PERIODICA POLYTECHNICA SER. MECH. ENG. VOL. 36, NO. 1, PP. 3–16 (1992)

LONGITUDINAL PRESSURE VARIATION IN THE LOWER AIR DISTRIBUTION SPACE IN AN AIR-SLIDE CONVEYOR WITH OPEN TOP AIR SPACE (Part One)

L. KOVÁCS and S. VÁRADI

Department of Fluid Machinery
Technical University, H–1521, Budapest

Received: May 15, 1992.

Abstract

A computation method is described for determining hydraulic parameters of lower air distribution space of an air-slide conveyor with open top air space. Equations are of a form directly solvable if friction is omitted. Solving the differential equations by the Runge–Kutta method for the case of friction, variation of parameters as a function of the duct length is presented.

In a duct long enough, velocity of the fluidizing air flowing through the air distribution layer assumes a minimum, taking the friction into consideration.

Equations help plotting the characteristic curve of the duct, and intersection with the fan's characteristic curve defines the common work point of the duct.

Keywords: Pneumatic conveying, air-slide conveying.

Longitudinal Velocity Distribution in the Air Distribution Space of the Duct. No-Friction Case

The mathematical-physical model for determining velocity and pressure distribution in the air distribution space of a duct, and longitudinal distribution of velocity v_f of fluidizing air flowing through the air distribution layer involves some simplifying assumptions, such as:

— Air compressibility is ignored, reckoning with approximation $\rho =$ const.
— Air flow over the flowing material layer in the open top duct is omitted, reckoning with pressure $p_0 =$const. along the duct.
— Velocity profile variation in the air distributing space of the duct is omitted, only the prevailing mean velocity is reckoned with.
— In writing pressure drop of air flowing through the air distribution layer, resistance of the overlaying material layer is omitted. This causes no significant error since in designing the duct, resistance at

the distribution layer is chosen as multiple (6 to 15 times) of the material layer. It is, of course, feasible to reckon in the following with a distribution layer of a resistance increased by that of the material layer flowing along the duct, of a thickness m_0 considered constant at a fair approximation.

— The air distribution duct is mostly confined on three sides by metal sheets, while on the top by the air distributing fabric or some other porous layer. The duct resistance is reckoned as if all the four sides were made of material of the same quality.

— Effect of the material layer proceeding on the air distribution layer is omitted.

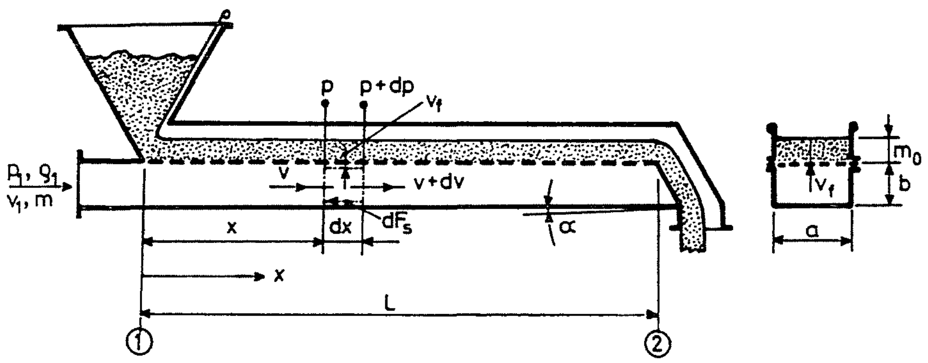

Fig. 1.

Continuity equation for the elementary duct section cut out at x of the aeration duct in *Fig. 1*:

$$(v + \mathrm{d}v)\rho ab - v\rho ab + v_f \rho a \mathrm{d}x = 0\,. \tag{1}$$

After possible reductions, *Eq.* (1) becomes:

$$\mathrm{d}x = -b\frac{\mathrm{d}v}{v_f}\,. \tag{2}$$

Making use of simplexes below:

$$x^* = \frac{x}{L}; \quad b^* = \frac{b}{L}; \quad v^* = \frac{v}{v_1}; \quad v_f^* = \frac{v_f}{v_1}; \quad p^* = \frac{p}{p_0}\,. \tag{3}$$

