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EFFECT OF THE DEFORMATION OF YARN CROSS SECTION ON KNITTED BASIC FABRIC OF REGULAR STRUCTURE

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The basic structural element of knitted fabrics is the loop. The different fabrics are built up by connections of loops. A number of researchers have dealt with the examination of loops and with the possibilities of their connections using the considerations of fabric geometry.

In the case of this method accepting the yarn properties measured by instruments and based on the position and connection of the yarn parts assumed as a flexible cylindrical body long compared to the dimensions of its cross section each property of fabrics is determined a priori.

The statements obtained on the basis of fabric geometry undoubtedly involve a certain inaccuracy because the idealization of the real structure of fabric is unavoidable. If the fabric is considered as simple connections of bodies independent of the physical-mechanical properties of fabrics, the stress state of fabric is not taken into consideration, which comes into being at connection places of loops because of the elastic and plastic deformations of the yarn during forming that into loop. It is influenced by the friction and varies in the function of time. In comparison with the reality all of these can lead to essential differences in relation to the real situation.

The objections to the method and approach of fabric geometry are reasoned to a certain extent. There cannot be doubted the modifying effect of the difference between the assumed, and the real fabric structures and the necessity of taking it into consideration.

In spite of that it would be incorrect to undervalue the method of fabric geometry because based on that it succeeded in finding such theoretical relationships expressed mathematically, which are very useful both for producing fabrics and for considering the usage properties of fabrics.

The better and more reliable the results are the smaller the difference between the real and idealized fabric structures is and the more properties of yarn are taken into consideration.

DELIDOVICH, A. C. (1954), the founder of knitted fabric geometry in the USSR, considered e. g. the elongation of yarn in tighten knitted fabric and the reduction of diameter owing to the elongation (Poisson's effect) but he took the cross-section of yarn in fabric as round. This assumption does not square with the reality [1].

The yarns built up of filaments and especially the so-called spun yarns are flattened in consequence of connection of significant stress with each other during knitting and their cross-sections originally round deform into approximate ellipse. So the yarn takes up ribbon-like shape in every loop ribbon. This arrangement like a ribbon influences the mechanical properties of fabric.

Assuming the constant round cross-section of yarn in loop with regard to the so-called regular structure of weft-knitted basic fabric the English researcher Doyle did statements considered classical, which concern the ratio of the loop dimensions and the yarn diameter depending on the yarn count. Taking the deformations of yarn cross-sections into account, these relationships are in need for correction [2].

Suppose that the originally round cross-sections of yarn deform into ellipses with the traverse axis δ_b and conjugate axis δ_a then the half-circles of the needle loop and the sinker loop constructed of yarn mean lines deform into half-ellipses in consequence of increase of yarn cross-section in direction of loop column.

So the mean line of the yarn forming loop is composed of one half-ellipse and two quarter-ellipses linked with the straight-line loop legs. *Fig. 1* illustrates that showing the loop in 3 views.

For determining the parameters of fabric structure in the case of ribbon-like arrangement of yarn in loop it is necessary to know the initial undeformed yarn diameter δ, from which the axis of the elliptical yarn cross-section can be calculated using the following relations:

$$\frac{\delta_b \delta_a}{4}\pi = \frac{\delta^2 \pi}{4}\,, \tag{1}$$

$$\delta_b + \delta_a + \Delta_\delta = v_k\,. \tag{2}$$

Eq. 1 expresses that the cross-section area of yarn does not change after the deformation. *Eq.* 2 gives, however, the relationship between the two ellipse axes and the fabric thickness. The fabric thickness is to be measured by instrument beside standard surface load. Δ_δ in relationship (2) depends on the yarn count. In case of spun yarns with yarn count $N_m = 16/1 - 50/1$ the value of Δ_δ can be $2 \cdot 10^{-2} - 4 \cdot 10^{-2}$ mm. About its development *Fig. 1* gives information.

Owing to the changing of cross-sections along the yarn in the loop the tensed loop legs with elliptical cross-sections do not fill in completely the needle loop. So in the fabric with the so-called regular structure the

