PERIODICA POLYTECHNICA SER. MECH. ENG. VOL. 36, NO. 1, PP. 45–60 (1992)

CONTACT STATE AND STRESS ANALYSIS IN A KEY JOINT BY FEM

K. VÁRADI and D. M. VERGHESE

Institute of Machine Design
Technical University, H–1521 Budapest

Received: March 26, 1992

Abstract

The traditional way for selecting a key joint assumes a uniform pressure distribution between the shaft and key and the key and hub. In this paper the real pressure distributions have been evaluated furthermore the rigid body rotations of the shaft, key and hub fulfilling the equilibrium and geometric conditions of contact. The contact pressure distribution due to the initial interference fit between the shaft and key has also been calculated.

Keywords: contact problem, key joint, stress analysis, FEM.

Introduction

A key is a machinery component placed at the interface between a shaft and the hub of a power-transmitting element for the purpose of transmitting torque. The key is demountable to facilitate assembly and disassembly of the shaft system. It is installed in an axial groove machine into the shaft called a keyway or key seat. A similar groove in the hub of the power-transmitting element is usually the keyway.

General Assumptions

a) The displacements and deformations of the bodies in contact are assumed to be small.
b) The material of the bodies in contact is homogeneous and isotropic and obeys the Hooke's law. The material of the key, shaft and the hub is the same.
c) The force system distributed between the bodies in contact is normal and tangential, allowing dry friction.
d) Plane-strain state is assumed therefore displacements along the length of the key are ignored.
e) Only the major side faces of the hub and shaft are in contact with the key (i.e. there is no contact at the bottom of the keyway in the shaft

or the top of the keyway in the hub or the minor sides of the hub and shaft). The reason for this assumption will be discussed later.

f) The axis of the key is a line which lies in the middle of the shear plane and is parallel to the axis of shaft (*Fig. 1*).

Traditional Way for Selecting the Key

The key and the key seat for a particular application are usually designed after the shaft diameter is specified by methods commonly used for shafting [1], [2]. Then with the shaft diameter as a guide, the size of the key is selected from standards which give the key size for a range of shaft diameters. The only remaining variables are the length of the key and its material. One of these can be specified and the other can be computed.

Because of the many uncertainties an exact analysis of the stresses is not usually made. Engineers commonly assume that the entire torque is carried by a tangential force F located at the shaft surface (*Fig. 1*). That is

$$F = \frac{T}{\frac{D}{2}},$$

where D is the diameter of the shaft and
T is the torque transmitted by the shaft.

There are two basic modes of potential failure for keys transmitting power: shear across the shaft-hub interface and compression failure due to the bearing action between the sides of the key and the shaft or hub material. The analysis for either failure mode requires an understanding of the forces that act on the key. *Fig. 1* shows the idealized case.

The key is in direct shear over its section $W \cdot L$. The assumed uniform shear stress is

$$\tau = \frac{F}{A_s} = \frac{T}{\left(\frac{D}{2}\right)(W \cdot L)} = \frac{2T}{D \cdot W \cdot L}.$$

The assumed uniform compressional stress is

$$\sigma = \frac{F}{A_c} = \frac{T}{\left(\frac{D}{2}\right)(L)\left(\frac{H}{2}\right)} = \frac{4T}{D \cdot L \cdot H}.$$

where A_c is the bearing area.

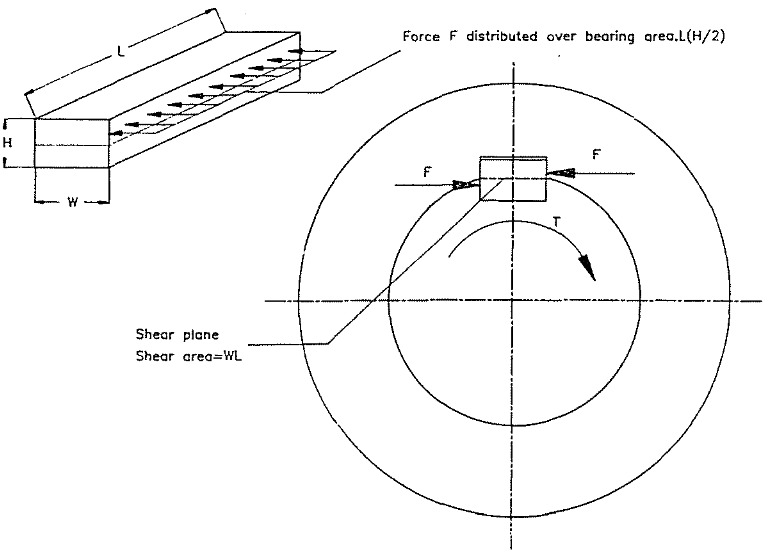

Fig. 1. Idealised force distribution during torque transmission

In typical industrial applications a safety factor of $N=3$ is used. This is a fairly high safety factor compared to for example 1.5 which is commonly used for engineering practice.

The reason for this is the *assumed uniform pressure* distribution between the shaft and key and the key and hub at their respective contact surfaces. Instead of the uniform contact pressure along the keyway, in the shaft there is a *non-uniform local contact stress distribution* located at the top of the keyway in the shaft. Between the hub and the key the problem is similar.

