:

$$\varepsilon = \text{const.}; \qquad \varrho_h = \text{const.}$$

– The friction of the transport gas along the pipe wall is not taken into consideration.

– To accelerate the mass of gas passing through the plug is disregarded.

– The friction force influencing the material is considered to be proportional to the weight of the plug both in the horizontal and vertical pipe sections.

– The changing of the gas density passing through the pipe is regarded as an isothermal one.

– The friction factor 'k' is regarded constant so that the dependence from the Reynolds number is neglected.

– The air friction is neglected in the pipe section before the moving plug. This indicates that on the surface of the plug ② the pressure is regarded atmospheric; we are calculating with the value

$$p_2 \approx p_0.$$

– The whole cross-section of the pipe is filled by the moving plug.

– The front and the back area of the plug are perpendicular to the movement direction.

1.1. The Force Acting on the Permeable Plug

The transporting gas flow passing through the gaps of the permeable plug goes under a pressure drop. From this pressure drop rises the force moving the plug forward.

In the *Fig. 1* for the elemental pipe tract cut out in an arbitrary place of the plug, following Welschof's idea [1] this equation can be described:

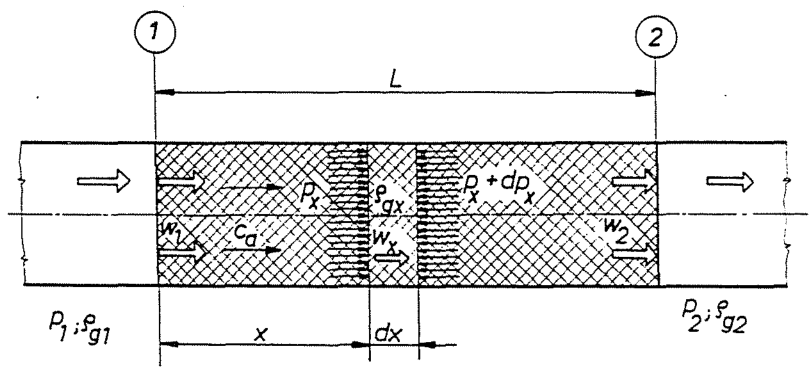

Fig. 1. Plug moving in a straight section of the pipe. Marks of pressures, density, and velocities are on the frontal and back areas of the plug, respectively, in the cross-section marked by 'x' of the plug.

$$p_x - (p_x + dp_x) = k\frac{dx}{d_h}\frac{\varrho_{gx}}{2}w_x^2. \tag{1}$$

In the equation by 'w_x' the relative velocity is marked in the free cross-section of the volume element cut out in the place 'x' of the tube, that is:

$$w_x = c_{gx} - c_a. \tag{2}$$

With the help of the voids volume proportion, that can be determined from the note $\varepsilon = (V - V_{ad})/V$ connection, the relative velocity 'w_x' can be converted into the free cross-section of the pipe as follows:

$$w_{x0} = \varepsilon w_x. \tag{3}$$

In the *Eq* . (1) the hydraulic diameter in Barth's [2] view is given by the following formula:

$$d_h = 4\frac{V - V_{ad}}{A_{ad}} = \frac{2}{3}\frac{\varepsilon}{1-\varepsilon}d. \tag{4}$$

The friction factor 'k' can be determined experimentally and represented in function of the Reynolds number.

In *Fig. 2* the change of resistance coefficients of alumina balls named AGELON is represented in function of the Reynolds number. The Reynolds number will be defined with the formula:

$$Re = \frac{1}{1-\varepsilon}\frac{(c_{g2} - c_a)}{\nu}. \tag{5}$$

In *Fig. 2* is seen that in the case $Re > 1000$ the friction factor 'k' for this material can be regarded with a good approximation as constant.

The change of state of the gas flowing through the plug is supposed to be isothermal. It can be described:

$$\varrho_{gx} = \varrho_{g1}\frac{p_x}{p_1} = \varrho_{g2}\frac{p_x}{p_2} = \varrho_{g0}\frac{p_x}{p_0}. \tag{6}$$

The mass flow of the transporting gas in an arbitrary place 'x' of the plug is:

$$\dot{m}_g = \varepsilon A w_x \varrho_{gx} = \varepsilon A(c_{gx} - c_a)\varrho_{gx} \tag{7}$$

that is the same size within the plug on the front and the back area that is:

$$\dot{m}_g = \varepsilon A(c_{g1} - c_a)\varrho_{g1} = \varepsilon A(c_{g2} - c_a)\varrho_{g2}. \tag{8}$$

Regarding the mentioned formulas the *Eq* . (1) can be written so:

$$-p_x dp_x = \frac{k}{d_h}\frac{\varrho_{g2}p_2}{2}(c_{g2} - c_a)^2 dx. \tag{9}$$

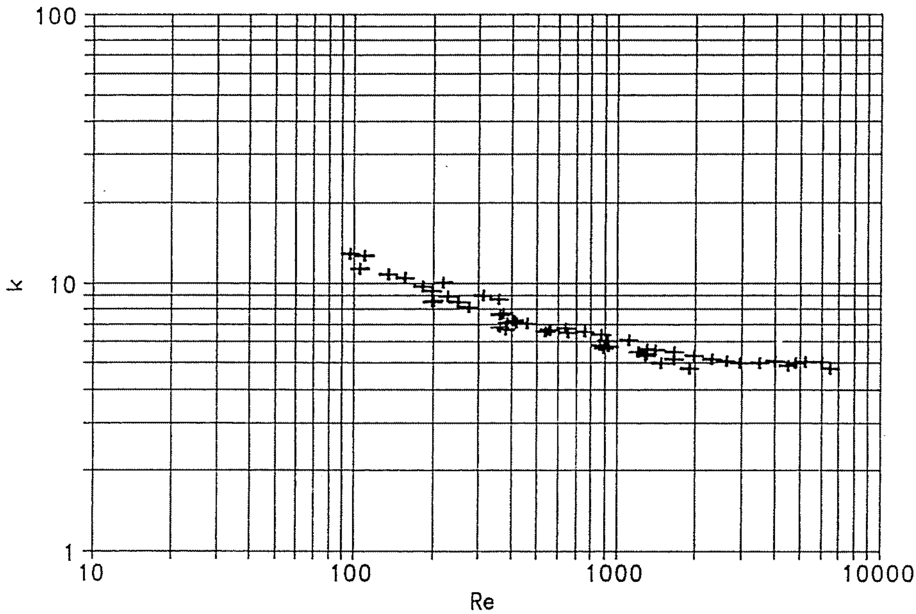

Fig. 2. The change of the factor 'k' in function of the Reynolds number in case of a plug with a different length L', ($L = 0.4 - 1,2$ m) and consisting of alumina balls with a diameter ($d = 0.8 - 5$ mm). Experimental data.