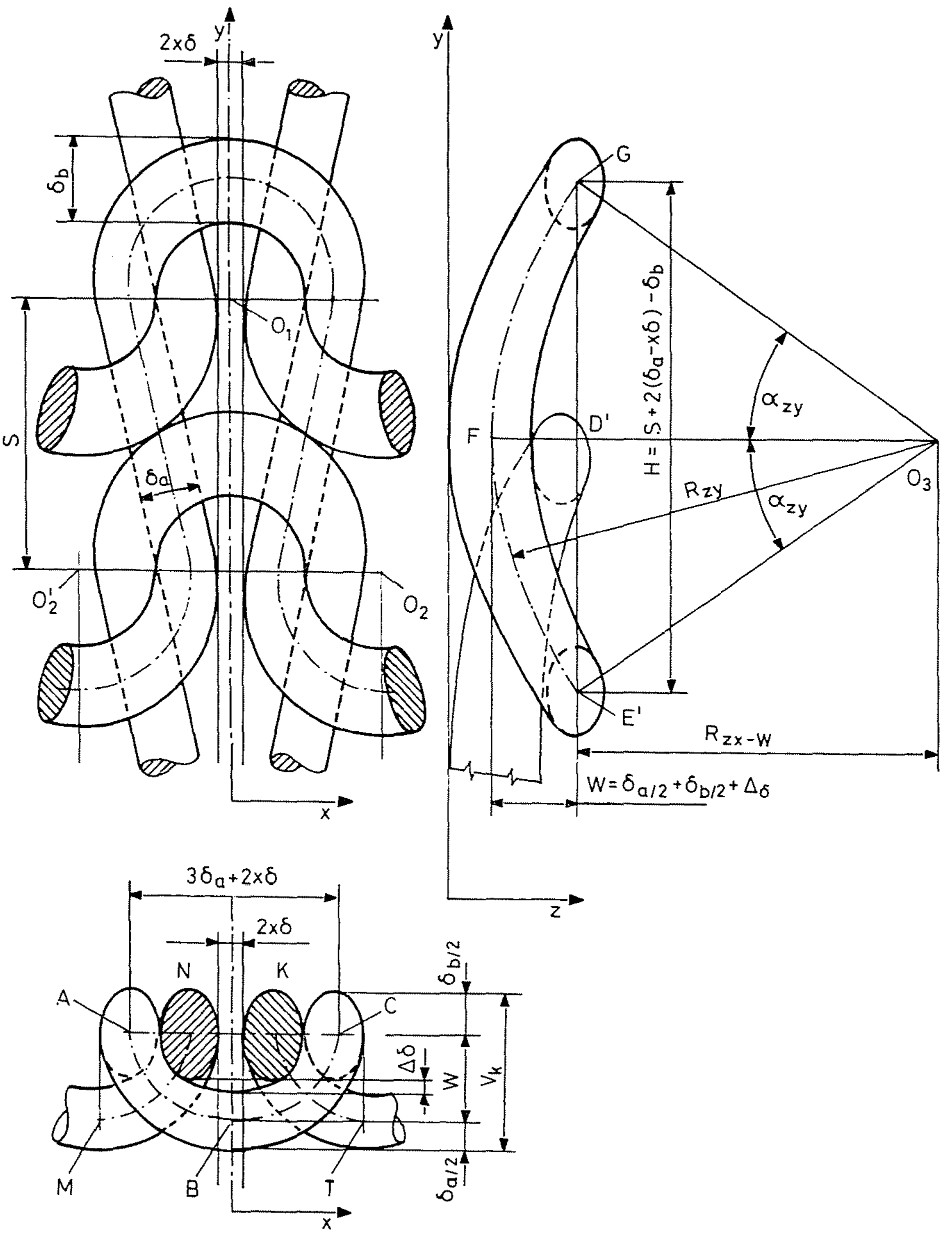
2xδ
y
x
δb
O1
S
δa
O'2
O2
y
G
H = S + 2(δa - xδ) - δb
αzy
F
D'
Rzy
O3
E'
Rzx - W
W = δa/2 + δb/2 + Δδ
z
3δa + 2xδ
2xδ
N
K
A
C
δb/2
Δδ
W
Vk
M
B
T
δa/2
x

Fig. 1.

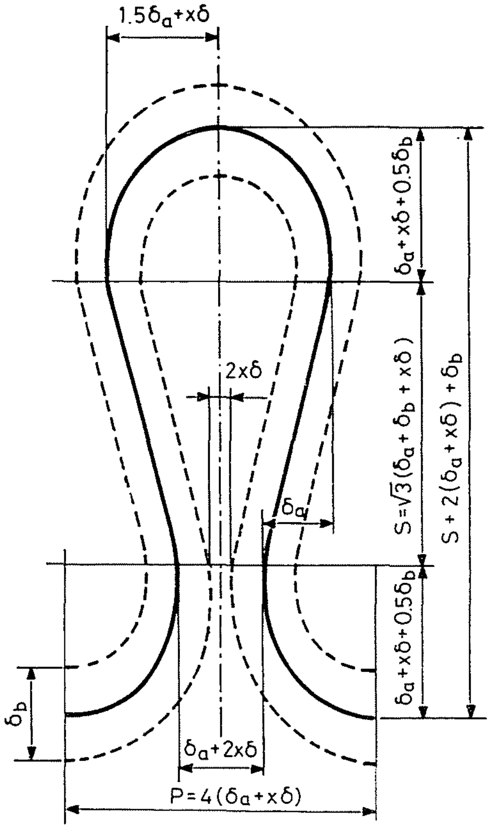

Fig. 2.

clearance[1] with a value of $2x\delta$ referred to the yarn diameter comes into being between the two loop legs converging on the needle loop.

The clearance $2x\delta$ can be computed from the assumption that the length of loops does not change in spite of their ribbon-like arrangement. If the loop legs do not join tangentially the needle loops and the sinker loops become elliptical like in *Fig. 2,* then from the point of view of loop length an inaccuracy of negligible magnitude is made, which will be proved later. So according to *Fig. 1* the length of yarn line is:

$$16.64\,\delta = \left[(1.5\,\delta_a + x\delta) + \left(\delta_a + x\delta + \frac{\delta_b}{2}\right)\right]\pi + 2\sqrt{S^2 + \delta_a^2}\,. \qquad (3)$$

[1] In general, this clearance disappears by means of the relaxation process during the finishing operations.

δ_a and δ_b determined by means of *Eqs.* (1) and (2) can be expressed with the proportions of the undeformed yarn diameter, that is:

$$\delta_a = \varepsilon\delta \qquad \text{and} \qquad \delta_b = \eta\delta\,.$$

Substituting them into *Eq.* 3, we obtain

$$16.64\,\delta = \left[(1.5\,\varepsilon\delta + x\delta) + \left(\varepsilon\delta + x\delta + \frac{\eta}{2}\delta\right)\right]\pi + \frac{2\delta}{\delta}\sqrt{S^2+\varepsilon^2\delta^2}\,. \tag{4}$$

After reducing and dividing that by δ

$$16.64 = \left(2.5\,\varepsilon + \frac{\eta}{2}\right)\pi + 2x\pi + \frac{2}{\delta}\sqrt{S^2+\varepsilon^2\delta^2}\,. \tag{5}$$

The distance of loop rows can be obtained on the basis of the condition for touch of ellipses of needle loop and sinker loop. This assumption is reasoned because the largest extension can be just found parallel to the plain of fabric. The conditions of touch shows *Fig. 3,* where the connection of external ellipses is magnified from *Fig. 1.*

Considering the denotations in *Fig. 3,* it can be written on the basis of fundamental relationships for the ellipses that:

$$r_1 + r_2 = b_k\,, \qquad F = \frac{\sqrt{b_k^2 - a_k^2}}{2}$$

and the axes:

$$b_k = 2(\delta_b + \delta_a + x\delta)\,, \qquad a_k = 2(2\delta_a + x\delta)\,.$$

