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A UNIFIED MATHEMATICAL FRAMEWORK OF RELIABILITY CONTROL[1]

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Abstract

The paper details the different types of system variables. A system vector is composed which is suitable to describe the whole system and, at the same time, satisfies a Markov type condition. Accelerated tests, pattern recognition methods and maintenance optimization are shown to make use of this approach.

Keywords: reliability, model, pattern recognition.

1. Introduction

The present paper sets out to give a general mathematical framework for the theoretical description, statistical inference and optimal control of operation, deterioration and maintenance processes.

The approach applied in the paper is a synthesis of earlier works of the authors in the field of system modelling ([1], [2], [3], [4], [5], [6]).

The central idea of the model is a choice of the system state vector in such a way that from its earlier observed values it 'remembers' only the one observed last, and the rules governing its behaviour are time independent.

With this way of description the operation and/or reliability of a wide range of different products, complex systems can be characterized in spite of many, in itself also very complicated subproblems.

2. Dynamic Model of the Operation, Deterioration and Maintenance Process

Examining the reliability of some industrial products or complex systems we try to describe possible system states as completely as possible.

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Independently of the given object it is reasonable to distinguish the following types of states:

- *quality state* characterizing the ability to perform the proper job;
- *operational state* describing the realization of the proper job and subject to quick changes in any instant;
- *inner state* consisting of slowly changing inner parameters connected to the components of the system.

During the operational phase the objective of influencing, controlling the system is to maintain a prescribed value of some reliability characteristics, or to reach an optimal state, e.g. concerning the accessibility, availability of the system, or the minimum of the life cycle cost, etc.

In order to give an exact description of the system structure and operation we introduce the following variables:

u– the vector of external effects influencing the system. These factors constitute the policy of operation. They usually mean either stress or control. These two types of external effects are, however, not always clearly separable from each other.

e– the vector representing the stress type external effects.

d– the vector representing the control.

y– the vector of parameters characterizing the technical state of the system (quality, aging). These parameters are chosen and observed by us. It depends on basic properties of constituting elements, on the inner structure of the system, on the operational policy (stress and control).

x– the vector giving a satisfactory representation of inner physical state characteristics from the point of view of the given model. We choose it in such a way that it provides the necessary information for the description of the inner physical, chemical, etc. processes.

v– the vector representing all output signals characterizing the functional operation of the system. Its undesirable values represent erroneous operation, while the trend of its trajectory may point to functional disturbances.

b– the vector containing the full set of parameters of the system (inner parameters, basic properties of components). It is not directly dependent on **x**, only on the long-term effect of operation.

w– the vector representing the operational state of the system.

We introduce the notation

$$z = (\mathbf{w}^T, \mathbf{b}^T, \mathbf{u}^T)^T .$$

We suppose that the operation of the system is described for any $t > 0$ by the relations

$$z(t) = \Psi\left(z(t_0); \mathbf{u}(s), 0 \leq t_0 < s < t\right) , \tag{1}$$

$$\Xi(z(t)) = 0 \ , \tag{2}$$

$$\Lambda(z(t)) \leq 0 \ , \tag{3}$$

where Ξ, Λ are appropriate functions, and relations (2)–(3) describe the hypersurface of feasible system states at time t, while Ψ is an appropriate operator, and relation (1) characterizes the time behaviour of the system.

It follows from (1)–(3) that the response to any feasible control policy depends only on the initial state and the control applied but not on the time when we begin to apply the given control. This means that the input-output relation is independent both of the time and the history preceding the given system state.

We remark that if

$$\Psi\left(z : (t_0); \mathbf{u}(s), 0 \leq t_0 < s \leq t\right) =$$

$$= z(t_0) + \psi(z(t_0))(t - t_0) + o(\|t - t_0\|) \ ,$$

where $\psi(\cdot)$ is an appropriate function, then (1) can be replaced by the differential equation

$$\dot{z} = \psi(z) \ .$$

The parameter vector $\mathbf{y}$ intended to qualify the given device is formed on the basis of technical and economic considerations. The fact that the device is faultless, i.e. the system is able to perform all of its prescribed functions, will be defined in such a way that the vector $\mathbf{y}$ stays within a given domain.

