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A NUMERICAL METHOD FOR SOLUTION OF LINEAR TRANSIENT HEAT CONDUCTION EQUATIONS

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Abstract

A new numerical integrating method is compared in this paper with the most popular two-level schemes, as the Crank–Nicolson (C.–N.), the Galerkin (G.), the EulerCauchy (E.–C.), the Backward Difference (B.–D.), and the 4th order Runge–Kutta (R–K.4). This procedure, the Weighting-Function Method (W.–F. M.) uses not a constant weighting factor (like 1/2 in C.–N. scheme) but a weighting function. The weighting function depends on the actual problem and on the time step. The approximating weighting function is calculated in the first few steps until it reaches a constant value; after that, the calculation will be continued using this constant weight. The W.–F. M. was tested on different simple examples, and was compared with the analytical solution and with the results of other schemes. The W.–F. M. has the best accuracy.

Keywords: numerical method, temperature fields calculation, differential equations, finite element method, heat-flow network method.

1. Introduction

After having discretized (either by finite element or by heat-flow network method) the equation of heat conduction in space co-ordinates, the system of first-order differential equations (S.D.E.) to be solved becomes the following form:

$$\mathbf{C} \cdot \dot{\mathbf{T}}(\tau) + \mathbf{K} \cdot \mathbf{T}(\tau) + \mathbf{K}_A \cdot [\mathbf{T}(\tau) - \mathbf{T}_A] = \dot{\mathbf{Q}}\,, \tag{1.a}$$

where $\mathbf{T}(\tau)$ is the vector of the temperatures, $\mathbf{C}$ is the (diagonal) heat-capacity matrix, $\mathbf{K}$ is the conductance matrix, $\mathbf{K}_A$ is the conductance matrix of the boundary conditions, $\mathbf{T}_A$ is the ambient temperature, and $\dot{\mathbf{Q}}$ is the vector of heat sources.

The initial values to the S.D.E. (1.a) are

$$\mathbf{T}(\tau = 0) = \mathbf{T}_0\,. \tag{1.b}$$

Various methods are known for the numerical integration of the *Eq.* (1.a). The most favourable two-level schemes have the following form:

$$\mathbf{C}\frac{\mathbf{T}^{k+1}-\mathbf{T}^{k}}{\Delta\tau}+\hat{\mathbf{K}}\left[\mathbf{T}^{k}(1-\gamma)+\gamma\mathbf{T}^{k+1}\right]=\hat{\dot{\mathbf{Q}}}, \tag{2}$$

where k denotes the k-th time step, $T^{k+1}=T(\tau_k+\Delta\tau)$ are the unknown temperatures at the end of the time step, and $T^k=T(\tau_k)$ the known temperatures at the beginning of the time step. At the beginning of the calculations is: $\mathbf{T}^k=\mathbf{T}(\tau=\tau_k=0)=\mathbf{T}_0$, in other time steps T^k means the temperatures at the end of the previous time step, $\hat{\mathbf{K}}=\mathbf{K}+\mathbf{K}_A$, $\hat{\dot{\mathbf{Q}}}=\dot{\mathbf{Q}}+\mathbf{K}_A\mathbf{T}_A$, and γ is the weighting factor.

We have to solve the items in the S.D.E. (2) one after other until the increase of temperatures becomes less than a given value or the time domain is over. The accuracy of the calculation depends not only on the chosen time step but on the chosen method, too.

The weighting factor in the S.D.E. (2) differs from method to method e.g.:

$\gamma=0$ – Forward difference schemes (Euler–Cauchy),
$\gamma=1/2$ – Crank–Nicolson schemes,
$\gamma=2/3$ – Galerkin method,
$\gamma=1$ – Backward difference schemes.

Stability, oscillation and accuracy determine the time step to be chosen for all methods [1, ..., 6]. The time step depends on the largest eigenvalue (λ_{max}) [3]:

	no oscillation, if	stable, if
$\gamma=0$	$\Delta\tau\leq 1/\lambda_{max}$,	$\Delta\tau\leq 2/\lambda_{max}$,
$\gamma=1/2$	$\Delta\tau\leq 2/\lambda_{max}$,	$\Delta\tau\leq\infty$,
$\gamma=2/3$	$\Delta\tau\leq 3/\lambda_{max}$,	$\Delta\tau\leq\infty$,
$\gamma=1$	$\Delta\tau\leq\infty$,	$\Delta\tau\leq\infty$.