Since the touching ellipses with the same parameter have a common tangent, on the basis of *Fig. 3* it follows from the right-angled triangles O_4O_2M and O_1NO_3, that:

$$(2r_2)^2 = (S+2F)^2 + \left(\frac{a_k}{2}\right)^2\,, \tag{6}$$

$$(2r_1)^2 = (S-2F)^2 + \left(\frac{a_k}{2}\right)^2\,. \tag{7}$$

So eliminating r_2 the density of loop rows can be calculated because *Eq.* 6 can be formed as follows:

$$[2(b_k - r_1)]^2 = (S+2F)^2 + \frac{a_k^2}{4}\,,$$

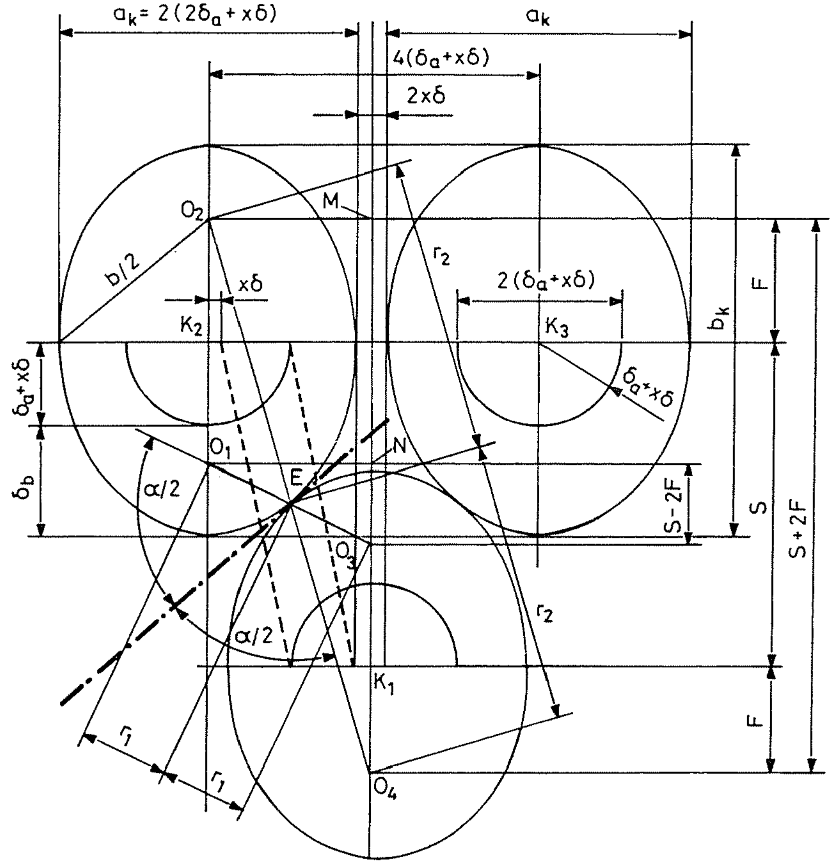

Fig. 3.

if performing the operations given in the left side, we obtain:

$$b_k^2 - 2b_k r_1 + r_1^2 = \frac{(S+2F)^2}{4} + \frac{a_k^2}{16}. \tag{8}$$

Expressing the value of r_1 from *Eq.* 7:

$$r_1 = \frac{\sqrt{4(S-2F)^2 + a_k^2}}{4}.$$

After performing the operations and the possible reduction in the equation:

$$b_k^2 - \frac{1}{2} b_k \sqrt{4(S-2F)^2 + a_k^2} = 2SF \tag{9}$$

from that:

$$(b_k^2 - 2SF)^2 = \frac{b_k^2[4(S-2F)^2 + a_k^2]}{4}.$$

Doing the operations, the simplification and the rearrangement:

$$S^2(4F^2 - b_k^2) = 4b_k^2F^2 + \frac{a_k^2 b_k^2}{4} - b_k^4.$$

Consequently:

$$S = \sqrt{\frac{4b_k^2F^2 + \frac{a_k^2 b_k^2}{4} - b_k^4}{4F^2 - b_k^2}}.$$

Substituting the value of F depending on the yarn diameter:

$$S = \sqrt{\frac{4b_k^4 - 4a_k^2b_k^2 + a_k^2b_k^2 - 4b_k^4}{4b_k^2 - 4a_k^2 - 4b_k^2}}.$$

Thus

$$S = \frac{\sqrt{3}}{2} b_k. \tag{10}$$

If substituting the value of b_k then:

$$S = \frac{\sqrt{3}}{2} 2(\delta_b + \delta_a + x\delta) = \sqrt{3}(\delta_b + \delta_a + x\delta). \tag{11}$$

This result as a limit is identical with Doyle's formula because in the case of the yarn with round cross-section $\delta_b = \delta_a = \delta$ and $x = 0$, consequently $\delta_b + \delta_a + x\delta = 2\delta$ that is the known relationship for the distance of loop can be obtained.

So, rearranging formula (5) and substituting the value of S we arrive at the following result:

$$16.64 - \left(2.5\,\varepsilon + \frac{\eta}{2}\right)\pi - 2x\pi = \frac{2}{\delta}\sqrt{3(\delta_b + \delta_a + x\delta)^2 + \varepsilon^2\delta^2}.$$

Employing the notation $2B = 16.64 - (2.5\varepsilon + \eta/2)\pi$ and expressing δ_b and δ_a with their factors:

$$B - x\pi = \frac{1}{\delta}\sqrt{3(\eta\delta + \varepsilon\delta + x\delta)^2 + \varepsilon^2\delta^2},$$

from which

$$(B - x\pi)^2 = 3(\eta + \varepsilon + x)^2 + \varepsilon^2 ,$$

Performing the operations and rearranging the equation by x:

$$x^2(\pi^2 - 3) - x[2B\pi + 6(\eta + \varepsilon)] + B^2 - 3(\eta + \varepsilon)^2 - \varepsilon^2 = 0 ,$$

from that as a mixed quadratic equation we obtain if $-[2B\pi+6(\eta+\varepsilon)]=M$ and $B^2-3(\eta+\varepsilon)^2-\varepsilon^2=N$

$$x_{12} = \frac{-M \pm \sqrt{M^2 - 4(\pi^2 - 3)N}}{2(\pi^2 - 3)} .$$

In a particular case, e. g. if the count of the cotton yarn is 36/1, the yarn diameter in undeformed state $\delta=20.83\cdot 10^{-2}$ mm. The fabric thickness got by measurement $v_k=46\cdot 10^{-2}$ mm.