Eq. (2) may be written also in dimensionless form:

$$\mathrm{d}x^* = -b^* \frac{\mathrm{d}v^*}{v_f^*}\,. \tag{4}$$

Pressure drop of air flowing through the air distribution layer, and the fluidized material layer:

$$\Delta p = \Delta p_{lb} + \Delta p_m\,. \tag{5}$$

In conformity with those stated above, it is no great error to apply approximation:

$$\Delta p \cong \Delta p_{lb}\,. \tag{6}$$

With this approximation, pressure drop of air flowing through the air distribution layer:

$$\Delta p = p - p_0 = k\rho v_f\,. \tag{7}$$

Making this equation dimensionless, then expressing from it the v_f value:

$$v_f^* = \frac{p_0}{k\rho v_1}(p^* - 1) = \pi_1(p^* - 1)\,. \tag{8}$$

Combining *Eqs.* (4) and (8)

$$\mathrm{d}x^* = -\frac{b^*}{\pi_1}\frac{\mathrm{d}v^*}{p^* - 1}\,. \tag{9}$$

Momentum equation written for the elementary section cut out at x yields:

$$pab - (p + \mathrm{d}p)ab - \mathrm{d}F_r = (v + \mathrm{d}v)\rho\, ab(v + \mathrm{d}v) - v\rho\, abv + v_f a\mathrm{d}x\rho\, v\,. \tag{10}$$

Elementary mass flow across the elementary distribution layer in the last term of the right-hand side of *Eq.* (10):

$$\mathrm{d}\dot{m} = v_f \rho\, a\, \mathrm{d}x\,. \tag{11}$$

Thereby component in direction x of the impulse of air flowing out of elementary surface $a\,\mathrm{d}x$:

$$\mathrm{d}I_x \cong \mathrm{d}\dot{m}v \cong v_f \rho\, a\, \mathrm{d}xv\,. \tag{12}$$

Reckoning with condition

$$\mathrm{d}F_r = \frac{\lambda}{2d_h}\rho\, v^2 \mathrm{d}x = 0 \tag{13}$$

in *Eq.* (10), that is, considering the lower air distribution space of the dust as exempt from losses, *Eq.* (2) may be directly integrated, taking *Eq.* (2) into consideration.

Integrating for no-friction case yields:

$$p = -\frac{\rho}{2}v^2 + C. \tag{14}$$

Eliminating integration constant C at the duct inlet under conditions $p = p_1$ and $v = v_1$:

$$p + \frac{\rho}{2}v^2 = p_1 + \frac{\rho}{2}v_1^2. \tag{15}$$

Introducing total pressure $p_t = p + \frac{\rho}{2}v^2$

$$p_t = p_{t1} = \text{const.} \tag{16}$$

meaning that in no-loss case, the air distribution duct acts as a diffusor.

Equation (16) becomes in dimensionless form:

$$p_t^* = p^* + \frac{v_1^2\rho}{2p_0}v^{*2} = p^* + \pi_2 v^{*2} = \text{const.} \tag{17}$$

Combining *Eqs.* (9) and (17) yields the dimensionless equation:

$$\mathrm{d}x^* = -\frac{b^*}{\pi_1\left[(p_t^* - 1) - \pi_2 v^{*2}\right]}\mathrm{d}v^* \tag{18}$$

that can be integrated. The integral value becomes:

$$x^* = -\frac{b^*\pi_3}{2\pi_1\pi_2}\ln\frac{1+\pi_3 v^*}{1-\pi_3 v^*} + C. \tag{19}$$

Integration constant C of *Eq.* (19) may be determined from the condition that at the duct inlet, at $x^* = 1$, velocity along the duct $v^* = 0$, the duct outlet being closed. Under this condition $C = 1$.

Equation (19) expressing relationship $x^* = f(v^*)$ - – taking the integration constant into consideration and expressing from it the v^* value may be written as:

$$v^* = \frac{e^{\pi_4(1-x^*)} - 1}{\pi_3\left[e^{\pi_4(1-x^*)} + 1\right]}. \tag{20}$$

Distribution along the Duct of Air Flow Velocity across the Air Distribution Layer