With the boundary conditions in place $x = 0$; $p_x = p_1$ and with $x = L$ $p_x = p_2$ the solution of the *Eq.* (9) is the following:

$$p_1^2 - p_2^2 = \frac{k}{d_h} \varrho_{g2} p_2 (c_{g2} - c_a)^2 L. \tag{10}$$

The pressure drop of the gas flow passing through the plug, $\Delta p = p_1 - p_2$, considering that $p_1^2 - p_2^2 = \Delta p(\Delta p + 2p_2)$ can be written as follows:

$$\Delta p = -p_2 + \left[p_2^2 + \frac{k}{d_h} \varrho_{g2} p_2 (c_{g2} - c_a)^2 L \right]^{1/2}. \tag{11}$$

The form of the equation without dimension is:

$$\Delta p^* = -p_2{}^* + \left[p_2{}^{*2} + \frac{k}{d_h{}^*} \pi_1 \varrho_{g2}{}^* p_2{}^* (c_{g2}{}^* - c_a{}^*)^2 \right]^{1/2}. \tag{12}$$

The resultant force 'F_e' influencing the plug moving in the pipe is composed of the force '$A\Delta p$' and of the friction force 'F_s', as well as, in case of a vertical pipe of the plug weight '$A\ L\varrho_h g$'. In case of vertical pipe:

$$AL\varrho_h \frac{dc_a}{dt} = F_e = A\Delta p - AL\varrho_h g(1+\mu). \tag{13}$$

In case of a plug moving in a horizontal pipe:

$$AL\varrho_h \frac{dc_a}{dt} = F_e = A\Delta p - AL\varrho_h g\mu. \tag{14}$$

The form without dimensions of the equation after the separation of the variables, for a vertical pipe:

$$dt^* = \frac{2}{\pi_2 \Delta p^* - (1+\mu)} dc_a{}^*. \tag{15}$$

The *Eq.* (15) can be integrated by the substitution of the *Eq.* (12). Considering the condition $t^* = 0$; $c_a{}^* = 0$ for the time without dimension we have:

$$t^* = -K_0[K_1 \ln(K_2 T_2) + \ln(A_1)]. \tag{16}$$

In the *Eq.* (16) the independent variable is contained in the expressions 'A_1' and 'T_2'.

The acceleration of the plug can be determined from the relation:

$$a_a = 2g\frac{dc_a{}^*}{dt^*} = 2ga_a{}^*. \tag{17}$$

After the differentiation of the *Eq.* (16) we have

$$\frac{dt^*}{dc_a{}^*} = -K_0\left[\frac{K_1 T_2{}'}{T_2} + \frac{A_1{}'}{A_1}\right]. \tag{18}$$

The acceleration of the plug in a form without dimensions is:

$$a_a{}^* = -\frac{A_1 T_2}{K_0(A_1 K_1 T_2{}' + A_1{}' T_2)}. \tag{19}$$

The terminal velocity '$c_{a\infty}$' can be determined by the butting transition $t^* \to \infty$ from the *Eq.* (16) or regarding the condition $\frac{dc_a}{dt} = 0$ from the *Eq.* (13).

Replacing the previous condition into the equation for the terminal velocity '$c_{a\infty}$' of the plug moving in the vertical pipe we have:

$$\pi_2 \Delta p^* = 1 + \mu. \tag{20}$$

Replacing the velocity without dimension $c_a{}^* = c^*_{a\infty}$ into the *Eq.* (12) after rearranging the value of the terminal velocity can be determined from the relation:

$$c^*_{a\infty} = c_{g2}{}^* - \left[\frac{d_h{}^*(1+\mu)}{\pi_1\pi_2 k p_2{}^* \varrho_{g2}{}^*}\left(2p_2{}^* + \frac{1+\mu}{\pi_2}\right)\right]^{1/2}. \tag{21}$$

The path made by the plug can be written as follows:

$$s^* = \int c_a{}^* dt^*. \tag{22}$$

Using the *Eq.* (18) we have that:

$$s^* = -\int K_0 \left[\frac{K_1 T_2{}'}{T_2} + \frac{A_1{}'}{A_1}\right] c_a{}^* dc_a{}^*. \tag{23}$$

The expression standing on the right side of the equation cannot be integrated so we have used an approximate method for the solution.

2. The Equation of Motion of the Plug in a Bend of 90° Connecting Straight Pipe Sections

During describing the equation of motion the case will be treated when the length of the plug is more than the one of the bend of an angle 90° that is: $L > R\pi/2$.

Depending on the case of the plug three cases can be distinguished:

a) The moving plug charges the bend. This lasts as long as the frontal area of the plug marked ② reaches the central angle $\alpha = 90°$.

b) Motion in the bend. This lasts as long as the horizontal section before the bend discharges. Then the third phase, the discharge of the bend begins.

c) The discharge of the bend. This finishes when the back of the plug ① reaches the end of the bend characterized by the angle $\alpha = 90°$.

2.1 The Moving Plug Charges the Bend

In the scheme demonstrated in *Fig. 3* the frontal area of the plug marked ② arrived into the bend to the angle position 'α'.

To determine the friction force of the plug moving in the bend regarding the forces influencing the elemental mass in an arbitrary position 'φ'

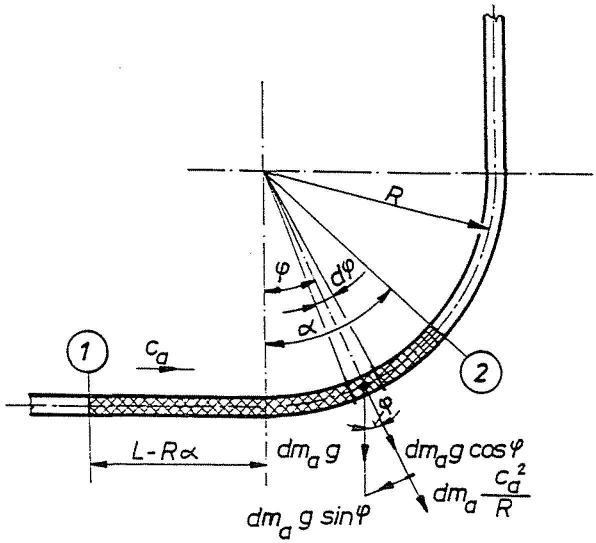

Fig. 3. The charge of the bend of vertical plane. The determination of forces acting on the elemental cut away in the angle position 'φ' of the partially charged bend.