From the point of view of qualification of proper operation we have to distinguish between *failure* ($\mathbf{y}$ leaves its tolerance domain) and *erroneous operation* ($\mathbf{w}$ leaves its tolerance domain but $\mathbf{y}$ does not). Diagnosis is actually performed by means of a statistical test: if the phenomenon turns up several times in a given time interval, then we accept failure as hypothesis. There may exist, however, *reversible failure* when the system returns into the tolerance domain by itself and does not show any kind of failure for some time. This latter case is hard to be handled by a hypothesis test.

The vector $\mathbf{x}$, describing the inner physical state of the device, changes in time according to the operational policy $\mathbf{u}$ – control and environmental effects – and to the complete set of inner parameters contained in $\mathbf{b}$.

The change of state vector $\mathbf{x}$ has in turn an effect on the inner parameters of the system, which after all leads to a change in the functional (technical) parameters, therefore also in the quality of the system.

The connection between the full set of inner parameters and the quality parameters is described by the relation

$$\mathbf{y}(t) = \kappa(\mathbf{b}(t)) \ , \tag{4}$$

where the function κ is a representation of the technical structure and operational principle of the device. So κ is considered to be independent of both time and individual inner parameter values of the given object. With other words: quality is equal to inner parameter values substituted into structure.

We choose the controllable part of $\mathbf{u}$ (the actual policy of operation) in such a way that some functionals of argument $\mathbf{w}$ or $\mathbf{y}$ are optimal according to some predetermined assumptions, e.g. maximum life cycle or accessibility, minimal operational cost.

3. Stochastic Version of the Model

The processes taking place within the device can be very complicated depending on the structure of the device and the components of the parameter vector $\mathbf{b}$. The change of state vector $\mathbf{x}$ can be often described only through stochastic relations

If we allow the system variables to be random, then $z(t)$ will be a stochastic process. Its nature is described by relations (1)–(3).

(2) and (3) in the stochastic case mean that the state space is actually limited to a hypersurface.

(1) means that if we have several observations on a trajectory, the process will later 'remember' only the preceding one. Together with the given form of time independence this defines a homogeneous Markov process. Under mild regularity assumptions $z(t)$ is specified to a diffusion process. The statistical inference and optimal control of such processes have an extensive literature, see e.g. the classical work [7].

4. Accelerated Reliability Tests

Let the life time distribution function of a device operating under stress level $\mathbf{e} = (e_1, \dots, e_k)$ be $F(t; \mathbf{e})$.

The main objective of accelerated reliability testing is to make inference on the life time distribution under usual (nominal) stress level $F(t; \mathbf{e}_u)$ from observation of the life time under an accelerating stress level $\mathbf{e}_a$. The result of the increased stress level will be

$$F(t; \mathbf{e}_u) \leq F(t; \mathbf{e}_a) .$$

If $F(\cdot; \mathbf{e})$ is a member of a parametric family $G(\cdot; \theta)$, acceleration can be described by the dependence of the parameter θ on $\mathbf{e}$, i.e.

$$\theta(\mathbf{e}) = \psi(\mathbf{e}; \alpha, \beta, \ldots) ,$$

where $\alpha, \beta, \ldots$ are constants to be estimated.

The estimation of the constants in an assumed or known model can be based on the regression analysis of observed life testing data under different test levels.

However, one cannot avoid making a decision on the hypothesis that the physical model of deterioration does not change with the increased stress level.

This problem is simplified by the Markov property, which in this case means that the reliability of the system depends only on the lost amount of operational reserve irrespective of the way it has been lost. With other words: the damages are linearly accumulated, the system does not 'remember'.

5. Pattern Recognition Methods in Reliability Forecast

When forecasting reliability characteristics we have to give an estimation of the future trend on the basis of the history of the device.

The stochastic model of the problem is exposed in the sequel.

Suppose that at $t = t_i$ $(i = 0, 1, 2, \ldots, n)$ we have observations $\mathbf{W}_i$ on the vector $\mathbf{w}$ describing the operational state of the device, and we have also the empirical distribution functions $R_i(\mathbf{w})$. We have to determine

$$L_{n+j}(\varepsilon) = P(|\mathbf{W}_{n+j} - \mathbf{w}_u| < \varepsilon) ,$$

where

$\mathbf{w}_u$– vector of the nominal (usual) values

ε– tolerance bound

The application of pattern recognition methods is justified by the fact that we have no satisfactory information needed to apply parametric multivariate statistical methods (e.g. regression analysis). This is usually the case in practical situations. Therefore nonparametric classification methods must be preferred.