On the basis of the comparison of the solutions a lot of different problems [4] comes to the following conclusions:

$\gamma=0$ – the time step is the smallest, the results have the less accuracy,
$\gamma=1$ – the most stable, but less accurate than the methods below,
$\gamma=1/2$ – the truncation error is $O(\Delta\tau^3)$, (in all other cases $O(\Delta\tau^2)$), in some cases it is perhaps the most accurate method, but it tends to oscillate,
$\gamma=2/3$ – stable, good accuracy for a wide range of time steps.

[4] proposes to use $\gamma = 2/3$ for the first time steps and for the continuation $\gamma = 1/2$

[1] finds the Galerkin process significantly better for the treatment of fast-varying boundary conditions than the usual Crank–Nicolson scheme in matters of short time accuracy.

Three different techniques are presented in [5] to treat the oscillations of the case $\gamma = 1/2$. It states that the Crank–Nicolson scheme with a simple averaging process (using the average of the temperatures at the beginning and end of the first time step as a new, modified initial condition and using the Crank–Nicolson scheme for the calculation) seems to be an effective way of dealing with oscillations.

The number of freedom influencee the time step to be chosen [5].

If the number of degrees of freedom is large, it is not convenient to decrease the noise by reducing of the time step extremely [5].

The method given in [8] shows a weak analogy to the weighting function method.

In [9] there are a lot of reasonable points of view. [10] has found $\gamma = 0.878$ as the best weighting factor.

[7] uses a weighting function (like in this paper Chapter 2.1), too, which depends on Fourier number.

In practice we have no data about the accuracy of the results since we know neither the analytical solution nor the largest eigenvalue.

To overcome some of the difficulties we do not use a constant weighting factor (like 1/2 in C.–N. scheme) but a weighting function.

2. The Weighting-Function Method

In order to allow the comparison with the analytical solution the method is derived for problems of single-degree of freedom systems. Similarly, this simple case we have made is a generalization for N-degree of freedom, in which the analytical solution is substituted with more accuracy solution produced by a usual numerical method (like C.–N.) but with a reduced time step.

2.1 Single Degree-of-Freedom Systems

The differential equation to be solved is,

$$C \cdot \dot{T}(\tau) + K_A \cdot [T(\tau) - T_A] = \dot{Q}\,, \tag{3.a}$$

the initial value

$$T(\tau = 0) = T_A\,. \tag{3.b}$$

The analytical solution of *Eq.* (3.a), within (3.b)

$$T = (T_\infty - T_A)\left(1 - e^{-\tau/\Theta}\right) + T_A\,, \tag{4}$$

where $\Theta = C/K_A$, and T_∞ is the steady-state temperature.

By substituting *Eq.* (4) into *Eq.* (2), then expressed γ,

$$\gamma = \frac{1}{1 - e^{-x}} - \frac{1}{x}\,, \tag{5}$$

where $x = \Delta\tau/\Theta$.

The weighting function is defined by *Eq.* (5), and has values $\approx 0.5 \leq \gamma \leq 1.0$, linked with $\approx 10^{-6} \leq x \leq \infty$, see *Fig. 1.*

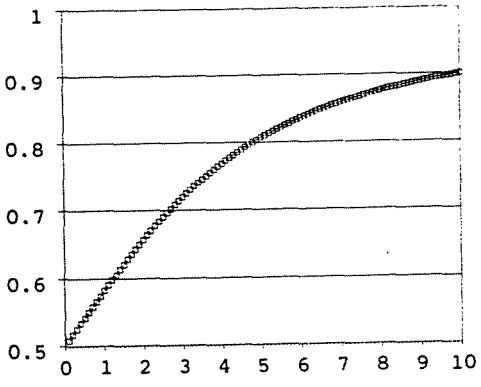

Fig. 1. γ weighting function depends on $x = \Delta\tau/\Theta$

Reversing this sequence of thoughts we can substitute γ with *Eq.* (5) into *Eq.* (2) so *Eq.* (2) gives the analytical solution and the time step may be chosen in a wide range.