for the elemental friction force the following equation can be written:

$$dF_{s1} = \mu \left[dm_a \, g \cos\varphi + dm_a \frac{{c_a}^2}{R} \right] \cdot \tag{24}$$

The elemental mass of '$d\varphi$' angle:

$$dm_a = Rd\varphi A\varrho_h \cdot \tag{25}$$

So the *Eq.* (24) becomes:

$$dF_{s1} = \mu R d\varphi A \varrho_h \left[g \cos\varphi + \frac{{c_a}^2}{R} \right] \cdot \tag{26}$$

After the integration of the equation we can get:

$$F_{s1} = \mu R A \varrho_h g \sin\alpha + \mu A \varrho_h {c_a}^2 \alpha. \tag{27}$$

The friction force acting on the plug moving in the horizontal straight section is:

$$F_{s2} = \mu (L - R\alpha) A \varrho_h g. \tag{28}$$

The sum of friction forces:

$$F_s = F_{s1} + F_{s2} = \mu A \varrho_h \left[Rg \sin\alpha + {c_a}^2 \alpha + (L - R\alpha) g \right]. \qquad (29)$$

The force component along the path is:

$$dF_p = dm_a g \sin\varphi = RA\varrho_h g \sin\varphi d\varphi. \qquad (30)$$

After the integration of the *Eq.* (30) we have:

$$F_p = RA\varrho_h g [1 - \cos\alpha]. \qquad (31)$$

The equation describing the motion of the plug can be written in the form:

$$m_a \frac{dc_a}{dt} = F_e = A\Delta p - (F_s + F_p). \qquad (32)$$

The equation can be transformed. It can be written that:

$$c_a = \frac{d(R\alpha)}{dt} = R\frac{d\alpha}{dt}. \qquad (33)$$

Regarding that the value of $\frac{dc_a}{dt}$ is as follows:

$$\frac{d}{dt}\left(R\frac{d\alpha}{dt}\right) = R\frac{d^2\alpha}{dt^2}. \qquad (34)$$

With the equations (11), (29), and (31) we have the equation describing the charging phase of the bend as follows:

$$\begin{aligned} \frac{d^2\alpha}{dt^2} = C_1 + \frac{1}{L\varrho_h R}\left[\frac{k}{d_h}\varrho_{g2}p_2\left(c_{g2} - R\frac{d\alpha}{dt}\right)^2 L + {p_2}^2\right]^{1/2} - \\ - \frac{\mu g}{L}\sin\alpha + \frac{g}{L}\cos\alpha - \alpha\left[\frac{\mu R}{L}\left(\frac{d\alpha}{dt}\right)^2 - \frac{\mu g}{L}\right]. \end{aligned} \qquad (35)$$

After making into dimensionless form the *Eq.* (35) can be described in the form:

$$\begin{aligned} \frac{d^2\alpha}{dt^{*2}} = & C_{11} + \left[\frac{k}{{d_h}^*}{\varrho_{g2}}^*{p_2}^*\left({c_{g2}}^* - R^*\frac{d\alpha}{dt^*}\right)^2 \pi_1 + {p_2}^{*2}\right]^{1/2} \pi_0\pi_2 - \\ & - \frac{\mu}{2}\sin\alpha + \frac{1}{2}\cos\alpha - \alpha\left[\mu R^*\left(\frac{d\alpha}{dt^*}\right)^2 - \frac{\mu}{2}\right]. \end{aligned} \qquad (36)$$

The ordinary differential equation of the second order is equivalent to the two differential equations of the first order as follows:

$$\begin{aligned} \alpha' &= \frac{d\alpha}{dt^*} = z^*, \\ z^{*\prime} &= \frac{dz^*}{dt^*} = C_{11} + \left[\frac{k}{d_h{}^*}\varrho_{g2}{}^* p_2{}^* (c_{g2}{}^* - R^* z^*)^2 \pi_1 + p_2^{*2}\right]^{1/2} \pi_0 \pi_2 - \\ &\quad - \frac{\mu}{2}\sin\alpha + \frac{1}{2}\cos\alpha - \alpha\left[\mu R^* z^{*2} - \frac{\mu}{2}\right]. \end{aligned} \tag{37}$$

The equations (37) can be solved by the Runge–Kutta method.

The initial conditions are in the junction point in the straight pipe section connecting before the bend (regarding the dimensionless velocity '$[c_a{}^*]_a$') the following:

$$\begin{aligned} t^* &= t_a{}^*, & \alpha(t_a{}^*) &= 0, \\ t^* &= t_a{}^*, & z^*(t_a{}^*) &= \frac{[c_a{}^*]_a}{R^*}. \end{aligned} \tag{38}$$

2.2. Motion in the Bend

This period lasts as far as the end ① of the plug moving in the straight section before the bend reaches the beginning of the bend.

From the *Fig. 4* can be seen that the frontal area of the plug made a path 's' in the vertical section.

In the bend the value of the friction force can be determined from the *Eq.* (27) by replacement $\alpha = \pi/2$; $\sin(\pi/2) = 1$. Its value is:

$$F_{s1} = \mu R A \varrho_h g + \mu A \varrho_h \frac{\pi}{2} c_a{}^2. \tag{39}$$

The value of the force moving in the horizontal and vertical sections calculating from the friction 'Coulomb' we have:

$$F_{s2} = \left(L - R\frac{\pi}{2}\right) A \varrho_h g \mu. \tag{40}$$

The force component along the path acting on the plug moving in the bend can be calculated from the *Eq.* (31) by replacement $\alpha = \pi/2$. The equation is:

$$F_{p1} = R A \varrho_h g. \tag{41}$$

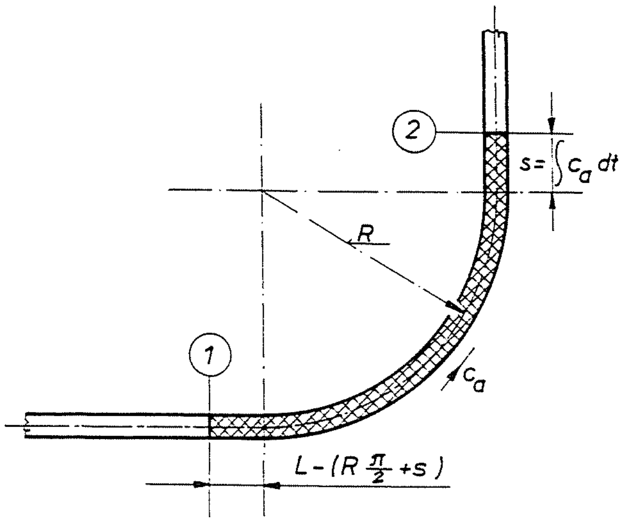

Fig. 4. Motion in the bend of vertical plane. This phase lasts as far as the back of the plug arrives to the beginning of the bend.