Suppose $\Delta_\delta = 3\cdot 10^{-2}$ mm we obtain $\delta_b=26.825\cdot 10^{-2}$ mm and $\delta_a = 16.175\cdot 10^{-2}$ mm and the factors are $\eta=1.287$ and $\varepsilon=0.776$.

Applying the previous data:

$$2B = 16.64 - (2.5 \cdot 0.776 + 0.6435)\pi = 8.528 ,$$

$$M = -39.16 \qquad \text{and} \qquad N = 4.812 ,$$

$$x_{12} = \frac{39.16 \pm \sqrt{1533.51 - 132.2}}{13.369}$$

$$= \frac{39.16 - 37.43}{13.369} = \frac{1.73}{13.369} = 0.1294 \cong 0.13.$$

It means that on the cross-section factors $\varepsilon = 0.776$ and $\eta = 1.287$ the clearance arising in the needle loop is $2x\delta = 0.26\,\delta$, namely it is about a quarter of the original yarn thickness.

The deformation of the yarn cross sections does not mean the change of number of the loops referred to a surface unit, e. g. 1 m^2 at the same time, because e. g. in the particular case just examined the area PS represented by one loop is on the basis of *Fig. 2,* as follows:

$$(\mathrm{PS})_{\mathrm{tor}} = 4(\delta_a + x\delta)\sqrt{3}(\delta_a + \delta_b + x\delta) =$$

$$= 4\sqrt{3}(0.776 + 0.13)(0.776 + 1.287 + 0.13)\delta^2 = 4\sqrt{3} \cdot 1.987\,\delta^2 \cong 8\sqrt{3}\,\delta^2 .$$

This is identical with the area represented by one loop for the fabric of regular structure, as well, because

$$(\mathrm{PS})_{\mathrm{szab}} = 4\delta\, 2\sqrt{3}\,\delta = 8\sqrt{3}\,\delta^2 .$$

Beside the morphological change the deformed cross-section arising in the loop causes the most essential deviation in the change of potential energy of the loop. It is the consequence of that the yarn of loop with deformed cross-section as a flexible fibre needs larger work for deformation, so the larger tension arising in it influences the relaxation of fabric which is an important property for knitted fabrics.

Potential Energy of the Loop

In order to show the deviation in the potential energy of loop D. Bernoulli's theorem referring to the potential energy of elastic fibres by that this is proportional to the following integral

$$\int_0^s \frac{\mathrm{d}s}{r^2} = \int_0^\alpha \frac{r\,\mathrm{d}\alpha}{r^2} = \int_0^\alpha \frac{\mathrm{d}\alpha}{r} = \frac{\alpha}{r} \tag{12}$$

and the elastic fibre strives to take the position for which the value of the expression above will be minimum [3].

On the basis of this theorem L. Euler determined the position of the elastic fibre with a number of diverse shapes. It can be shown that the Bernoulli's index is proportional to the bending work of the flexible fibres because the bending moment due to Navier is:

$$M = \frac{EI_x}{r} = \frac{1}{r}E\frac{\delta^4\pi}{64}$$

the bending work is, however:

$$L = \frac{M}{2}\alpha = \frac{1}{2}\frac{\alpha}{r}E\frac{\delta^4\pi}{64}\,.$$

From that it is clear that the bending work is different from the value referring to the potential energy only with the factor $\frac{1}{2}E\frac{\delta^4\pi}{64}$. Consequently the theorem set up a priori by Bernoulli is a kind of simplification of the Navier's theorem, where apart from the mechanical and physical properties of the elastic fibre (yarn) only the geometrical deformation resulted by the load is considered, but it is essentially proportional to the bending work. With respect to talking about a space curve the Bernoulli's index of only one loop of the knitted fabric can be computed using the principle of superposition.

Consider the space curve of the loop as if the 3 projections of that had come into being by 3 different actions of force being effective at the same

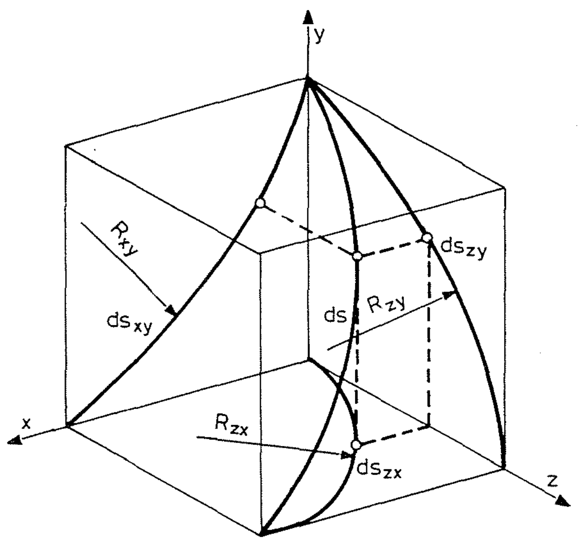

Fig. 4.

time. So the Bernoulli's index presents itself as a resultant of 3 components. Considering the *Fig. 4* the Bernoulli's index can be expressed as follows:

$$B_1 = \int \frac{\mathrm{d}s_{xy}}{R_{xy}^2} + \int \frac{\mathrm{d}s_{zy}}{R_{zy}^2} + \int \frac{\mathrm{d}s_{zx}}{R_{zx}^2} .$$

Since the arc lengths in the expression can be given with the angles and the curvature radii belonging to those, that is $\mathrm{d}s_{xy} = R_{xy}\alpha_{xy}$, $\mathrm{d}s_{zy} = R_{zy}\alpha_{zy}$ and $\mathrm{d}s_{zx} = R_{zx}\alpha_{zx}$ the Bernoulli's index can be obtained as below:

$$B_1 = \int \frac{\mathrm{d}\alpha_{xy}}{R_{xy}} + \int \frac{\mathrm{d}\alpha_{zy}}{R_{zy}} + \int \frac{\mathrm{d}\alpha_{zx}}{R_{zx}} . \tag{13}$$