2.2 N-Degree of Freedom Systems

We coupled the weighting functions with each node (to each degree of freedom), so over the time interval $\tau_k \leq \tau \leq \tau_{k+1} = \tau_k + \Delta\tau$ the weighted average temperature of ith node is

$$\overline{T}_i = \left(1 - \gamma_i^k\right) T_i^k + \gamma_i^k T_i^{k+1}\,, \tag{6}$$

where γ_i^k = constant (during the time step), and $i = 1, \ldots, j, \ldots, N$.

Replacing $(1-\gamma)\mathbf{T}^k + \gamma\mathbf{T}^{k+1}$ in *Eq.* (2) with the right side of *Eq.* (6), *Eqs.* ith and jth can be rewritten in the following form

$$\begin{bmatrix} \frac{C_i}{\Delta\tau} + \gamma_i^k \widehat{K}_{i,i} & \cdots & -\gamma_j^k \widehat{K}_{i,j} \\ \vdots & \ddots & \vdots \\ -\gamma_i^k \widehat{K}_{j,i} & \cdots & \frac{C_i}{\Delta\tau} + \gamma_j^k \widehat{K}_{j,j} \end{bmatrix} \begin{bmatrix} T_i^{k+1} \\ \vdots \\ T_j^{k+1} \end{bmatrix} =$$

$$= \begin{bmatrix} \widehat{Q}_i + \frac{C_i}{\Delta\tau} T_i^k - \widehat{K}_{i,i}\left(1-\gamma_i^k\right) T_i^k + \widehat{K}_{i,j}\left(1-\gamma_j^k\right) T_j^k \\ \vdots \\ \widehat{Q}_j + \frac{C_j}{\Delta\tau} T_j^k - \widehat{K}_{j,i}\left(1-\gamma_i^k\right) T_i^k + \widehat{K}_{i,j}\left(1-\gamma_j^k\right) T_j^k \end{bmatrix}. \tag{7}$$

If the correct weighting functions γ_i^k $(i = 1, \ldots, j, \ldots, N)$ were known the accurate solution T^{k+1} would be given by *Eq.* (7).

In this case there is no point in calculating the weighting functions γ_i^k by the analytical solution.

Substituting the analytical solution with more accuracy calculations, for example, using R.–K.4, C.–N. or G. methods with a reduced time step $\Delta\tau^* = \Delta\tau / N_{\Delta\tau}$, where $N_{\Delta\tau} = 5 \div 50$. Since we have to know the analytical solution, only during the first time steps until the weighting-functions become the steady-state values, it is enough to calculate the first time steps with more accuracy.

For determination of γ_i^k weighting functions *Eq.* (9) is to be solved. We get *Eq.* (9) by the rewriting *Eq.* (7),

$$\begin{bmatrix} \widehat{K}_{i,i} & \cdots & -\widehat{K}_{i,j} \\ \vdots & \ddots & \vdots \\ -\widehat{K}_{j,i} & \cdots & \widehat{K}_{j,j} \end{bmatrix} \begin{bmatrix} \gamma_i^k \Delta T_i \\ \vdots \\ \gamma_j^k \Delta T_j \end{bmatrix} =$$

$$= \begin{bmatrix} \widehat{Q}_i - \frac{C_i}{\Delta\tau}\Delta T_i \\ \vdots \\ \widehat{Q}_j - \frac{C_j}{\Delta\tau}\Delta T_j \end{bmatrix} - \begin{bmatrix} \widehat{K}_{i,i} & \cdots & -\widehat{K}_{i,j} \\ \vdots & \ddots & \vdots \\ -\widehat{K}_{j,i} & \cdots & \widehat{K}_{j,j} \end{bmatrix} \begin{bmatrix} T_i^k \\ \vdots \\ T_j^k \end{bmatrix}, \tag{8}$$