In our example we will analyze the following key joint:

shaft diameter	100 mm
hub outer diameter	175 mm
selected key size	16×28 mm
coefficient of friction	0.1
shaft-key fit	P9/h9
hub-key fit	J9/h9

The material parameters of the elements are as follows:

The modulus of elasticity	$E = 200000$ MPa
The Poisson's ratio	$\nu = 0.3$

Load Transmission in the Shaft-Key-Hub System

The load transmitted in the shaft-key-hub system is far more complicated than that assumed while selecting the key.

The behaviour of the key in the keyway is dependent on a number of factors such as: the initial clearance at the top of the key, the initial fit between the key and the key seat of the shaft, the geometry of the key, the torque being transmitted and the material of the three elements.

Now let us observe the real behavior of the key joint when a certain load is being transmitted by the shaft. The torque on the shaft creates a force on the left side of the key (*Figs. 1* and *2*). The key in turn exerts a force on the right side of the hub key seat. The reaction force of the hub back on the key and the force of the shaft on the key both produce a set of opposing forces that cause a couple. This couple tends to rotate the key.

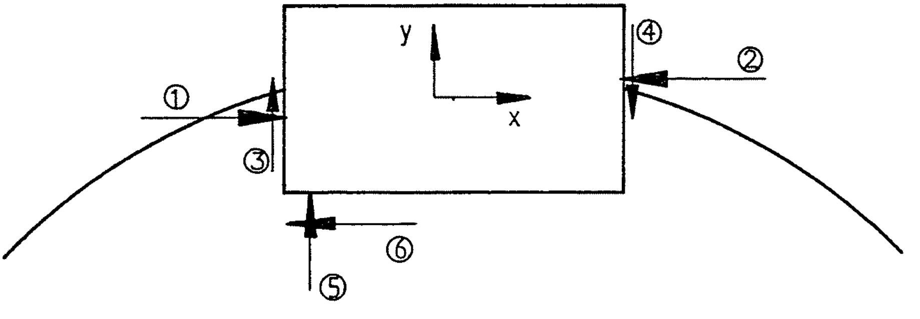

Fig. 2. Resultant forces acting on the key

Since the key is in equilibrium, the sum of the forces and moments on the key should be equal to zero. That is, there should be another force system to counter this couple. In reality there is a very important role played by friction due to these normal forces. The key is pressed into the keyway of the shaft, so that at the left side of the key there are two vertical forces. One is force 5 between the bottom of the key and the keyway in the shaft the other is force 3 (*Fig. 2*). The equilibrium equations are:

$$\sum F_x = 0\,,$$
$$\sum F_y = 0\,,$$
$$\sum M = 0\,.$$

According to the vertical equilibrium force 3 + 5 and force 4 are the same in magnitude. *Fig. 2* shows the equilibrium conditions of the key in this case. Force 6 is the friction force of force 5, so force 6 is the smallest force in the system. The dominant forces are force 1 and 2 and force 3 and 4. In the later analysis force 5 and 6 are ignored. We will only consider the case when the friction component is enough to keep the key in equilibrium. That is the torque transmitted by the key joint produces a couple on the key due to the normal forces which in this case is countered by frictional resistance arising between the surfaces of contact.

The direction of the normal forces is trivial. The direction of the friction forces can be clarified by considering the tendency of key rotation in the absence of these forces or by considering the relative motion of the surfaces in contact. These forces together keep the key in equilibrium and create a local contact pressure distribution. The task is to determine this non-uniform pressure distribution.

Numerical Solution of the Contact Problem

In this chapter a brief explanation of the solution process for the contact pressure distribution is given. The following points give the main requirements for achieving the real contact pressure distribution.

(a) the contact pressure distribution and the tangential force system in the two zones should keep the key in equilibrium,
(b) there should be zero gap between the elements in contact in the area where the pressure is applied,
(c) the torque that is transmitted by the shaft should be the same as that received by the hub.

Let's derive the geometric conditions of contact for the shaft-key-hub system [3], [4].

During torque transmission each element of the system not only deforms elastically but performs certain rigid body rotations as well (*Fig. 3*). For satisfying the geometric contact condition requirement it is clear that the three bodies must rotate with respect to each other. The relative position of the three elements as seen in *Fig. 3* illustrates the need for these rigid body rotations.

Let us introduce the following rigid body rotations:

ε_k: rigid body rotation of the key about its assumed axis of rotation,
ε_s: rigid body rotation of the shaft about its axis, ,
ε_h: rigid body rotation of the hub about its axis.

Due to the contact pressure distribution there are elastic deformations on both sides of the key and in the keyway of the shaft and the hub. These elastic displacements are obtained from finite element analysis (see later).