Accordingly the 3 components of the Bernoulli's index are:

$$B_1 = B_{xy} + B_{zy} + B_{zx} .$$

In order to show the effect of deformation of yarn cross-section the 3 components above can be determined in the case of supposing the yarn in loop

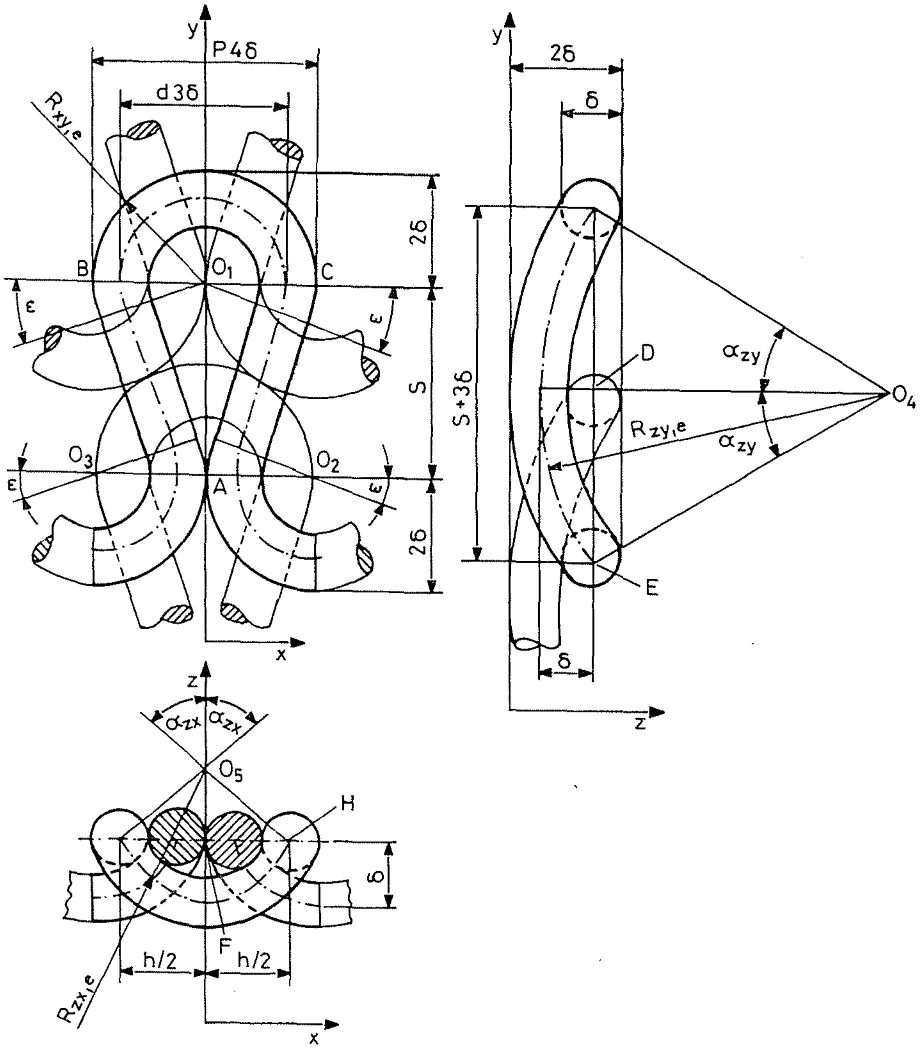

Fig. 5.

does not deform, namely it keeps its original round cross-section. So express the arc lengths and the curvature radii belonging to previous on the basis of view 3 in *Fig. 5.*

For the yarn with deformed cross-section it can be performed on the basis of *Fig. 1.* In both cases the calculation of component B_{xy} requires a different concept. Since e. g. connecting the loop legs to the circles of the needle loop and sinker loop not tangentially like e. g. in *Fig. 6,* then the potential energy of the 2 loop legs with length $\sqrt{S^2+\delta^2}$ will be zero because the curvature radius R belonging to these is ∞. In the case of connection tangentially like in *Fig. 7* above, the part of zero potential energy will be smaller because $a_0 < \sqrt{S^2+\delta^2}$.

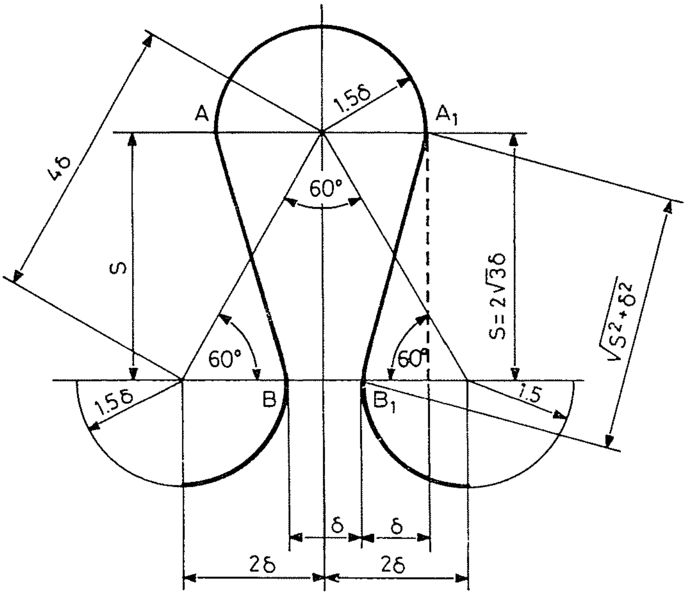

Fig. 6.

On the other hand, the potential energy of circles of the needle loop and sinker loop will grow larger on account of the increases of arc length belonging to the angles ε.