To determine longitudinal distribution of air flow velocity v_f through the air distribution layer, v_f^* is expressed from *Eq.* (4):

$$v_f^* = -b^* \frac{\mathrm{d}v^*}{\mathrm{d}x^*} . \tag{21}$$

Substituting $\mathrm{d}v^*/\mathrm{d}x^*$ from *Eq.* (18) in (21) yields $v_f^* = f(v^*)$ for air flow velocity through the air distribution layer, to be written as:

$$v_f^* = \pi_1(p_t^* - 1)(1 - \pi_3^2 v^{*2}) , \tag{22}$$

which, substituting the v^* value from *Eq.* (20), yields relationship $v_f^* = f(x^*)$, namely:

$$v_f^* = \pi_1(p_t^* - 1)\left\{1 - \frac{\left[\mathrm{e}^{\pi_4(1-x^*)} - 1\right]^2}{\left[\mathrm{e}^{\pi_4(1-x^*)} + 1\right]^2}\right\} . \tag{23}$$

Determination of the Total Air Volume Flowing through the Air Distribution Layer. Volume of Air Supply to the Duct

$\mathrm{d}q^*$ flowing across the elementary air distribution layer cut out at x from the duct:

$$\mathrm{d}q^* = \frac{v_f^*}{b^*}\mathrm{d}x^* . \tag{24}$$

Taking *Eq.* (23) into consideration, *Eq.* (24) may be integrated to determine q^*.

There is also a direct, simpler way to obtain the equation for the air volume supplying the duct, namely by determining velocity v_1 involved in factor π_3 in *Eq.* (20). Product of this velocity by the inlet cross section yields air volume supply to the duct. Substituting in *Eq.* (20) $x^* = 0$ for the duct inlet, here $v^* = 1$.

Substitution yields:

$$v^* = 1 = \frac{\mathrm{e}^{\pi_4} - 1}{\pi_3[\mathrm{e}^{\pi_4} + 1]} , \tag{25}$$

hence:

$$\pi_3 = \frac{e^{\pi_4} - 1}{e^{\pi_4} + 1} . \tag{26}$$

Since:

$$\pi_3 = \left(\frac{\pi_2}{p_t^* - 1}\right)^{\frac{1}{2}} = \frac{v_1}{\left[\frac{2p_0}{\rho}(p_t^* - 1)\right]^{\frac{1}{2}}} . \tag{27}$$

Furthermore:

$$\pi_4 = \frac{\left[\frac{2p_0}{\rho}(p_t^* - 1)\right]^{\frac{1}{2}}}{kb^*} . \tag{28}$$

Since at the duct outlet, $v_2^* = 0$, here also:

$$p_t^* = p_2^* . \tag{29}$$

Then *Eq.* (28) takes the form:

$$\pi_4 = \left[\frac{2p_0}{\rho k^2 b^{*2}}(p_t^* - 1)\right]^{\frac{1}{2}} . \tag{30}$$

Velocity v_1 of air entering the duct inlet may be obtained by determining π_3 from *Eq.* (20) substituting $x^* = 0$. Velocity v_1 is being involved in π_3 in conformity with *Eq.* (27), hence inlet velocity:

$$v_1 = kb^* \pi_4 \frac{e^{\pi_4} - 1}{e^{\pi_4} + 1} . \tag{31}$$

Hence, air volume supplying the duct, taking *Eq.* (30) into consideration:

$$q = abv_1 = abkb^* \pi_4 \frac{e^{\pi_4} - 1}{e^{\pi_4} + 1} . \tag{32}$$

Equation of the Duct Characteristic Curve

Characteristic curve of the duct is understood as relationship $p_{t1} - p_{t0} = p_{t1} - p_0 = f(q)$. In knowledge of it, as well as of the air supply mechanism for the duct, the work point can be marked cut, and the duct operation adjusted to it. After *Eq.* (17), non-dimensional total pressure

$$p_t^* = \text{const.} \tag{33}$$

throughout the duct.

At 1 at the duct inlet, $v^* = 1$, while at 2 at the duct inlet — being closed — $v^* = 0$, hence:

$$p_{t1}^* = p_1^* + \pi_2 = p_2^*, \tag{34}$$

that is:

$$p_{t1} = p_1 + p_0 \pi_2 = p_2 . \tag{35}$$

p_2^* (equal to p_{t1}^*) being inexpressible from factor π_4 in *Eq.* (32), relationship $p_{t1} - p_0 = f(q)$ cannot be given analytically. Assuming fluidizing velocity v_f at e. g. $x^* = 1$, the characteristic curve can be computed point-wise by means of *Eqs.* (7) and (32). *Fig. 2* shows a set of characteristic curves for a frictionless duct of length $L = 50$ m. Parameters are constant characteristic of the distribution layer, and fluidization velocity v_f at $x^* = 1$. It appears from the diagram that relationship $p_{t1} - p_0 = f(q)$ is but approximately linear.