or written in matrix form

$$\widehat{\mathbf{K}} \begin{bmatrix} \gamma_i^k \Delta T_i \\ \vdots \\ \gamma_j^k \Delta T_j \end{bmatrix} = \hat{\mathbf{Q}} - \frac{1}{\Delta\tau}\mathbf{C} \cdot \Delta\mathbf{T} - \widehat{\mathbf{K}} \cdot \mathbf{T}^k, \tag{9}$$

where $\Delta T_i = T_i^{k+1} - T_i^k$, and ΔT_i is the ith element of $\Delta\mathbf{T}$ vector.

The main steps of W.F. method are below:

1. choosing of the time step ($\Delta\tau$),
2. calculating a reduced time step by dividing the (first) time step into an equidistant time interval $\Delta\tau^* = \Delta\tau/N_{\Delta\tau}$, ($N_{\Delta\tau} = 5 \div 50$),
3. calculating the temperatures with reduced time step, then calculating the increase of each temperature at the end of original ($\Delta\tau$) time step

$$\Delta T = T^{k+1}(\tau_k + \Delta\tau) - T^k(\tau_k)\,,$$

4. solving *Eq.* (9), then ($\gamma_i^k \Delta T_i$ and ΔT_i are known) each γ_i^k, $i = 1, \ldots, N$ weighting function is to be calculated,
5. calculation is to be continued from point 3, until all the weighting functions convergent to the steady-state value:

$$\max_{\substack{i=1,\ldots,N \\ j=1,\ldots,N \\ i\neq j}} (\gamma_i - \gamma_j) \leq \varepsilon \left(\approx 10^{-3}\right)\,,$$

6. calculation is continued from point 1. if the weighting functions in the first time step are out of the prescribed interval $0 \leq \gamma_i \leq 1$, the reduced time step ($\Delta\tau^*$) must be chosen smaller,
7. the time step meets the requirements if all steady-state values of each weighting function (coupled with each node) lie in the interval

$$0.5 \leq \gamma_i \leq 0.7 \div 0.95\,.$$

We remark here that the calculations aren't unconditionally convergent. If a calculation is divergent we have to reduce the time step or we have to take another more accurate method to find the steady-state value of weighting functions. In the given examples we have to change the methods to solve the problems or to solve the problems with a larger time step.

The W.–F. method was in our cases sometimes divergent, but in all of our cases the calculations get convergent by reducing the time step or changing the method for calculation of first time steps.

3. Numerical Examples

3.1 Example 1

We assume the following differential equation to be solved

$$\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 - T_A \\ T_2 - T_A \end{bmatrix} = \begin{bmatrix} 4.5 \\ 25 \end{bmatrix}\,,$$

Table 2

Time [s]	Errors, K C.–N.		Errors, K W.–F.	
($\Delta\tau = 10$ s)	$E1$	$E2$	$E3$	$E4$
0	0.000	0.000	0.000	0.000
10	1.400	14.026	0.000	0.003
20	-1.007	9.672	0.003	0.002
30	0.640	-0.762	0.000	-0.003
40	-0.519	4.597	-0.000	-0.003
50	0.265	-3.283	-0.005	-0.004
60	-0.287	2.164	0.003	0.004
70	0.088	-1.614	-0.002	-0.004
80	-0.175	0.998	-0.005	-0.002
90	0.006	-0.816	-0.004	0.004
100	-0.126	0.436	0.004	-0.004
200	-0.059	-0.051	0.001	-0.002
300	-0.040	-0.038	0.000	0.002
400	-0.201	-0.022	-0.000	-0.002
500	-0.005	-0.014	0.005	-0.004
600	-0.006	-0.008	0.004	0.002
700	-0.005	0.001	-0.004	0.001
800	0.003	0.001	0.003	0.001

($E1 = T_{1,anal} - T_{1,C.-N.}$,
$E2 = T_{2,anal} - T_{2,C.-N.}$,
$E3 = T_{1,anal} - T_{1,W.-F.}$,
$E4 = T_{2,anal} - T_{2,W.-F.}$)

Figs. 4 and *5* show that the k_j values depend on time stochastically, so it is not applicable in our cases.