If connecting the loop legs to the needle loops and sinker loops not exactly tangentially like in *Fig. 6,* then it does not cause essential difference at calculating the length of yarn in the loop. The difference is sized by 0.001 mm. However, at computing the potential energy, as mentioned, it is already essential. So it is necessary to determine the position of the touch point belonging to the loop leg at the elliptical needle loops and sinker loops. In the case of the weft-knitted basic fabric of regular structure the

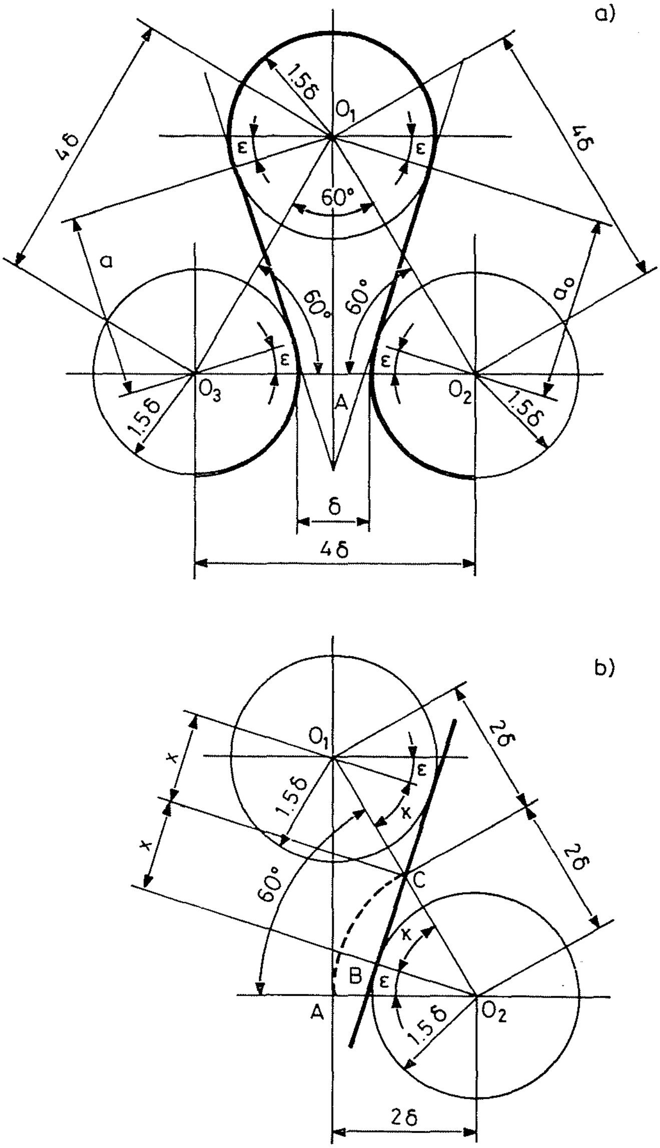
a)
1.5δ
O_1
ε
ε
60°
4δ
4δ
a
a_0
60°
60°
ε
ε
O_3
A
O_2
1.5δ
1.5δ
δ
4δ
b)
O_1
x
x
1.5δ
ε
x
2δ
2δ
60°
C
x
B
ε
A
O_2
1.5δ
2δ

Fig. 7.

increase of angle ε can be calculated on the basis of *Fig. 7.* Namely, using the notations in *Fig. 7:*

$$2x = a_0 \qquad \text{and} \qquad x = \sqrt{(2\,\delta)^2 - (1.5\,\delta)^2} = \delta\sqrt{1.75}\,.$$

From the triangle O_2BC:

$$\cos\kappa = \frac{1.5\,\delta}{2\delta} = 0.75\,, \qquad \text{and} \qquad \kappa = \arccos 0.75 = 41^\circ 24'\,,$$

$$\varepsilon = 60^\circ - \kappa = 18^\circ 36'\,, \qquad \varepsilon = 0.324631\,\text{rad}\,.$$

So the length of loop referred to the yarn diameter:

$$l = 1.5\,\delta(2\pi + 4\,\varepsilon) + 4\,\delta\sqrt{1.75} = 16.66\,\delta\,.$$

The methods of two kinds show just a difference of $0.02\,\delta$ in the length of loop referred to the yarn diameter. This difference showing in projection, to the plain of fabric is usually negligible because also the yarn diameter is some kind of fraction of one mm, but from the point of view of the potential energy represented by the fabric it is considerable.

This increase of angle for the needle loops and sinker loops deformed into ellipses shows itself in the arc length Δs of the ellipse. In order to determine that, the position of the tangent drawn from an external point to the ellipse must be calculated.

If pointing out the ellipses belonging together constructed of the mean line of yarn and drawing them in a position enlarged and turned round with 90°, so we get *Fig. 8* there, the common tangent e means the tangential connection of the loop leg to the needle loop and the sinker loop.

Since the ellipses can be consider identical the common tangent may be assumed to be drawn from the midpoint P_1 of the distance $\overline{O_1O_2}$.

The slope $m = \operatorname{tg}\omega$ of tangent e can be computed from the system of equations below:

$$y - y_1 = m(x - x_1)\,,$$
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\,.$$

From the second equation we obtain $y^2 = b^2 - kx^2$, where $k = \frac{b^2}{a^2}$. Substituting that into the first equation:

$$\sqrt{b^2 - kx^2} - y_1 = m(x - x_1)\,.$$

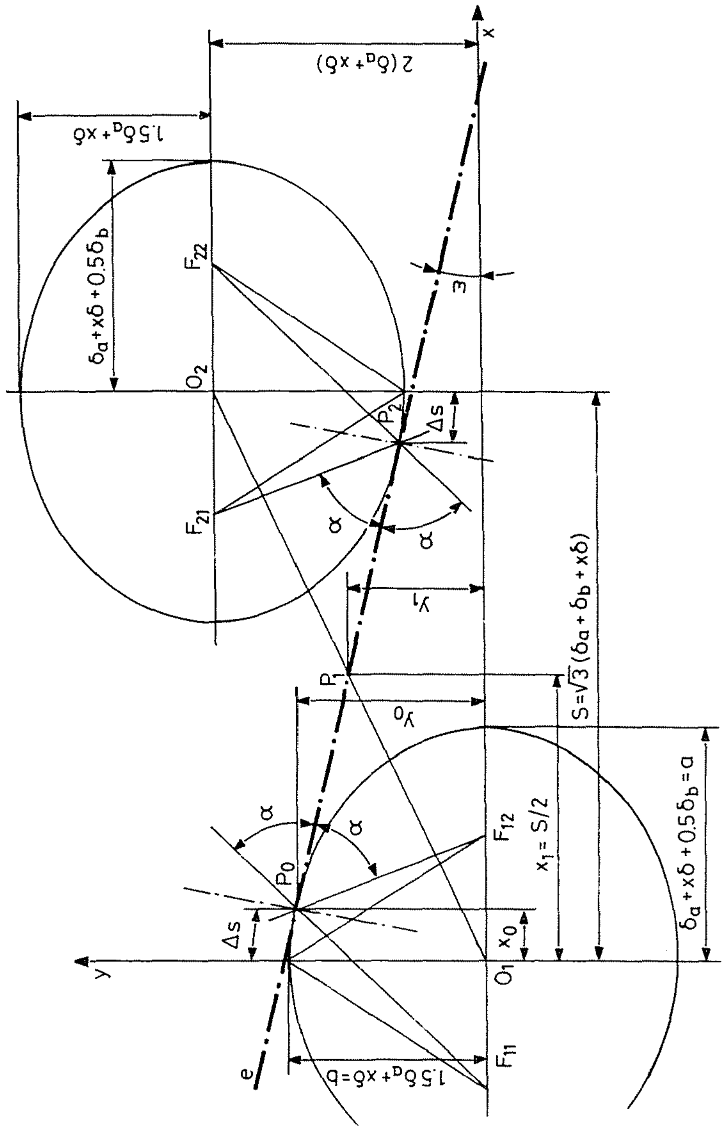

Fig. 8.

Raising that to the second power and rearranging it:

$$x^2(m^2+k) - x(2m^2x_1 - 2my_1) + m^2x_1^2 - 2mx_1y_1 - b^2 + y_1^2 = 0\,.$$

Since the tangent may have only one common point with the ellipse, the discriminant of the equation above must be zero. So it can be written that:

$$(2m^2x_1 - 2my_1)^2 - 4(m^2 + k)(m^2x_1^2 - 2mx_1y_1 - b^2 + y_1) = 0 .$$

Performing the operations and rearranging:

$$m^2(b^2 - kx_1^2) + m(2kx_1y_1) + k(b^2 - y_1^2) = 0 ,$$

from which:

$$m_{12} = \frac{-2kx_1y_1 \pm \sqrt{(2kx_1y_1)^2 - 4(b^2 - kx_1^2)(b^2 - y_1^2)k}}{2(b^2 - kx_1^2)} .$$

If it is about a yarn with the count of 36/1 and with the diameter $\delta = 0.2083\,\mathrm{mm}$, so substituting the numerical values of the quantities into the formula above, then:

$$\begin{aligned} b &= 1.5\,\delta_a + x\delta = 26.95 \cdot 10^{-2}\,\mathrm{mm}, \\ a &= \delta_a + x\delta + 0.5\,\delta_b = 32.276 \cdot 10^{-2}\,\mathrm{mm}, \\ x_1 &= \frac{\sqrt{3}}{2}(\delta_a + \delta_b + x\delta) = 39.56 \cdot 10^{-2}\,\mathrm{mm}, \\ y_1 &= \delta_a + x\delta = 18.872 \cdot 10^{-2}\,\mathrm{mm}, \\ \frac{b^2}{a^2} &= 0.6972 , \end{aligned}$$

$$m_{12} = \frac{-1034.64 \pm \sqrt{1034.64^2 + 4(762.3 - 1091.12) \cdot 0.6972 \cdot 370.15}}{2(726.3 - 0.6972 \cdot 1565)} ,$$

$$m_1 = \frac{-1034.64 + 1202.94}{-729.64} = -0.23066 , \qquad \omega = \arctan 0.23066 = 12.98^\circ .$$

Knowing the slope of the common tangent of the ellipses for the needle loop and the sinker loop, the coordinates of the touch point P_1 can already be calculated. Since x_0 and y_0 must satisfy the equations of both the tangent and the ellipse thus their values can be obtained from the equations below:

$$y_0 - y_1 = m(x_0 - x_1) ,$$

$$\frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} = 1 .$$

Expressing y_0 from the second equation and resubstituting it into the first one, we obtain:

$$\sqrt{b^2 - kx_0^2} = m(x_0 - x_1) + y_1 .$$

Substituting the value $m = -0.23$:

$$m^2 + k = 0.7504 \qquad \text{and} \qquad 2m^2 x_1 - 2my_1 = 12.9155 .$$

In the case of the constant slope therefore only one tangent can be drawn from the point $P(x_1, y_1)$ to the ellipse, so:

$$x_0 = \frac{2m^2 x_1 - 2my_1}{2(m+k)} = \frac{12.9155}{1.5008} = 8.606 .$$

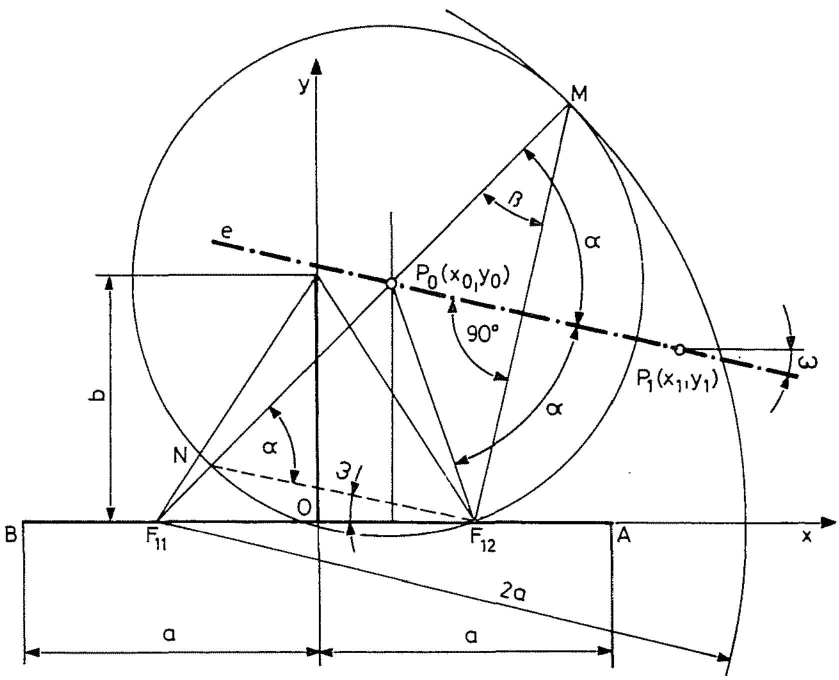

Fig. 9.