The component along the path in the vertical section:

$$F_{p2} = A\varrho_h g s = A\varrho_h g \int c_a dt. \tag{42}$$

The equation describing the motion of the plug is as follows:

$$m_a \frac{dc_a}{dt} = F_e = A\Delta p - (F_s + F_p) = A\Delta p - (F_{s1} + F_{s2} + F_{p1} + F_{p2}). \tag{43}$$

Regarding the equations (11), (39), (40), (41) and (42), the (43) one is after transformation and rearranging as follows:

$$\begin{aligned} \frac{dc_a}{dt} = &-\frac{p_2}{\varrho_h L} - \mu g\left(1+\frac{R}{L}\right) - g\frac{R}{L}\left(1-\frac{\mu\pi}{2}\right) + \\ &+ \frac{1}{\varrho_h L}\left[\frac{k}{d_h}\varrho_{g2}p_2(c_{g2}-c_a)^2 L + {p_2}^2\right]^{1/2} - \\ &- \frac{\mu\pi}{2L}{c_a}^2 - \frac{g}{L}\int c_a dt. \end{aligned} \tag{44}$$

Derivating both sides of the *Eq.* (44) we can get the following equation:

$$\begin{aligned}\frac{d^2c_a}{dt^2} = &-\frac{k}{\varrho_h d_h}\frac{\varrho_{g2}p_2(c_{g2}-c_a)}{\left[\frac{k}{d_h}\varrho_{g2}p_2(c_{g2}-c_a)^2L+p_2{}^2\right]^{1/2}}\frac{dc_a}{dt}- \\ &-\frac{\mu\pi}{L}c_a\frac{dc_a}{dt}-\frac{g}{L}c_a.\end{aligned} \tag{45}$$

After making into a dimensionless form of the *Eq.* (45) we have:

$$\begin{aligned}\frac{d^2c_a{}^*}{dt^{*2}} = &-\frac{k}{d_h{}^*}\frac{\varrho_{g2}{}^*p_2{}^*(c_{g2}{}^*-c_a{}^*)\frac{dc_a{}^*}{dt^*}\frac{\varrho_{g0}}{\varrho_h}}{\left[\frac{k}{d_h{}^*}\varrho_{g2}{}^*p_2{}^*(c_{g2}{}^*-c_a{}^*)^2\pi_1+p_2{}^{*2}\right]^{1/2}}- \\ &-\mu\pi c_a{}^*\frac{dc_a{}^*}{dt^*}-\frac{c_a{}^*}{2}.\end{aligned} \tag{46}$$

The *Eq.* (46) is equivalent to the following differential equation of first order:

$$\frac{dc_a{}^*}{dt^*} = a_a{}^*,$$

$$\begin{aligned}a_a{}^{*\prime} = \frac{da_a{}^*}{dt^*} = &-\frac{\frac{k}{d_h{}^*}\varrho_{g2}{}^*p_2{}^*(c_{g2}{}^*-c_a{}^*)a_a{}^*\frac{\varrho_{g0}}{\varrho_h}}{\left[\frac{k}{d_h{}^*}\varrho_{g2}{}^*p_2{}^*(c_{g2}{}^*-c_a{}^*)^2\pi_1+p_2{}^{*2}\right]^{1/2}}- \\ &-\mu\pi c_a{}^*a_a{}^*-\frac{c_a{}^*}{2}.\end{aligned} \tag{47}$$

The equations (47) can be solved by the Runge–Kutta method. The initial conditions can be calculated from the solution of the *Eq.* (37) described during the charging of the bend mentioned in the part 2.1 from the values of the angular velocity and angular acceleration at the angle position $\alpha = \pi/2$. If they will be marked $[z^*]_\alpha$ respectively $[z^{*\prime}]_\alpha$ then:

$$\begin{aligned} t^* &= t_b{}^*, \qquad & c_a{}^*(t_b{}^*) &= R^*[z^*]_\alpha, \\ t^* &= t_b{}^*, \qquad & a_a{}^*(t_b{}^*) &= R^*[z^{*\prime}]_\alpha. \end{aligned} \tag{48}$$

2.3 The Discharge of the Bend

This period lasts as far as the cross-section marked ① in *Fig. 5* of the moving plug reaches the position characterized by the $\alpha = \pi/2$.

The friction force acting on the cut out elemental part is equivalent to the value calculable from the *Eq.* (26). Integrating the equation between

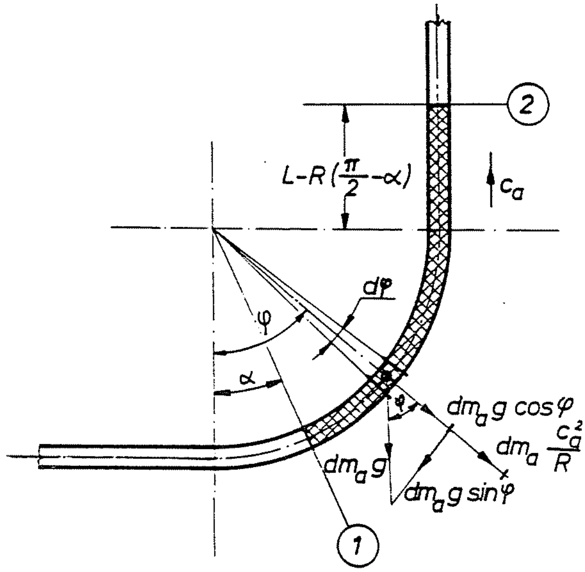

Fig. 5. The discharge of the bend of vertical plane. The determination of forces acting on the elemental cut away in the angle position 'φ' of the partially charged bend.