All the rows of this table are linked with serial number of time steps. So the number of different rows with weighting function values are identical with the number of first time steps.

For the weighting function values (in last rows of different cases in *Table 3*) the individual criteria were

$$|\gamma_1 - \gamma_2| \leq 10^{-3}.$$

All the other tables are like *Table 3*.

We remark here that our calculations using the 4th order Runge–Kutta method were sometimes unsuccessful thereupon overflow. It is widely proposed to choose the time step for R.–K. 4, to increase the accuracy by

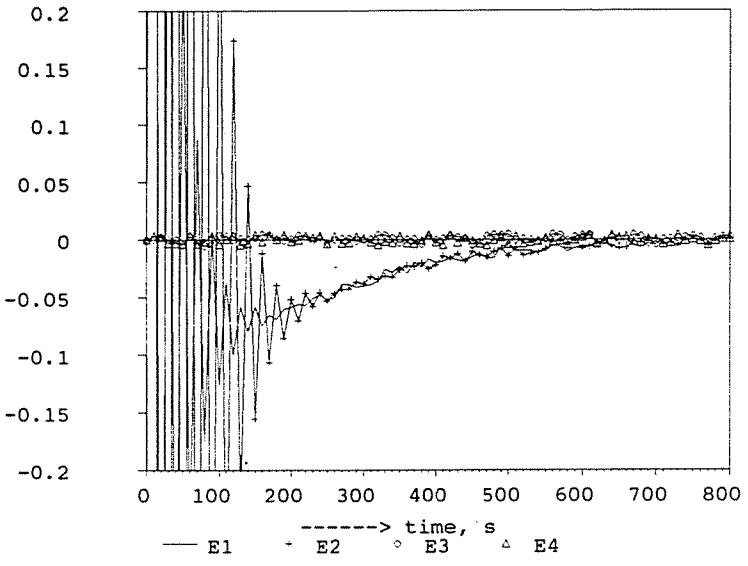

Fig. 2. The errors are in *Table 2*, $\Delta\tau = 10\,\mathrm{s}$
($E1 = T_{1,anal} - T_{1,C.-N.}$, $E2 = T_{2,anal} - T_{2,C.-N.}$,
$E3 = T_{1,anal} - T_{1,W.-F.}$, $E4 = T_{2,anal} - T_{2,W.-F.}$)

using the following equation:

$$k_j = \frac{|K_{2,j} - K_{3,j}|}{|K_{1,j} - K_{2,j}|} \leq 0.2 \div 0.3\,,$$

where $j = 1, \ldots, N$, and $K_{i,j}$ is defined by the equations below ($K_{i,j}$ is the jth element of the $\mathbf{K}_i$ vector),

$$\mathbf{T}^{k+1} = \mathbf{T}^k + \frac{\Delta\tau}{6}\left(\mathbf{K}_1 + 2\mathbf{K}_2 + 2\mathbf{K}_3 + \mathbf{K}_4\right)\,,$$

where

$$\mathbf{K}_i = \mathbf{C}^{-1}\left(\hat{\hat{Q}} - \hat{\mathbf{K}}\mathbf{T}_i\right)\,, \qquad i = 1, \ldots, 4\,,$$

$$\mathbf{T}_1 = \mathbf{T}^k\,,$$

$$\mathbf{T}_i = \mathbf{T}^k + \frac{\Delta\tau}{2}\mathbf{K}_{i-1} \qquad (i = 2, \ldots, 4)\,.$$

Fig. 4 shows k_j ($j = 1, 2$) in this case.