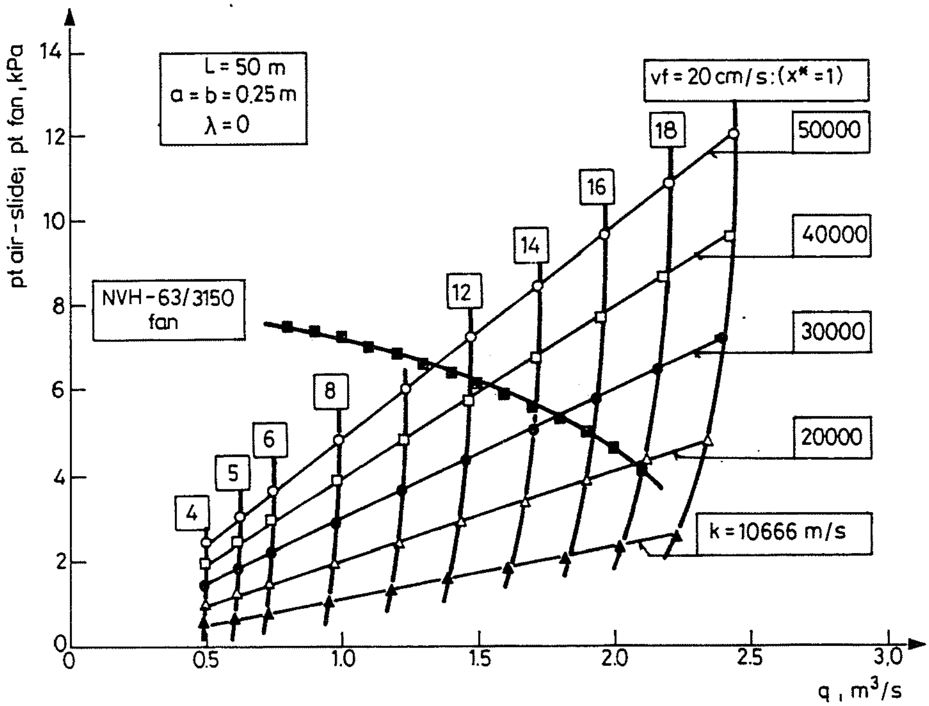

Fig. 2.

Longitudinal Velocity Distribution in the Air Distribution Space of the Duct. Frictional Case

Taking friction into consideration, resistance of the lower air space of the duct is computed as if the air distribution duct were confined on all four sides by sidewalls of the same quality. This is but partly true, since the duct is topped by an air distribution layer, different in roughness from the three — mostly metal sheet — sidewalls.

In *Eq.* (10) $\mathrm{d}F_r$ was zeroed, referring to a frictionless case. Thereby equations could be directly integrated. Omitting friction causes little error for short ducts. For long ducts, however, with $L > 25\,\mathrm{m}$ (taken to illustrate the kind of distributions for constant parameters in the model problem) friction was shown in tests not to be omissible any more.

Taking condition $\mathrm{d}F_r = 0$ as well as *Eq.* (9) into consideration, after transformation and arrangement, *Eq.* (10) can be written in non-dimensional form:

$$\frac{\mathrm{d}p^*}{\mathrm{d}v^*} = \lambda(\mathrm{Re})\pi_5 \frac{v^{*2}}{p^* - 1} - 2\pi_2 v^* . \tag{36}$$

Equation (36) may be solved e. g. by the Runge–Kutta method, relying on the initial condition

$$\Delta p_2^* = p_2^* - 1 , \qquad v^* = 0 \tag{37}$$

at the duct end.

For the solution, the program computes friction coefficient $\lambda = f(\mathrm{Re})$ point-wise, based on approximate relationships found in literature.

Taking solution of *Eq.* (36) into consideration, other parameters of the duct may be determined by *Eqs.* (8) and (4).

Fig. 3 shows a velocity distribution diagram $v_f = f(x)$ obtained by solving *Eq.* (36).