'α' and '$\pi/2$' we will have for the value of the friction force acting on the mass part moving in the bend:

$$F_{s1} = \mu R A \varrho_h g[1 - \sin\alpha] + \mu A \varrho_h {c_a}^2 \left[\frac{\pi}{2} - \alpha\right]. \tag{49}$$

The friction force acting on the plug moving in the vertical section is:

$$F_{s2} = \left[L - R\left(\frac{\pi}{2} - \alpha\right)\right] A \varrho_h g \mu. \tag{50}$$

The force component along the path in the bend can be calculated from the *Eq.* (30). That is:

$$F_{p1} = R A \varrho_h g \cos\alpha. \tag{51}$$

The force component along the path in the vertical section is:

$$F_{p2} = \left[L - R\left(\frac{\pi}{2} - \alpha\right)\right] A \varrho_h g. \tag{52}$$

The equation describing the motion of the plug is:

$$m_a \frac{dc_a}{dt} = F_e = A\Delta p - (F_s + F_p) = A\Delta p - (F_{s1} + F_{s2} + F_{p1} + F_{p2}). \tag{53}$$

Regarding the *Eqs.* (11), (49), (50), (51) and (52) is after transformation and rearranging we obtain:

$$\frac{d^2\alpha}{dt^2} = C_2 + \frac{1}{L\varrho_h R}\left[\frac{k}{d_h}\varrho_{g2}p_2\left(c_{g2} - R\frac{d\alpha}{dt}\right)^2 L + {p_2}^2\right]^{1/2} + \\ + \frac{\mu g}{L}\sin\alpha - \frac{g}{L}\cos\alpha + \alpha\left[\frac{\mu R}{L}\left(\frac{d\alpha}{dt}\right)^2 - \frac{g}{L}(1+\mu)\right] - \\ - \mu\frac{R\pi}{2L}\left(\frac{d\alpha}{dt}\right)^2. \tag{54}$$

The *Eq.* (54) appears after making into dimensionless form as follows:

$$\frac{d^2\alpha}{dt^{*2}} = C_{21} + \left[\frac{k}{{d_h}^*}{\varrho_{g2}}^*{p_2}^*\left({c_{g2}}^* - R^*\frac{d\alpha}{dt^*}\right)^2\pi_1 + {p_2}^{*2}\right]^{1/2}\pi_0\pi_2 + \\ + \frac{\mu}{2}\sin\alpha - \frac{1}{2}\cos\alpha + \alpha\left[\mu R^*\left(\frac{d\alpha}{dt^*}\right)^2 - \frac{1}{2}(1+\mu)\right] - \\ - \frac{\mu R^*\pi}{2}\left(\frac{d\alpha}{dt^*}\right)^2. \tag{55}$$

The differential equation of the second order (55) is equivalent to the following two differential equations of the first order:

$$\alpha' = \frac{d\alpha}{dt^*} = z^*,$$

$$z^{*\prime} = \frac{dz^*}{dt^*} = C_{21} + \left[\frac{k}{{d_h}^*}{\varrho_{g2}}^*{p_2}^*({c_{g2}}^* - R^*z^*)^2\pi_1 + {p_2}^{*2}\right]^{1/2}\pi_0\pi_2 + \\ + \frac{\mu}{2}\sin\alpha - \frac{1}{2}\cos\alpha + \alpha\left[\mu R^* z^{*2} - \frac{1}{2}(1+\mu)\right] - \\ - \frac{\mu R^*\pi}{2}z^{*2}. \tag{56}$$

The *Eq.* (56) can be solved by the Runge–Kutta method.

The initial conditions derive from the solution of the equation system (47) describing the motion mentioned under the part 2.2 when the cross-section ① of the plug reaches the beginning of the bend that is $s = L - R\pi/2$.

Let us sign the values $[{c_a}^*]_s$ then:

$$\begin{aligned} t^* &= {t_c}^*, & \alpha({t_c}^*) &= 0, \\ t^* &= {t_c}^*, & z^*({t_c}^*) &= \frac{[{c_a}^*]_s}{R^*}. \end{aligned} \tag{57}$$

3. Diagrams Showing the Characteristics of the Moving Plug

Motion characteristics got from solutions of deduced equations are shown in *Figs. 6–12*. As example we have a plug with a length $L = 1$ m and a bend with a radius of $R = 0.5$ m. Before the bend the plug starts from a horizontal distance $s = 10$ m and with a starting velocity $c_a = 0$. After the bend the motion characteristics of the moving plug in the vertical section are shown by the figures up to the path $s = 8$ m.

The plug consists of the already mentioned alumina balls 'AGELON'. The following data are:

$$k = 5, \quad \varepsilon = 0.514, \quad \varrho_h = 1128 \text{ kg/m}^3, \quad \varrho_{g2} = 1.188 \text{ kg/m}^3,$$
$$\mu = 0.48, \quad p_0 = 10^5 \text{ Pa}, \quad d_h = 2,82 * 10^{-3} \text{ m}, \quad c_{g2} = 5 \text{ m/s}.$$

In *Fig. 6* the change of the resultant force 'F_e' is shown in function of time. As soon as the frontal area of the plug arrives to the beginning of the bend it moves with a constant velocity with a good approximation, so that the resultant force is $F_e \approx 0$.

As soon as begins the charge of the bend the resultant force becomes negative on the influence of the friction force, the plug begins to move decelerately. This results the growing of the pressure drop on the plug so that the force acting on the plug grows. The resultant force grows to the maximum in function of time, later it begins to diminish.

The friction force, acting on the plug left the bend, moving in the vertical section, is constant, so the velocity of the plug in the vertical section will have a value, in order to the resultant force – like the horizontal section – be $F_e \approx 0$. This takes place with the chosen for example data after $t \approx 5.5$ s.

In *Fig. 7* the velocity 'c_a' of the plug is seen in function of the time. In the basis of all that told so far the value of the velocity 'c_a' is before the bend and after the $t \approx 5.5$ s is constant with a good approximation while the velocity decreases rapidly in the bend. In *Fig. 8* the connection velocity-time $c_a = f(t)$ is shown magnified in the bend.

The *Fig. 9* shows the acceleration in function of time. From the figure may be seen that in the straight section before the bend after $t \approx 1.5$ s the value of the acceleration is close to zero. The value of the acceleration is also zero after the already mentioned $t \approx 5.5$ s in the vertical section.

In *Fig. 10* the diagram acceleration-time can be seen magnified in the bend. In *Figs. 11* and *12* the path made is represented in function of time.