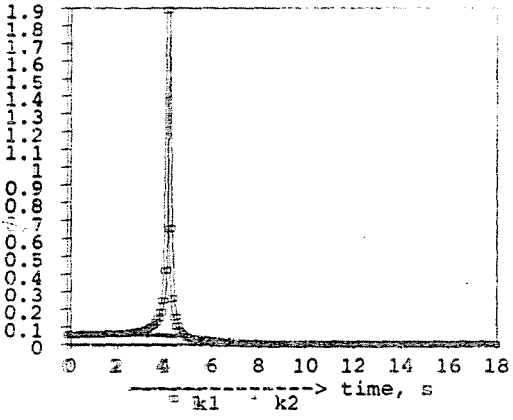

Fig. 4. $k_{j,n}$ $(j = 1, 2)$, R.–K.4, $\Delta\tau = 0.1$ s

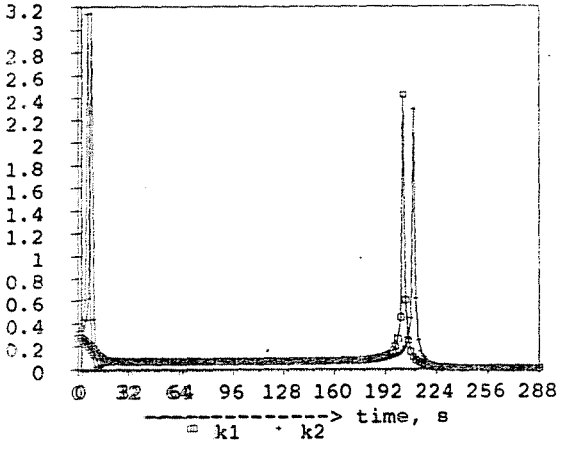

Fig. 5. $k_{j,n}$ $(j = 1, 2)$, R.–K.4, $\Delta\tau = 1.6$ s

3.2 Example 2

...ment discretization in space dimension of a unit length wire ...lowing equilibrium equations for two nodes at the ends

$$\begin{bmatrix} \cdot \\ T_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} 1+\frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} T_1 - T_A \\ T_2 - T_A \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

...al values $T_1(\tau = 0) = T_2(\tau = 0) = 0$, °C.

The prescribed boundary conditions are for the cover of the wire

$$\dot{Q}_p = \alpha A_p [T(x) - T_A] = T(x) - T_A ,$$

for the first node

$$\dot{Q}_1 = T_1 - T_A ,$$

for the 'last' node

$$\dot{Q}_N = -\lambda \left(\frac{\partial T}{\partial x} \right) = 0 ,$$

the ambient temperature is

$$T_A = 0 ,$$

and the other data are given by the equation.

We calculate the two nodes model, and N nodes model which is produced by dividing into $N - 1$ equidistant sections of the unit length wire.

The results are given in *Tables 4 - 7*. In these tables the parameters of the calculation $\Delta\tau$, $N_{\Delta\tau}$, N, and the method for the first time steps to determine the steady-state values of W.–F. are given.

Tables 4 - 6 compare the results and it can be established that neither the R.–K.4 method nor the C.–N. scheme give the best results.

Table 7 shows the results of G. method. In this case the time step was the largest, and the number of first time steps was the less.

Table 4

Example 2	$\Delta\tau = 10\,s$ $N_{\Delta\tau} = 10$ R.–K.4, $N = 2$		$\Delta\tau = 10\,s$ $N_{\Delta\tau} = 20$ R.–K.4, $N = 2$		$\Delta\tau = 0.5\,s$ $N_{\Delta\tau} = 40$ R.–K.4, $N = 10$	
Serial number of time steps	γ_1	γ_2	γ_1	γ_2	γ_1	γ_{10}
1	1.094	0.821	0.894590	0.880045	0.553390	0.527463
2	-2.914	-2.871	0.884106	0.884106	0.537077	0.535493
3	divergent	divergent			0.536199	0.536095

Fig. 6 shows the temperature difference $D(\tau)$ between the results produced by G. method and produced by C.–N. scheme in the Nth node, the parameters are the same.

Note that all the values of weighting functions coupled with 2, ..., $N - 1$ nodes were less in our cases than the value for the 1st node and greater than the values for the Nth node.

Fig. 6 shows that if the steady-state values of W.–F. are different (caused only by different methods) the results are different too, but the differences are less.