Fluidizing air velocity varies at maximum for perlon fabrics of the least resistance ($k = 10666\,\mathrm{m/sec}$). The velocity first decreases from the duct outlet value $v_f = 10\,\mathrm{cm/sec}$, then it increases and assumes the maximum value $v_f = 10.8\,\mathrm{cm/sec}$ at the duct inlet. Velocity v_f is seen from the diagram to be the lowest, namely $v_f = 9.88\,\mathrm{cm/sec}$, for a duct length $x = 27.8\,\mathrm{m}$.

In *Fig. 4* pressure distribution along the duct length is seen, computed with constant, and with varying friction coefficients, respectively. The two diagrams deviate, showing that reckoning with an in fact variable friction coefficient, the pressure is higher at the duct inlet.

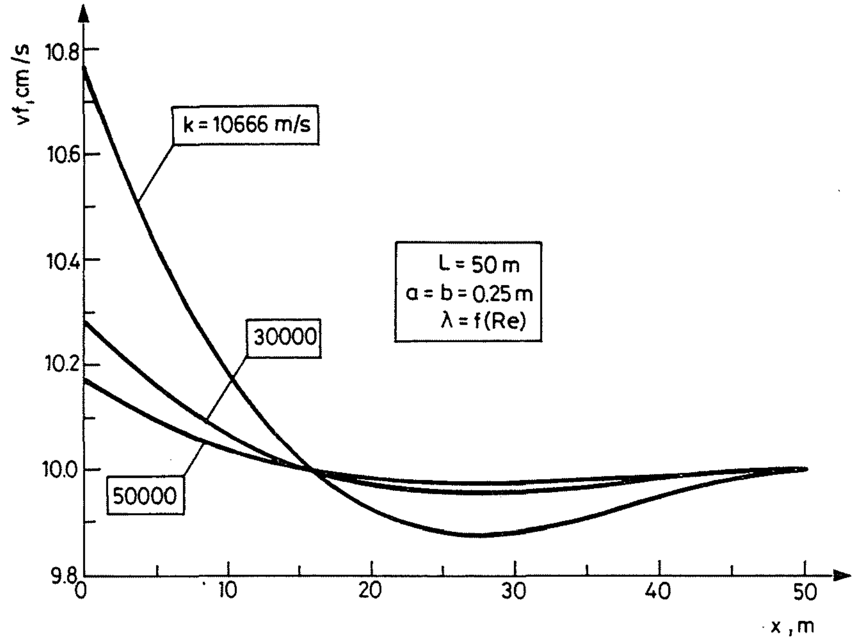

Fig. 3.

Two diagrams in *Fig. 5* are similar to those in *Fig. 4*. Obviously, since fluidizing velocity v_f is proportional to the pressure difference, relationship $v_f = f(x)$ has a course similar to that of $p = f(x)$.

The set of curves in *Fig. 6* is similar to that in *Fig. 2* excepted that the set of curves has been plotted as solution of *Eq.* (36) taking the friction effect into consideration.

Numerical Example for Determining Parameters of an Air-Slide Conveyor Omitting Friction

Duct data:

$a = b = 0.25\,\mathrm{m}$
$\rho = 1.2\,\mathrm{kg/m^3}$
$L = 50\,\mathrm{m}$
$k = 10666\,\mathrm{m/sec}$ (perlon fabric SZ8)
$v_f = 0.05\,\mathrm{m/sec}$ at $x^* = 1$, where $v^* = 0$

From *Eq.* (7):