In order to forecast by means of pattern recognition methods we observe $\mathbf{w}$ at the initial time or time interval and from the observed data we compute characteristics which are essential from the point of view of the failure or deterioration process. On the basis of these characteristics we can now classify the given device and we forecast according to the class.

It is possible to forecast a physical characteristic, quality parameter, complex reliability feature or the event that the operation process does not leave the tolerance domain.

The methods facilitating the proper choice of essential parameters and classes as well as the decision on the classification of the given device belong to the mathematical pattern recognition (statistical classification).

The problem is not solved but made somewhat easier that we have actually a multidimensional Markov (diffusion) process. The observability is anyway incomplete because of the inner parameters contained in **b**. This incompleteness can be more or less serious according to the structure and the measurement methods applied.

6. Optimization of Maintenance

Suppose that the time behaviour of a given system is described by the vector valued stochastic process $z(t)$. At given discrete times $t = t_1, t_2, \ldots$ we observe $z(t)$ and make corrective interventions (if necessary) in order to keep the system inside the tolerance domain. The intervention $\mathbf{d}(t_i)$ has the cost $\Phi(\mathbf{d}(t_i))$. We want to optimize the intervention policy, i.e.

$$\sum_i \Phi(\mathbf{d}(t_i)) \to \min \ .$$

Φ is supposed to mean costs or time losses involved by technical maintenance (regulation, change of units, etc.).

We have an optimal control problem relating again to a Markov (diffusion) process possibly with incomplete observations.

7. Some Closing Remarks

We demonstrate some system variables introduced in Chapter 2 on a car.

y– acceleration, braking distance, motor overwarming
w– motor temperature, speed
b– corrosion, deformation

In certain cases it may happen that $\mathbf{w} = (\mathbf{x}^T, \mathbf{v}^T)^T$ or $\mathbf{u} = (\mathbf{e}^T, \mathbf{d}^T)^T$. Nevertheless, this does not hold in general.

As we can see, the introduction of the vector **y** apart from **b** is not superfluous. The observation of the set of parameters **b** completely describing the system would be too difficult, even practically unaccomplishable, therefore **y**, chosen by us, does not necessarily carry full information on the proper operation of the given system. (Some authors fail to distinguish between the set of observable and that of actually observed parameters.)

There are cases when the system 'remembers' how it reached a given state, and later developments also depend on the earlier history. As an example, we can mention the process of neutron adsorption in a nuclear plant.

References

1. BARTA, G. (1979): Reliability Prediction of Electromechanical Systems by Pattern Recognition Method, Test Methods on Electronics Technology. 3rd *International Spring Seminar on Electronics Technology*, Balatonfüred.
2. BARTA, G. (1980): Accelerated Reliability Testing of Electromechanical Systems, Thesis, Budapest.
3. FARKAS, J. - SZÁSZ, G. (1976): Einige Probleme der Wahrscheinlichkeitsverteilung von periodisch systematischen Einflüssen in der Fertigungsmesstechnik, *Periodica Polytechnica*, Vol 20. No. 3.
4. KUN, I. (1977): On the Discounted Cost of Markovian Queuing Systems, *Math. Operationsforschung und Statistik*, Ser. Optimization, Vol. 8. No. 4, pp. 565–569.
5. KUN, I. (1978): The Mathematical Model of a Terminal System, in: *Proc. of the Coll. on Point Processes and Queuing Problems*, Bolyai-North Holland.
6. PETRIK, O. - BARTA, GY. - SZÁSZ, G. (1984): Some Experiences from Reliability Tests of Industrial Products, *Proc. of RELCOMEX Conference on Reliability and Exploitation of Computers*, Walbrzych.
7. STRATONOVICH, R.L. (1968): Conditional Markov Processes and their Application to the Theory of Optimal Control, Elsevier, New York-London-Amsterdam.