Table 5

Example 2	$\Delta\tau = 0.5\,s$ $N_{\Delta\tau} = 40$ R.–K.4, $N = 100$		$\Delta\tau = 0.5\,s$ $N_{\Delta\tau} = 40$ C.–N., $N = 100$		$\Delta\tau = 1\,s$ $N_{\Delta\tau} = 40$ C.–N., $N = 100$	
Serial number of time steps	γ_1	γ_{100}	γ_1	γ_{100}	γ_1	γ_{100}
1	overflow	overflow	0.552837	0.527464	0.585945	0.563566
2			0.537027	0.535495	0.571619	0.571531
3			0.536176	0.536078		

Table 6

Example 2	$\Delta\tau = 2\,s$ $N_{\Delta\tau} = 40$ C.–N., $N = 100$		$\Delta\tau = 5\,s$ $N_{\Delta\tau} = 40$ C.–N., $N = 100$		$\Delta\tau = 10\,s$ $N_{\Delta\tau} = 40$ C.–N., $N = 100$	
Serial number of time steps	γ_1	γ_{100}	γ_1	γ_{100}	γ_1	γ_{100}
1	0.648552	0.632208	0.788842	0.779913	0.888241	0.883521
2	0.638122	0.638115	0.783370	0.783151	0.923957	0.885224
3					divergent	divergent

Table 7

Example 2	$\Delta\tau = 2\,s$ $N_{\Delta\tau} = 40$ G., $N = 100$		$\Delta\tau = 5\,s$ $N_{\Delta\tau} = 40$ G., $N = 100$		$\Delta\tau = 10\,s$ $N_{\Delta\tau} = 40$ G, $N = 100$	
Serial number of time steps	γ_1	γ_{100}	γ_1	γ_{100}	γ_1	γ_{100}
1	0.651723	0.635524	0.789877	0.780995	0.888284	0.883580
2	0.641379	0.641378	0.784217	0.784217	0.885288	0.885288

3.3 Example 3

The problem to be solved is almost identical with example 2, but the boundary condition for the 1st node is $\dot{Q}_1 = 1000(T_1 - T_A)$. The equilibrium equations are for two nodes model

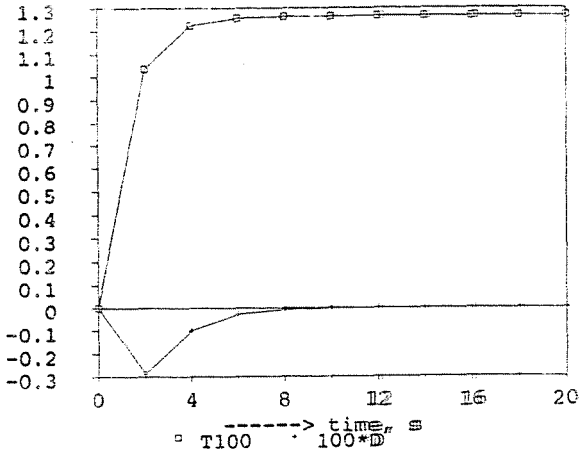

Fig. 6. $T_{100}(\tau)$ and $D(\tau) = T_{100}(\tau)$, [W.–F. (G.), $N = 100$, $\Delta\tau = 2\,\mathrm{s}$] – $T_{100}(\tau)$ (C.–N.), $N = 100$, $\Delta\tau = 2\,\mathrm{s}$]

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} + \begin{bmatrix} 100 + \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} T_1 - T_A \\ T_2 - T_A \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Divided the unit length wire in equidistant sections (like as we did ample 2), we get a model with N degree of freedom, too. The res in *Table 8.*

Table 8

Example 3	$\Delta\tau = 1\,\mathrm{s}$ $N_{\Delta\tau} = 40$ G., $N = 100$		$\Delta\tau = 5\,\mathrm{s}$ $N_{\Delta\tau} = 40$ G., $N = 100$	
Serial number of time steps	γ_1	γ_{100}	γ_1	γ_{100}
1	0.727668	0.626708	0.910283	0.880534
2	0.640743	0.640737	0.884701	0.884701