$$\Delta p = k\,\rho\,v_f = 10666 \cdot 1.2 \cdot 0.05 = 640\,\mathrm{Pa}\,,$$

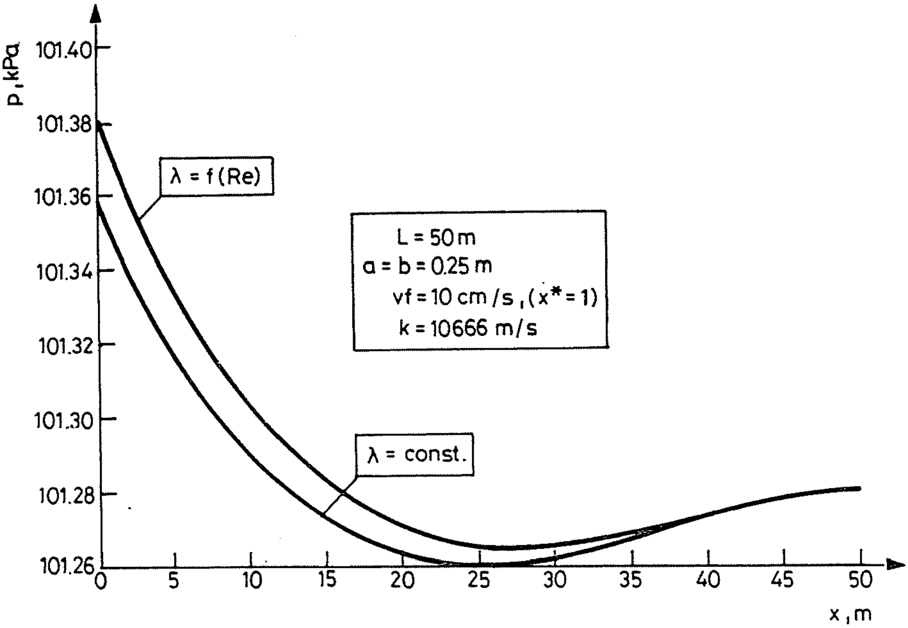

Fig. 4.

Hence:

$$p_t^* = p_2^* = \frac{\Delta p + p_0}{p_0} = 1.0064[-]$$

$$\pi_4 = \left[\frac{2p_0}{b^{*2}k^2\rho}(p_t^* - 1)\right]^{\frac{1}{2}} = \left[\frac{2 \cdot 10^5 \cdot 50^2}{0.25^2 \cdot 10666^2 \cdot 1.2} \cdot 0.0064\right]^{\frac{1}{2}} = 0.6124\,,$$

Air supply volume to the duct is obtained from *Eq.* (32), namely:

$$q = abkb^*\pi_4 \frac{e^{\pi_4} - 1}{e^{\pi_4} + 1}$$

$$= 0.25 \cdot 0.25 \cdot 10666 \cdot \frac{0.25}{50} \cdot 0.6124 \frac{e^{0.6124} - 1}{e^{0.6124} + 1} = 0.606\,\mathrm{m}^3/\mathrm{s}\,.$$

Hence, mean velocity of air entering at the duct inlet:

$$v_1 = \frac{q}{ab} = \frac{0.606}{0.25 \cdot 0.25} = 9.7\,\mathrm{m/s}\,,$$

Longitudinal velocity variation of air flowing in the air distribution duct can be point-wise determined by *Eq.* (20). So can be, e. g., the velocity value at the duct mid-section $x^* = 0.5$.

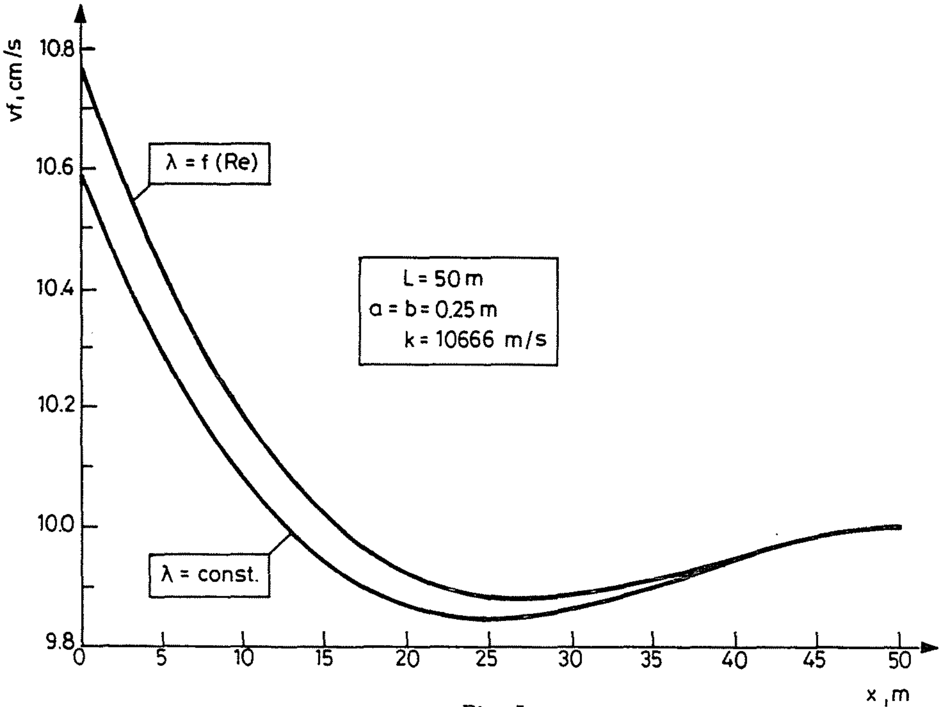

Fig. 5.