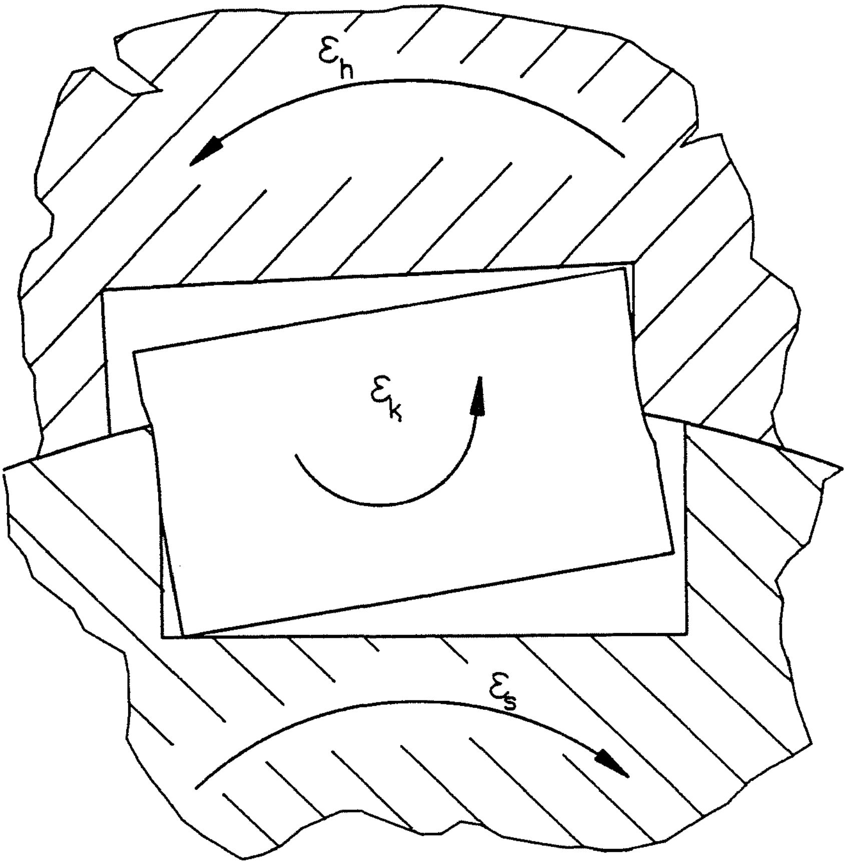

Fig. 3. The deformed elements with rigid body rotation. (The rotations and elastic deformations are enlarged)

The flow-chart in *Fig. 4* shows the process of iteration from which the real pressure distribution is obtained.

The applied force distribution fulfils the geometric contact conditions if the obtained length of the contact zone is the same as the assumed contact zone prescribed by the length of the contact pressure distribution.

The contact conditions should be checked over the two contact zones at the same time. If the obtained contact zone (from the rigid body rotations and the elastic displacements) is shorter than the assumed one, the length of the area having the force distribution (i. e. the length of the assumed contact zone) should be shortened and vice versa.