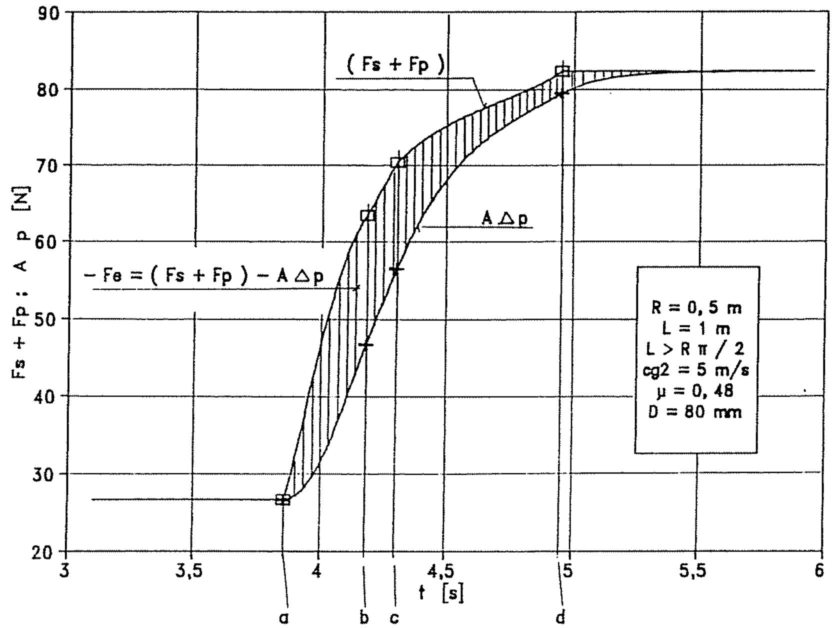

Fig. 6. Forces acting on the plug in the pipeline and in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend. The segments lined vertically are proportional to the braking force influencing on the plug.

Marks

$A = \frac{D^2\pi}{4}$ [m²] pipe cross-section

$A_0 = \alpha_3^{1/2} c_{g2}^* + (\alpha_3 c_{g2}^{*2} + 1)^{1/2}$ [–] constant

$A_1 = \alpha_3^{1/2}(c_{g2}^* - c_a^*) + [\alpha_3(c_{g2}^* - c_a^*)^2 + 1]^{1/2}$ [–] reduced variable

$A_1' = -(\alpha_3)^{1/2} - \frac{\alpha_3(c_{g2}^* - c_a^*)}{[\alpha_3(c_{g2}^* - c_a^*)^2+1]^{1/2}}$ [–] reduced variable

a_a [m/s²] acceleration of the plug

$a_a^* = \frac{a_a}{2g}$ [–] dimensionless acceleration of the plug

$C_1 = -\frac{p_2}{\varrho_h RL} - \frac{\mu g}{R} - \frac{g}{L}$ [1/s²] constant

$C_{11} = -\pi_0\pi_2 p_2^* - \frac{1}{2}\left(\frac{\mu}{R^*} + 1\right)$ [–] dimensionless constant

$C_2 = -\frac{p_2}{\varrho_h RL} + \frac{g}{L}\left[\frac{\pi}{2}(1+\mu) - \mu\right] - \frac{g}{R}(1+\mu)$ [1/s²] constant

$C_{21} = -\pi_0\pi_2 p_2^* - \pi_0(1+\mu) + \frac{1}{2}\left[\left(\pi\frac{1+\mu}{2} - \mu\right)\right]$ [–] dimensionless constant

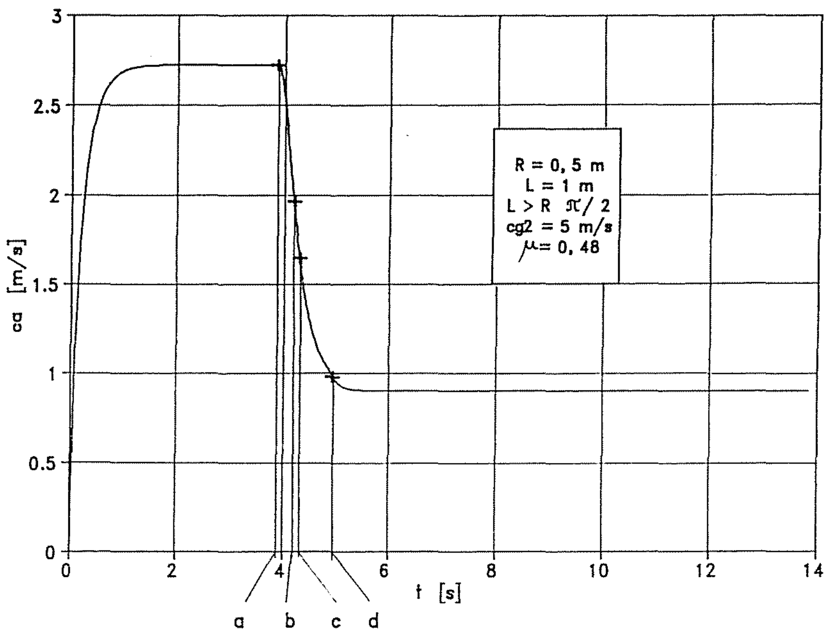

Fig. 7. The velocity of the plug in the pipeline and in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend.

c_a	[m/s]	material velocity
$c_a{}^* = \frac{c_a}{c_0}$	[–]	dimensionless material velocity
$c_0 = (2gL)^{1/2}$	[m/s]	velocity used for making into dimensionless form
c_{g1}	[m/s]	gas velocity in the cross-section ① of the porous plug
$c_{g1}{}^* = \frac{c_{g1}}{c_0}$	[–]	dimensionless gas velocity in the cross-section ① of the porous plug
c_{g2}	[m/s]	gas velocity in the cross-section ② of the porous plug
$c_{g2}{}^* = \frac{c_{g2}}{c_0}$	[–]	dimensionless gas velocity in the cross-section ② of the porous plug
$c_{a\infty}$	[m/s]	terminal velocity of the plug
$c_{a\infty}^* = \frac{c_{a\infty}}{c_0}$	[–]	dimensionless terminal velocity of the plug
D	[m]	diameter of the pipe