In the relationship, value of constant π_3, taking $p_t^* = p_2^*$ into consideration:

$$\pi_3 = \left[\frac{\pi_2}{p_2^* - 1}\right]^{\frac{1}{2}} = \left[\frac{\rho v_1^2}{2p_0(p_2^* - 1)}\right]^{\frac{1}{2}} = \left[\frac{1.2 \cdot 9.7^2}{2 \cdot 10^5 \cdot 0.0064}\right]^{\frac{1}{2}} = 0.297\,.$$

Thereby air flow velocity in the air distribution duct at $x^* = 0.5$:

$$v^* = \frac{e^{\pi_4(1-x^*)} - 1}{\pi_3[e^{\pi_4(1-x^*)} + 1]} = \frac{e^{0.6124(1-0.5)} - 1}{0.297 \cdot [e^{0.6124(1-0.5)} + 1]} = 0.5115\,,$$

that is,

$$v = v^* v_1 = 4.97\,\mathrm{m/sec}\,.$$

Hence, velocity of air flowing through the air distribution layer at $x^* = 0.5$ — taking value

$$\pi_1 = \frac{p_0}{k\rho v_1} = \frac{1 \cdot 10^5}{10666 \cdot 1.2 \cdot 9.7} = 0.805$$

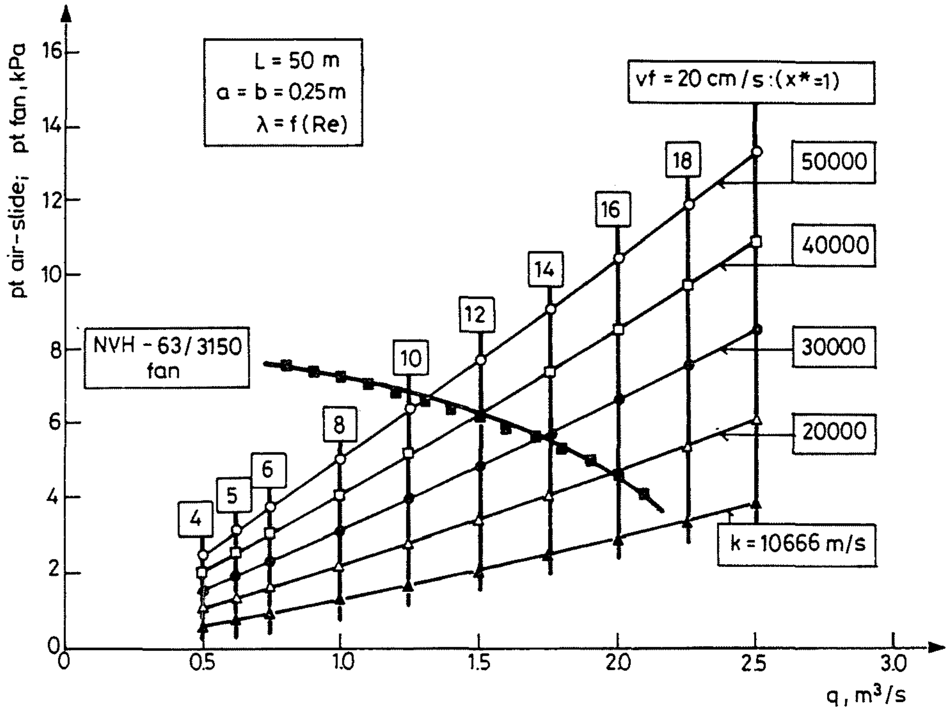

Fig. 6.