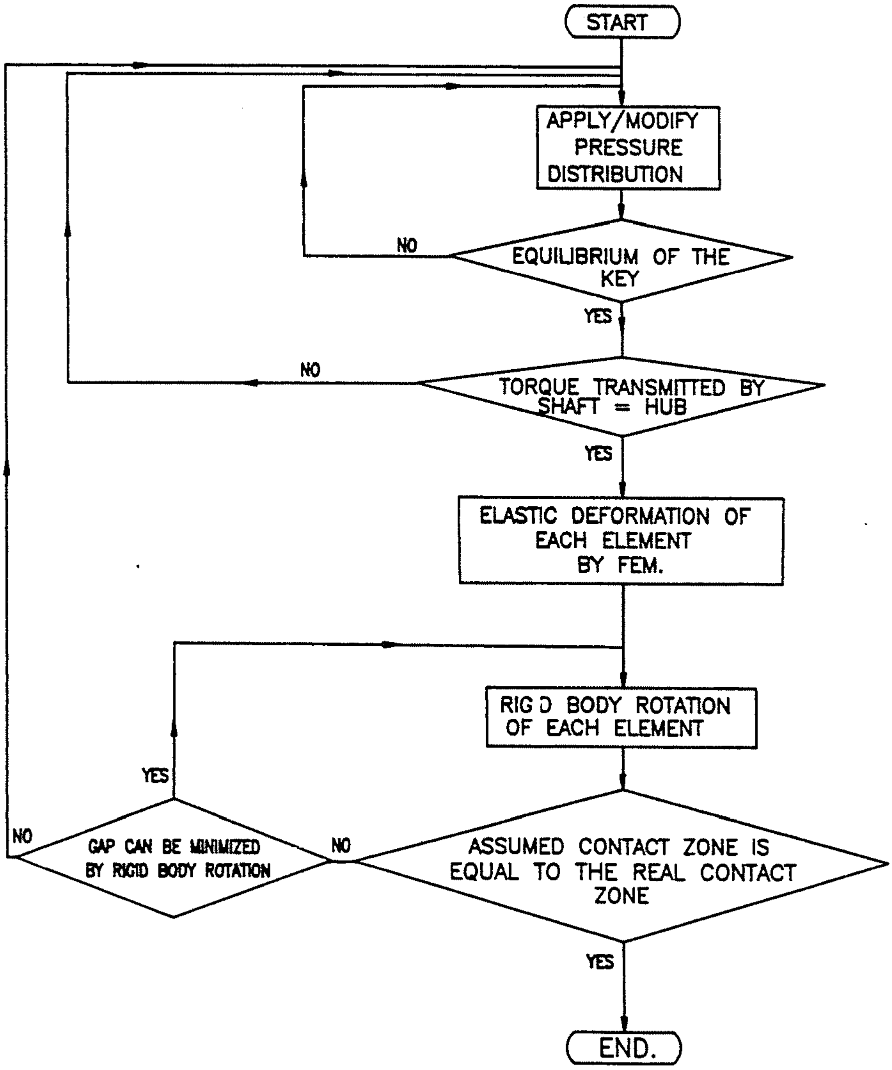

Fig. 4. Flowchart showing the solution process for pressure distribution

The Finite Element Models

The MTAB/SAP86 program system was used to create the FEM models of the key shaft and the hub.

The numerical solution requires working on the individual element models of the shaft-key-hub system separately (*Fig. 5*).

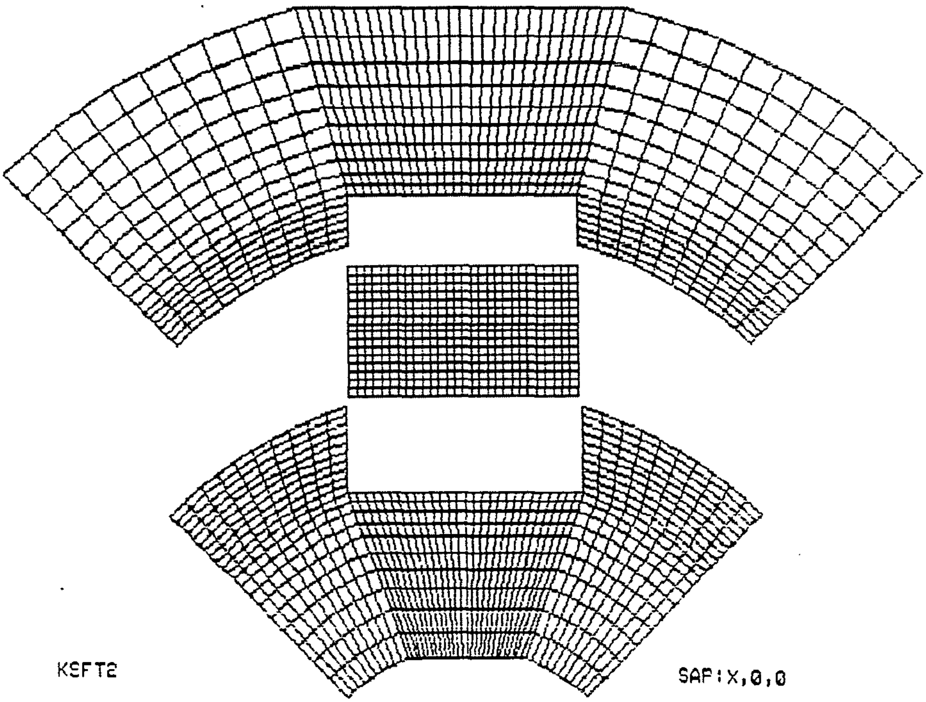

Fig. 5. Finite element mesh of the hub, key and the shaft

The model of the key shaft and hub as seen in *Fig. 5* are made up from a mesh of quadrilateral elements.

The mesh of the key consists of 493 nodes and 448 elements.

The model of the shaft is a section of the shaft close to the keyseat to satisfactorily analyse the displacements and stresses around the keyseat.

The mesh is made up from 680 elements and contains 759 nodes. The elements around the keyway are smaller than the ones towards the axis. This finer mesh gives an accurate stress distribution around the keyway. The model of the shaft is fixed at all boundary nodes. The mesh of the hub is similar to the mesh of the shaft in all aspects except the geometry. The mesh of the hub contains 600 elements and 671 nodes.

Contact Pressure Distribution and Contact Stresses in the Shaft-Key-Hub System

As discussed earlier, the pressure distribution of the shaft-key-hub system will be evaluated for the case when the key is loaded only on the two main contact zones. The pressure distribution on each of the two contact zones can be described as the resultant of two force systems. The first force system is the normal force system whose sum is 1000 N and the other is the tangential force system due to friction. Since the distribution of this force system is unknown initially, an arbitrary normal force distribution is applied with its corresponding tangential forces (frictional forces) to the two contact zones. The whole force system must satisfy the equilibrium conditions of the key. This force system is applied to the two contact zones on the shaft, key and hub models and then imposed on the finite element models. The output of the finite element analysis is the resultant deformations of the three models at their contact zones.

To achieve the requirement that a contact zone with zero gap must exist we have to apply rigid body rotations to the three elements.

The elements are rotated and by a process of trial and error we try to eliminate the gap on both surfaces of contact.