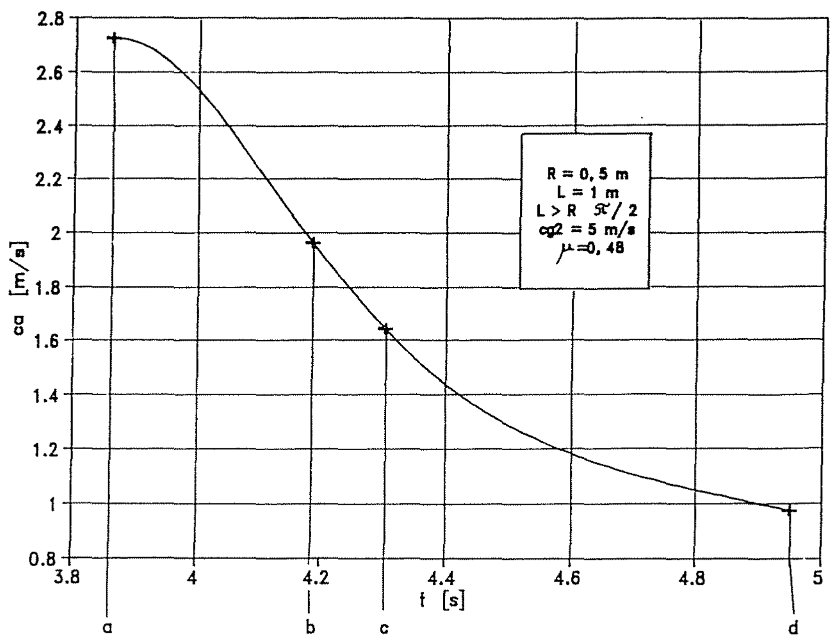

Fig. 8. The velocity of the plug in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend.

d	[m]	diameter of the particle
d_h	[m]	hydraulic diameter
$d_h{}^* = \frac{d_h}{L}$	[–]	dimensionless hydraulic diameter
F_e	[N]	resultant force
F_p	[N]	force component along the path
F_s; F_{s1}; F_{s2}	[N]	friction force
g	[m/s²]	gravity acceleration
$K_0 = \frac{2}{\alpha_2(\alpha_3)^{1/2}}$	[–]	constant
$K_1 = \frac{\alpha_1}{\alpha_0}$	[–]	constant
$K_2 = \frac{T_1}{A_0{}^{1/K_1}}$	[–]	constant
k	[–]	friction factor, experimental data
L	[m]	length of the plug
m_a	[kg]	material mass of the plug
$\dot{m}_g$	[kg/s]	gas mass flow

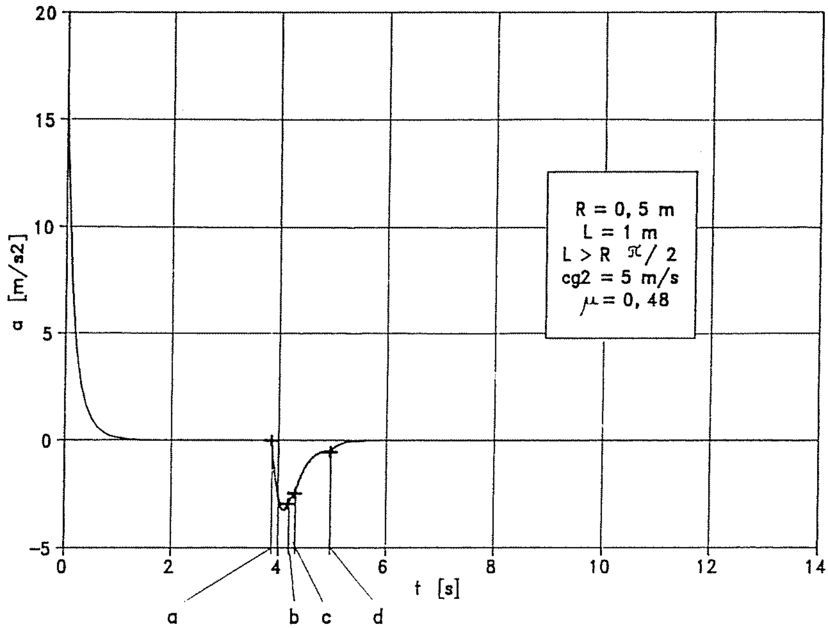

Fig. 9. The acceleration of the plug in the pipeline and in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend.

p	[Pa]	pressure
$p^* = \frac{p}{p_0}$	[–]	dimensionless pressure
p_0	[Pa]	atmospheric pressure
p_1	[Pa]	pressure on the surface ① of the plug
p_2	[Pa]	pressure on the surface ② of the plug
p_x	[Pa]	pressure on the place marked 'x' of the plug
Δp	[Pa]	pressure difference
$\Delta p^* = \frac{\Delta p}{p_0}$	[–]	dimensionless pressure difference
R	[m]	radius of the bend
$R^* = \frac{R}{L}$	[–]	dimensionless radius
Re	[–]	Reynolds number
s	[m]	path
$s^* = \frac{s}{L}$	[–]	dimensionless path
t	[s]	time

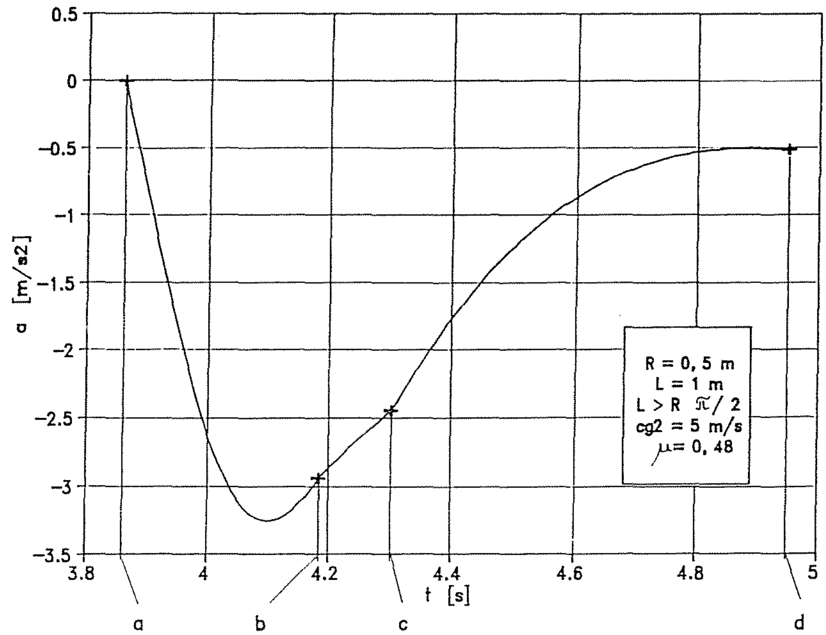

Fig. 10. The acceleration of the plug in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend.