Fig. 7 shows in the nodes the temperature differences betw results depending on $\Delta\tau = 1\,\mathrm{s}$ and $\Delta\tau = 5\,\mathrm{s}$, the time is $\tau = 5\,\mathrm{s}$. steady-state values of W.–F. are given in the last row of *Table 8.*)

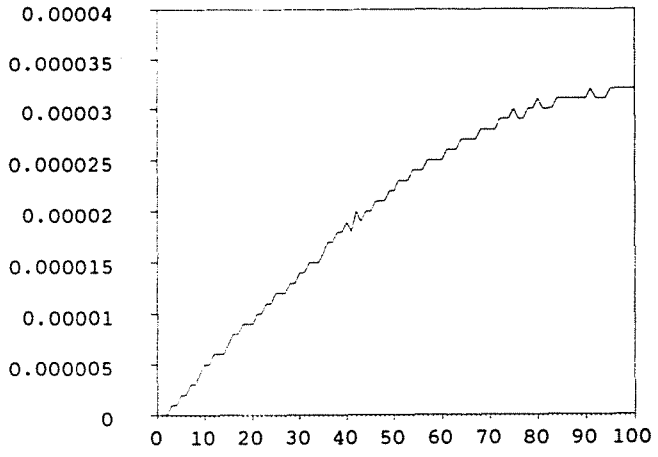

Fig. 7. $\longrightarrow i = 1, \ldots, 100$ (number of nodes)
$D_i = T_i$ ($\tau = 5$ s), [W.–F. (G.), $\Delta\tau = 1$ s] – T_i ($\tau = 5$ s), [W.–F. (G.), $\Delta\tau = 5$ s]

The temperature-time functions in Nth node calculated by W.–F. method and by original C.–N. scheme are shown in *Fig. 8.* The *Fig. 8* shows the differences between temperatures calculated by W.–F. method and by C.–N., too.

4. Conclusions

From the analysis of simple problems it seems that the weighting function method (W.–F. M.) is efficient for finding a near analytical solution.

The steady-state value of weightingfunctions depends only on actual problems, and chosen time steps.

Since the W.–F.M. differ from the other methods only in the weighting factor, branching from one scheme to the W.–F.M. is an easy matter.

It will be clear during the first time steps whether the calculation is convergent or not.

It's possible that the calculations are divergent caused by the chosen time step or caused by the accuracy of the method for the found of steady-state values of W.–F. In the last case it is necessary to change the method for a more accurate method in the first time steps.

In our examples the Galerkin method was the best for calculation of the first time steps.

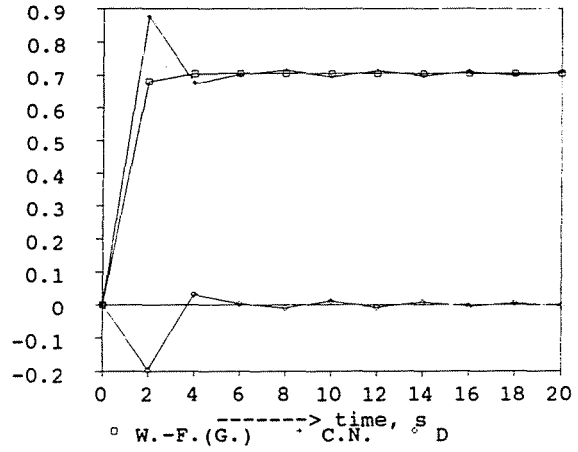

Fig. 8. $T_{100}(\tau)$ [W.–F. (G.), $N = 100$, $\Delta\tau = 2\,\mathrm{s}$], $T_{100}(\tau)$ [C.–N., $N = 100$, $\Delta\tau = 2\,\mathrm{s}$] and $D(\tau) = T_{100}(\tau)$, [W.–F.(G.), $N = 100$, $\Delta\tau = 2\,\mathrm{s}$] – $T_{100}(\tau)$, [C.–N., $N = 100$, $\Delta\tau = 2\,\mathrm{s}$]

The W.–F.M. can be used only for linear problems and the problem is to be calculated with a more accurate method during the first time steps.

The W.–F.M. only seldom needs to repeat the calculations to get some value about the accuracy.

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