After a number of iterations we arrive at the zero gap state and this gives the real pressure distribution. *Fig. 6* shows the final position of the three elements. The final rigid body rotations are $\varepsilon_s = 0.055$, $\varepsilon_k = 0.25$ and $\varepsilon_h = 0.028$ corresponding to the shaft, key and hub, respectively. The torque being transmitted is 47.47 Nm.

It must be noted that this pressure distribution and contact state corresponds to a specific torque applied by the shaft. In reality the torque to be transmitted is given rather than the sum of the forces. But fixing the forces to 1000 N simplifies the problem. The final pressure distribution is shown in *Fig. 7*.

Now that the final pressure distribution is achieved, the stresses in the shaft-key-hub system can be obtained through finite element analysis. The input parameters are the pressure distribution, the geometry and material data.

The horizontal stress for the shaft, key and hub are plotted in *Fig. 8*. They also show the resulting elastic deformations. The horizontal stress distributions represent high local compression in the key and local bending in the shaft and hub.

The Von Mises stress patterns can be seen in *Fig. 9* showing the stresses in the shaft, key and hub.

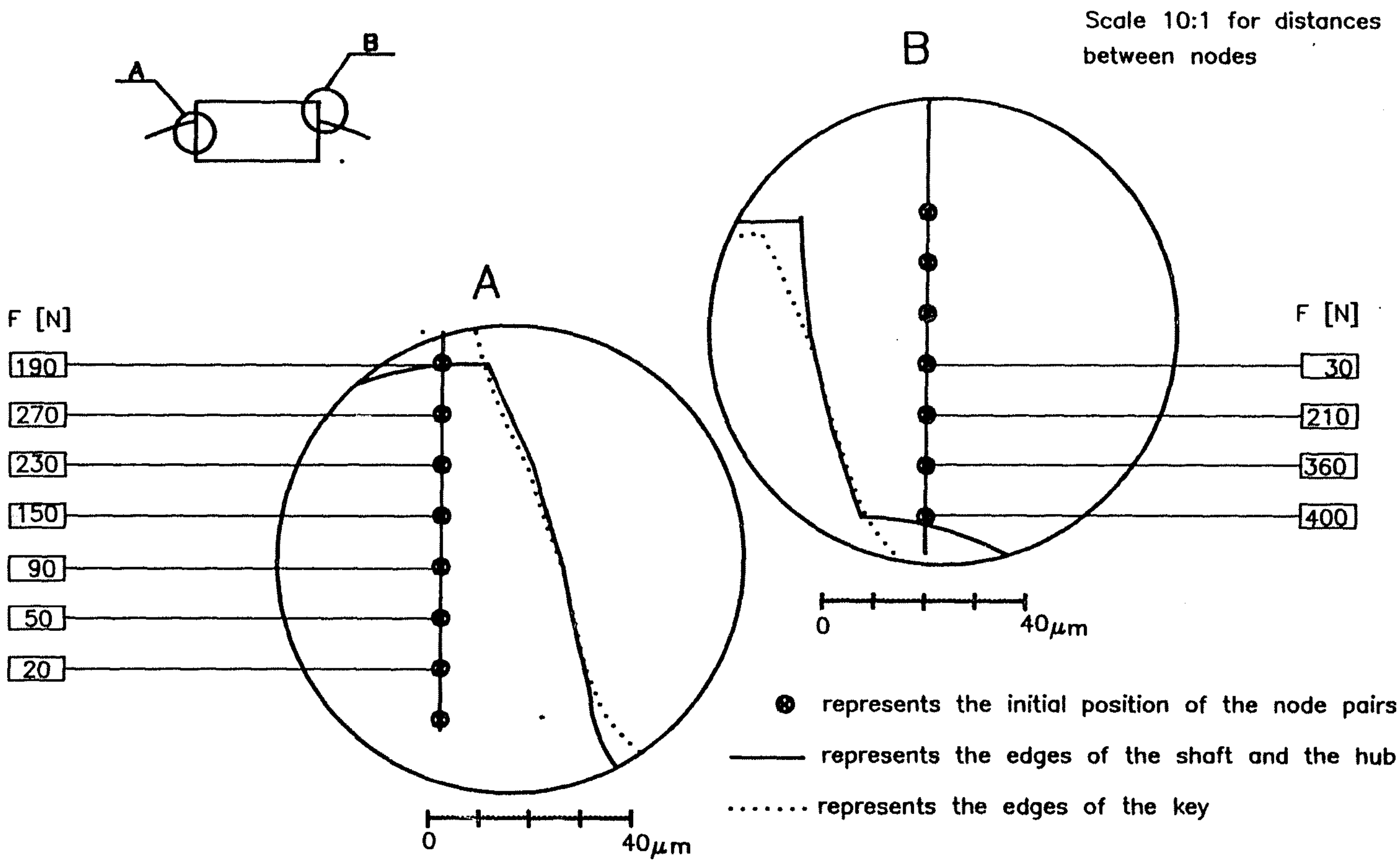

Fig. 6. Figure showing the final positions of the three elements and the magnitude of the normal forces