$t^* = \frac{t}{t_0}$	[–]	dimensionless time
$t_0 = \frac{L}{c_0}$	[s]	time used for making into dimensionless form
$T_1 = \frac{\beta(A_0+1)-\alpha_0(A_0-1)}{\beta(A_0+1)+\alpha_0(A_0-1)}$	[–]	constant
$T_2 = \frac{\beta(A_1+1)+\alpha_0(A_1-1)}{\beta(A_1+1)-\alpha_0(A_1-1)}$	[–]	reduced variable
$T_2' = \frac{4\alpha_0\beta A_1'}{[\beta(A_1+1)-\alpha_0(A_1-1)]^2}$	[–]	reduced variable
V	$[\mathrm{m}^3]$	volume
w_x	[m/s]	relative velocity in the place marked by 'x' of the plug

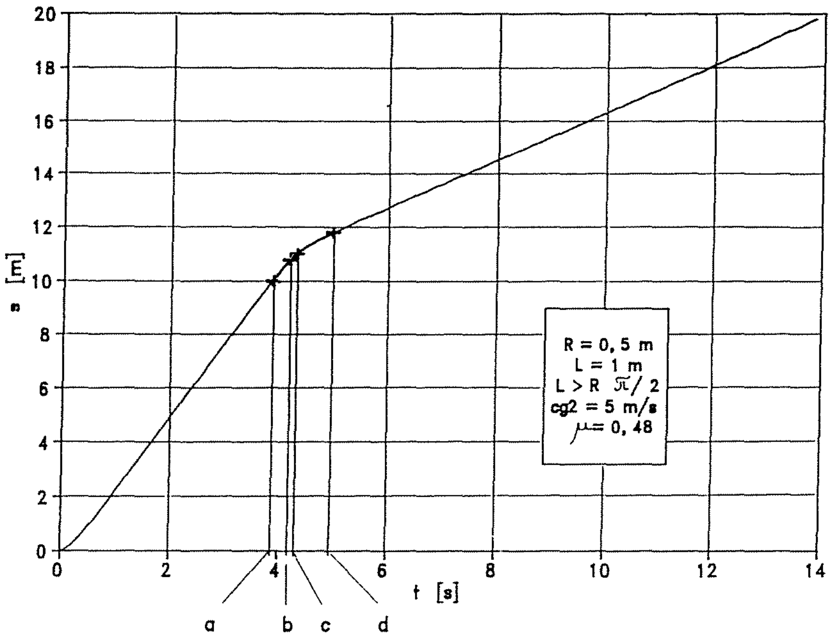

Fig. 11. The path made by the plug in the pipeline and in the bend conducting from the horizontal into the vertical plane in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the beginning of the bend. c)/ The back of the plug is at the end of the bend. d)/ The back of the plug is at the end of the bend.

$w_{x0} = w_x$ [m/s] relative velocity in the place marked by 'x' of the plug converting to the free cross-section of the pipe

x [m] coordinate of length within the plug

z^* [–] dimensionless variable

α [grad] angle

$\alpha_0 = ({\alpha_1}^2 - {\alpha_2}^2)^{1/2}$ [–] dimensionless constant

$\alpha_1 = \alpha_2 + (1 + \mu)$ [–] dimensionless constant

$\alpha_2 = p_2^* \pi_2$ [–] dimensionless constant

$\alpha_3 = \frac{k\pi_1}{d_h^*}$ [–] dimensionless constant

$\beta = \alpha_2 - \alpha_1$ [–] dimensionless constant

ε [–] voidage fraction

φ [grad] angle

μ [–] friction factor

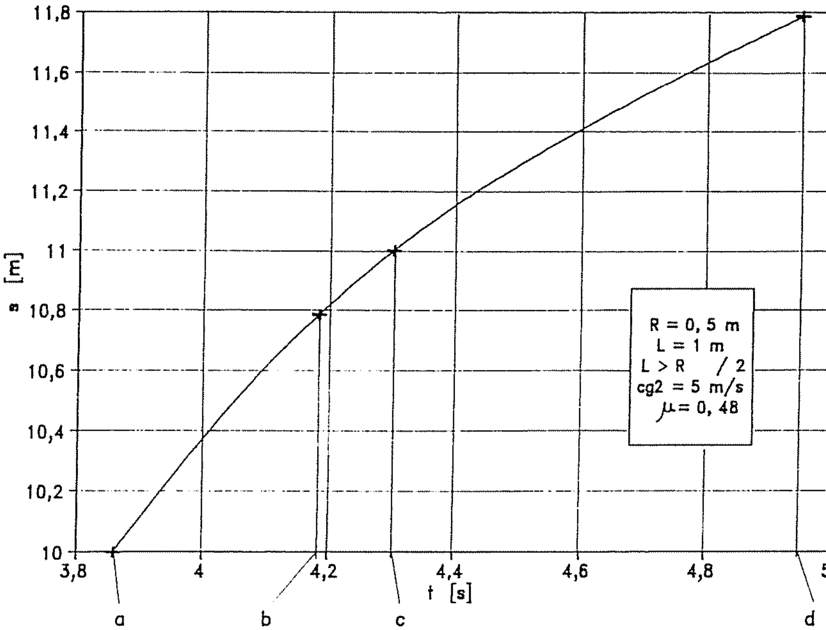

Fig. 12. The path made from the plug in the bend in function of time. a)/ The frontal area of the plug is at the beginning of the bend. b)/ The frontal area of the plug is at the end of the bend. c)/ The back of the plug is at the beginning of the bend. d)/ The back of the plug is at the end of the bend.

ν	$[\mathrm{m}^2/\mathrm{s}]$	kinematic viscosity
ϱ	$[\mathrm{kg}/\mathrm{m}^3]$	density
$\varrho_g{}^* = \frac{\varrho_g}{g_0}$	$[-]$	dimensionless gas density
π	$[-]$	Ludolphian number
$\pi_0 = \frac{1}{2R^*}$	$[-]$	constant
$\pi_1 = \frac{\varrho_{g0} c_0^2}{p_0}$	$[-]$	constant
$\pi_2 = \frac{p_0}{\varrho_h g L}$	$[-]$	constant

Indexes:

a material
ad material particle
e resultant
g gas
h bulk, hydraulic
o atmospheric state
x refers to the cross-section marked by 'x' of the plug
α angle
1 refers to the cross-section ① of the plug
2 refers to the cross-section ② of the plug

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