— can be determined from *Eq.* (23) as:

$$v_f^* = \pi_1(p_t^* - 1)\left\{1 - \frac{\left[e^{\pi_4(1-x^*)} - 1\right]^2}{\left[e^{\pi_4(1-x^*)} + 1\right]^2}\right\} =$$

$$= 0.805 \cdot (1.0064 - 1) \cdot \left\{1 - \frac{\left[e^{0.6124\cdot(1-0.5)} - 1\right]^2}{\left[e^{0.6124\cdot(1-0.5)} + 1\right]^2}\right\} = 0.00503\,,$$

$$v_f = v_f^* v_1 = 0.00503 \cdot 9.7 = 0.0488\,\mathrm{m/s}\,.$$

Thus, at the duct mid-section, velocity of air flow across the distribution layer has little changed compared to 5 cm/sec at the duct end, thanks to the distribution layer of relatively high resistance.

If e. g. the fan characteristic curve crosses point $k = 30000\,\mathrm{m/sec}$, $v_f = 12\,\mathrm{cm/sec}$ $(x^* = 1)$, then

$$p_{t1} - p_0 = 4.32\,\mathrm{kPa}\,, \qquad q = 1.46\,\mathrm{m^3/sec}\,.$$

Thus, the power needed to drive the fan at an efficiency $\eta_t = 0.68$:

$$P_t = \frac{q\Delta p_t}{\eta_t} = \frac{1.46 \cdot 4320}{0.68} = 9275\,\mathrm{W}\,.$$

Legend

a	[m]	duct width
$A = ab$	[m^2]	duct cross section
b	[m]	duct headroom
b^*	[–]	dimensionless headroom of air distribution space
C		integration constant
$d_h = \dfrac{4A}{K}$	[m]	hydraulic diameter
F_r	[N]	friction force
k	[m/s]	constant characteristic of the distribution layer
$K = 2(a+b)$	[m]	perimeter of the air distribution space
$\pi_1 = \dfrac{p_0}{k\rho v_1}$	[–]	constant
$\pi_2 = \dfrac{\rho v_1^2}{2p_0}$	[–]	constant
$\pi_3 = \left[\dfrac{\pi_2}{p_t^* - 1}\right]^{\frac{1}{2}}$	[–]	constant
$\pi_4 = \left[\dfrac{2p_0}{b^{*2}k^2\rho}(p_t^* - 1)\right]^{\frac{1}{2}}$	[–]	constant
$\pi_5 = \dfrac{b^*}{d_h^*}\dfrac{\pi_2}{\pi_1}$	[–]	constant
L	[m]	duct length
$\dot{m}$	[kg/s]	air mass flow
m_0	[m]	material layer depth
p	[Pa]	pressure
$p^* = \dfrac{p}{p_0}$	[–]	dimensionless pressure
p_t	[Pa]	total pressure
Δp_{lb}	[Pa]	pressure drop of air distribution layer
Δp_m	[Pa]	pressure drop of material layer
$\Delta p = p - p_0$	[Pa]	air pressure drop
$\Delta p^* = p^* - 1$	[–]	dimensionless pressure drop
q	[m^3/s]	air volume flow
$q^* = \dfrac{q}{Av_1}$	[–]	non-dimensional air volume flow

$\mathrm{Re} = \dfrac{d_h v}{\nu}$	[–]	Reynolds number
v	[m/s]	air velocity in the distribution duct
$v^* = \dfrac{v}{v_1}$	[–]	dimensionless velocity
v_f	[m/s]	air flow velocity across the air distribution layer
$v_f^* = \dfrac{v_f}{v_1}$	[–]	dimensionless velocity
x	[m]	longitudinal coordinate
$x^* = \dfrac{x}{L}$	[–]	dimensionless coordinate
λ	[–]	friction coefficient
ρ	[kg/m^3]	air density
ν	[m^2/s]	kinematic viscosity

Subscripts

1 duct inlet
2 duct outlet

References

MARSCHALL, J. – PRESZLER, L.: Strömungstechnische Untersuchungen an Filterschlauchen. *Periodica Polytechnica Ser. Mechanical Engineering,* Vol. 27, No. 1–2, pp. 46–65 (1983).

MUSCHELKNAUTZ, E.: Die Berechnung der pneumatischen Fliessförderung. *Pneumatische und hydraulische Förderung. Kongress "Transmatic 76"* Karlsruhe, Section C. pp. 29–43.

TALLIÁN, A.: Hydraulic Dimensioning of a Densely Branching-off Distribution Duct. Doctor's Thesis, Budapest, 1982 (In Hungarian).

Address:

Dr. László KOVÁCS
Dr. Sándor VÁRADI
Department of Fluid Machinery
Technical University Budapest
H–1521 Hungary