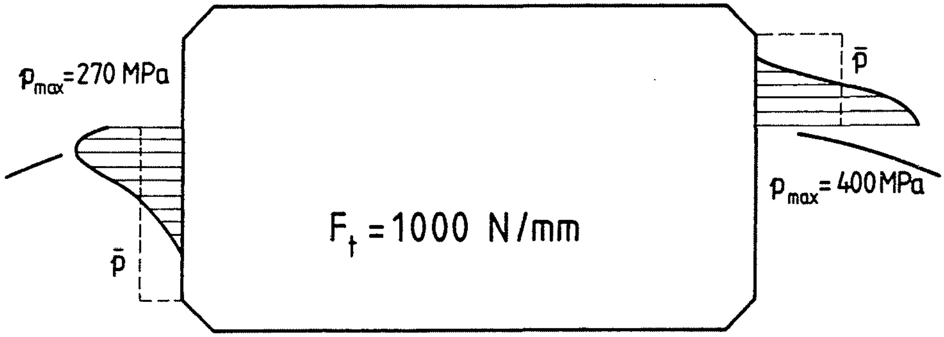

Fig. 7. Pressure distribution on the key (p is the average pressure)

Contact State of the Shaft and Key for the Initial Interference Fit

The interference fit between the keyway of the shaft and the key plays an important role in the behavior of the shaft-key-hub system.

For the standard key that is being investigated the interference fit between the key and the shaft is P9/h9.

For an interference fit of 28 P9/h9 the mean interference is 22 μm which exists between the width of the key and the key seat of the shaft horizontally. If we consider one side, that interference is 11 μm.

Therefore after assembly due to elastic deformation, the sum of the displacements of the node pairs (belonging to the shaft and key, respectively) should be equal to 11 μm. This is the key idea to solve for the contact pressure distribution.

The FEM models used here are the same models of the shaft and key used for torque transmission.

The iteration starts with an initial arbitrary pressure distribution.

This pressure distribution is applied to the proper sides of the shaft and the key. The output of the finite element calculation is the elastic displacements of the node pairs. The sum of the displacements of each node pair is calculated. Comparing the calculated displacements to the required total displacement of 11 μm we can modify the contact pressure distribution so that the sum of the displacements of the node pairs reaches the 11 μm. The final pressure distribution is shown in *Fig. 10.*

Now let us examine the stress in the shaft and key due to this pressure distribution. The horizontal stress patterns can be seen in *Fig. 11* and *Fig. 12* shows the Von Mises stress patterns.

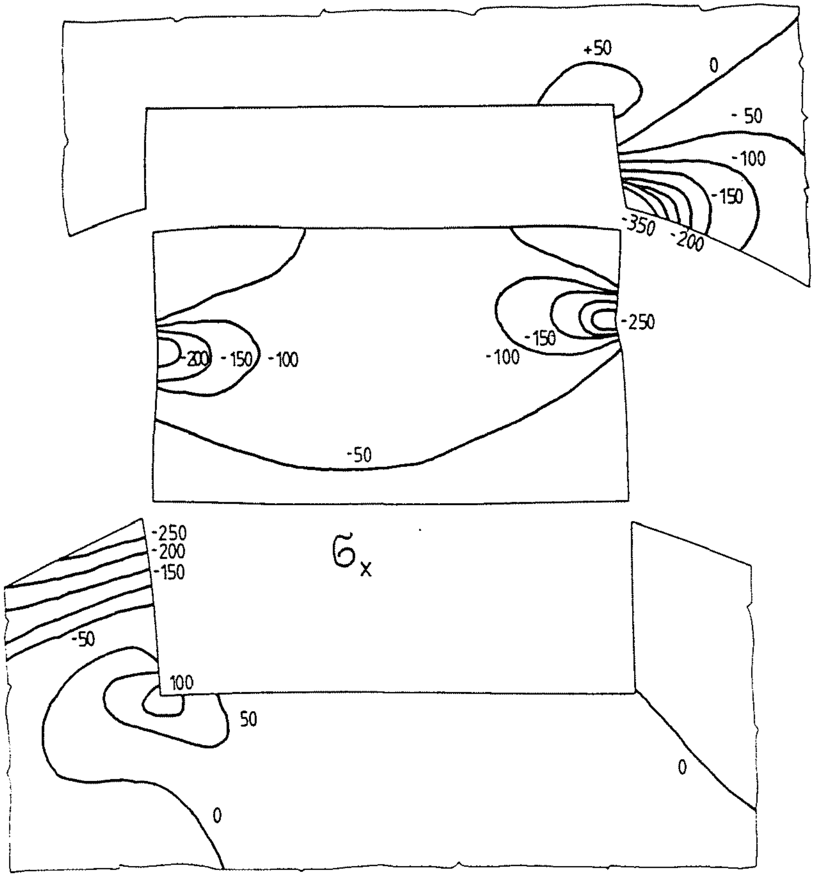

Fig. 8. Horizontal stresses in the shaft, key and hub due to torque transmission

No force was applied to the last node because the end of the key is usually chamfered. Therefore the last node to be loaded is the second node from the bottom of the keyway.

The stress level due to the interference fit is moderate relative to the stress distribution due to the torque transmission.

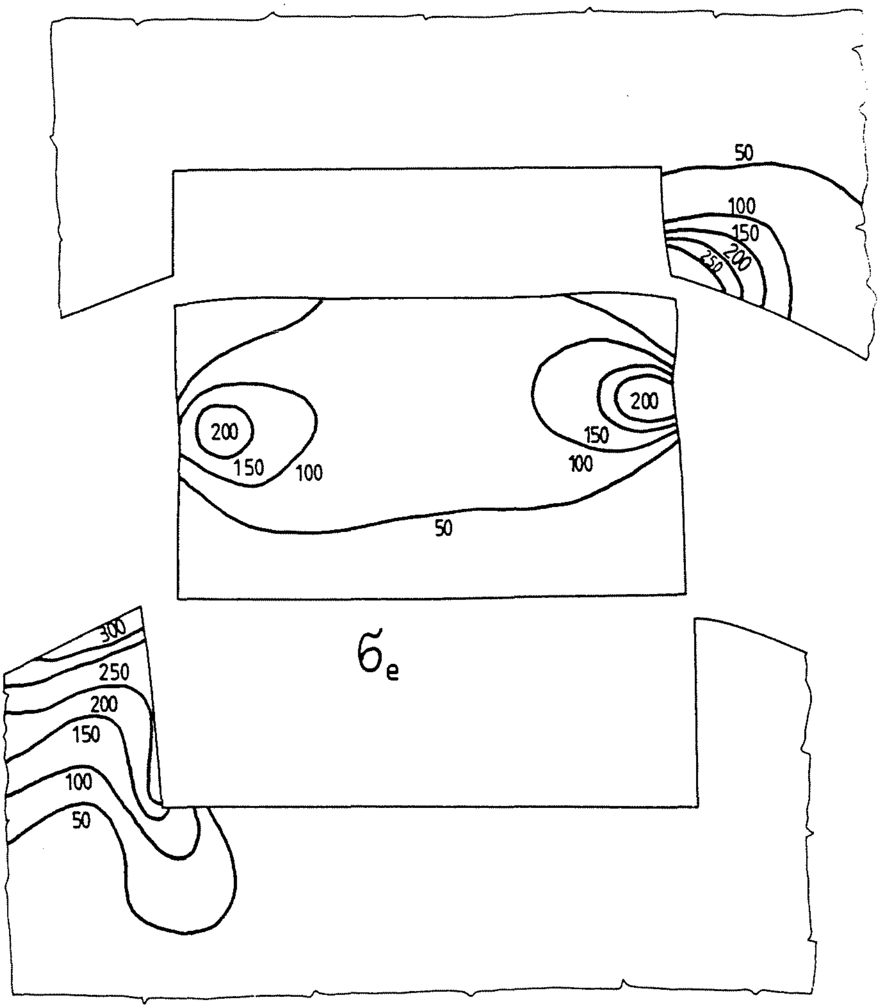

Fig. 9. Von Mises stresses in the'shaft, kry and hub due to torque transmission

Concluding Remarks

The results show clearly that the procedure used for determining the pressure distribution is viable and may be used for further analysis of the key joint.

The traditional way for selecting the key is highly approximate and the assumptions are far too many, especially assuming uniform pressure distribution between the shaft and key and hub and key, respectively. This

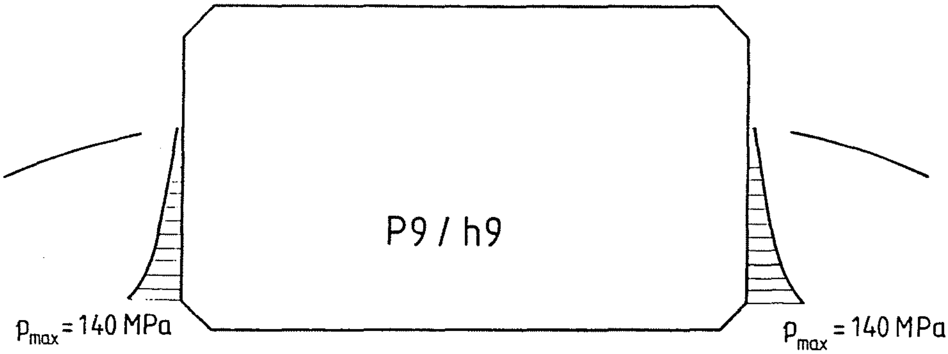

Fig. 10. The pressure distribution due to the interference fit

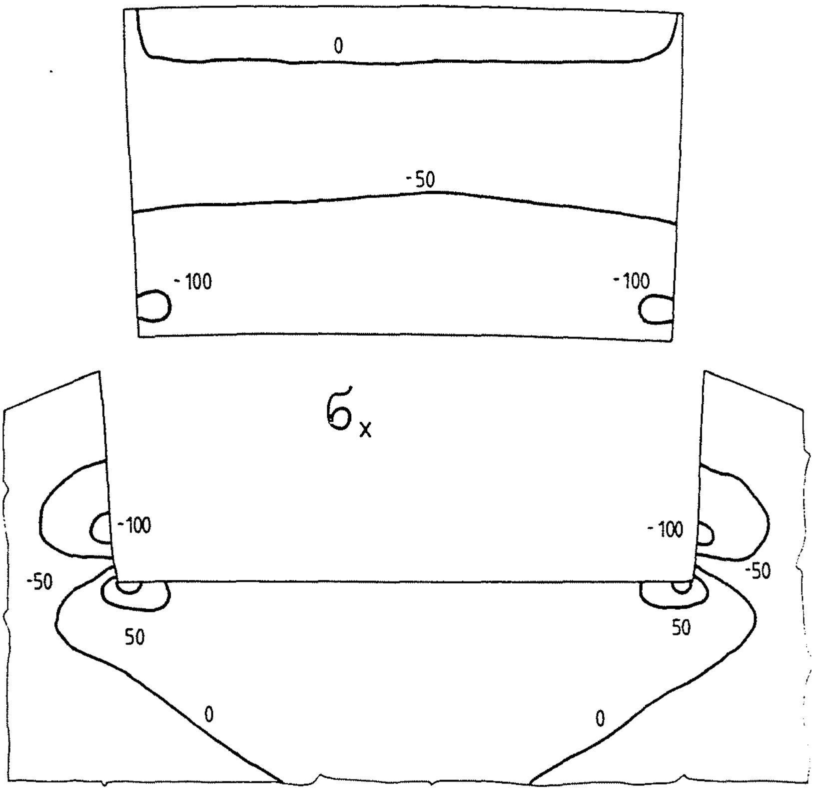

Fig. 11. Horizontal stresses in the shaft and key due to the interference fit

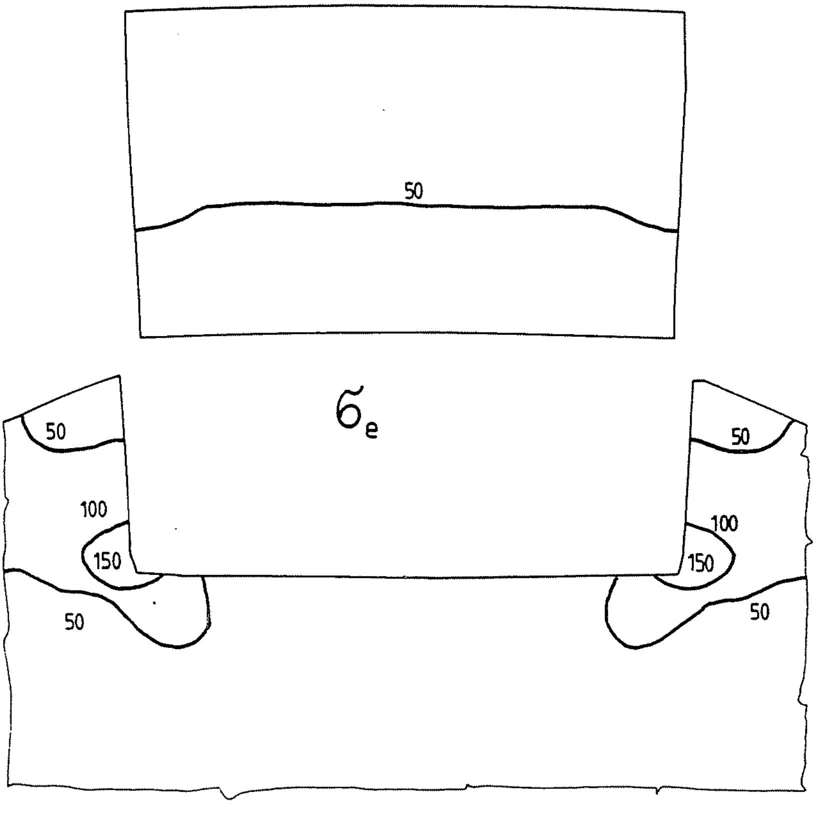

Fig. 12. Von Mises stresses in the shaft and key due to the interference fit

is the reason of the widely used high safety factor ($N = 3$). (See the real and the uniform pressure distributions in *Fig. 7*).

In reality there are a number of factors that affect the stress state of the key and keyway. Taking into consideration all these factors is far from the scope of this project but our intention was to initiate a process whereby the real stress distribution on the key is considered while selecting the key.

References

1. MOTT, L. C.: Machine Elements in Mechanical Engineering, Merrill, 1990.
2. SPOTTS, M. F.: Design of Machine Elements, Prentice Hall of India pvt., 1988.
3. VÁRADI, K. – POLLER, R.: Analysis of the Geometrical Condition of Contact, *Periodica Polytechnica, Mechanical Engineering*, Vol. 32, No. 1, (1988), pp. 37–49.
4. VÁRADI, K. – POLLER, R.: Analysis of Gear Teeth Contact by the Finite Element Method, *Acta Technica, Academy of Sciences Hungary*, Vol. 101, No. 4, 1988.

Addresses:

Dr. Károly VÁRADI
Institute of Machine Design
Technical University
H–1521 Budapest, Hungary

D. M. VERGHESE
(former postgraduate student)
India