Substituting the value of x_0 into relationship $y_0 = \sqrt{b^2 - kx_0^2}$, then we get the value $y_0 = 25.98$. The coordinates x_0 and y_0 can be obtained by construction, as well. *Fig. 9* shows the construction in enlarged scale based on bisecting the angle of the guide rays belonging to the focuses F_{11} and F_{12}. By applicating the sinus-theorem several times the coordinates x_0 and y_0 can be calculated, as well.

The increase Δs of the arc length on account of the tangential connection of the loop legs as the arc length determined by the parameters t_1 and t_2 can be computed on the basis of the parametric equation of the ellipse, that is:

$$\Delta s = a \int_{t_1}^{t_2} \sqrt{1 - \varepsilon^2 \cos^2 t}\, dt\,,$$

where $\varepsilon = \frac{\sqrt{a^2-b^2}}{a}$ is the eccentricity of the ellipse. On the basis of the coordinates x_0 and y_0 of the touch point the parameter t_1 can be calculated, because

$$x_0 = a \cos t_1 \qquad \text{and} \qquad y_0 = b \sin t_1\,.$$

So

$$\operatorname{tg} t_1 = \frac{y_0 a}{x_0 b} = \frac{25.98 \cdot 32.276}{8.606 \cdot 26.95} = 3.615\,,$$

$$t_1 = \arctan 3.615 = 74.54^\circ = 74^\circ 32' = 1.3008\,\text{rad}\,.$$

Considering only the terms containing $\cos^2 t$ and $\cos^4 t$ in the formula of the ellipse for the arc length after developing the term with square root into series, after integration the arc length determined by parameters t_1 just calculated and $t_2 = \frac{\pi}{2}$ is the following:

$$\Delta s = a\left(\frac{\pi}{2} - t_1\right) - \frac{a\varepsilon^2}{4}\left[t + \frac{1}{2}\sin 2t\right]_{t_1}^{\frac{\pi}{2}} - \frac{a\varepsilon^4}{8}\left[\frac{3}{8}t + \frac{1}{4}\sin 2t + \frac{1}{32}\sin 4t\right]_{t_1}^{\frac{\pi}{2}}\,.$$

The value of the eccentricity:

$$\varepsilon = \frac{\sqrt{a^2 - b^2}}{a} = \frac{\sqrt{1041.74 - 726.30}}{32.276} = \frac{17.76}{32.276} = 0.5502\,,$$

$$\frac{a\varepsilon^2}{4} = 2.4425 \cdot 10^{-2} \qquad \text{and} \qquad \frac{a\varepsilon^4}{8} = 3.6966 \cdot 10^{-3}\,.$$

The values of the terms integrated above:

$$a\left(\frac{\pi}{2} - t_1\right) = 32.276 \cdot (1.5708 - 1.3008) = 8.71452 \cdot 10^{-2}\,,$$

$$\frac{a\varepsilon^2}{4}\left[t+\frac{1}{2}\sin\left(180^\circ-2t\right)\right]_{t_1}^{\frac{\pi}{2}}=3.2\cdot 10^{-4}\,,$$

$$\frac{a\varepsilon^4}{8}\left[\frac{3}{8}t+\frac{1}{4}\sin\left(180^\circ-2t\right)-\frac{1}{32}\sin\left(360^\circ-4t\right)\right]_{t_1}^{\frac{\pi}{2}}=0.0166\cdot 10^{-4}\,.$$

and with the help of these:

$$\Delta s=8.71452\cdot 10^{-2}-3.2\cdot 10^{-4}-0.0166\cdot 10^{-4}=8.68236\cdot 10^{-2}\,\mathrm{mm}\,.$$

Therefore the second and third terms do not already influence essentially the value of Δs.

Calculating the Bernoulli's Index of the Loop in the Case of Yarn Deformation

Considering the yarn deformation the Bernoulli's index of the space curve of loop can be computed on the basis of the view 3 in *Fig. 1* and the auxiliary *Fig. 10* using the principle of superposition, as follows:

a. Calculating the Component $B_{xy,t}$ Considering the Arc Increase Δs

By this component as it can be seen in the first picture in *Fig. 1* as well as in *Fig. 2* and *10* it is about the parts of elliptical arc length. The ellipse can be substituted with its so-called basket-curve with a good approximation from point of view of calculating the arc length, the advantage of which is that it gives the angle belonging to the arc length as well as the curvature radius. Considering *Fig. 10* the curvature radii of the substituting arcs of circle:

$$\rho_{1,xy}=\frac{a_{xy}^2}{b_{xy}}=\frac{1.5495^2\delta^2}{1.294\,\delta}=1.855\,\delta\,,$$

$$\rho_{2,xy}=\frac{b_{xy}^2}{a_{xy}}=\frac{1.294^2\delta^2}{1.5495\,\delta}=1.0806\,\delta\,.$$

The angle ξ_{xy} belonging to the radius $\rho_{1,xy}$:

$$\xi_{xy}=\arctan\frac{b_{xy}}{a_{xy}}=\arctan\frac{1.294\,\delta}{1.5495\,\delta}=\arctan 0.835\,,$$

$$\xi_{xy}=39.86^\circ=39^\circ 50'=0.695\,\mathrm{rad}\,.$$

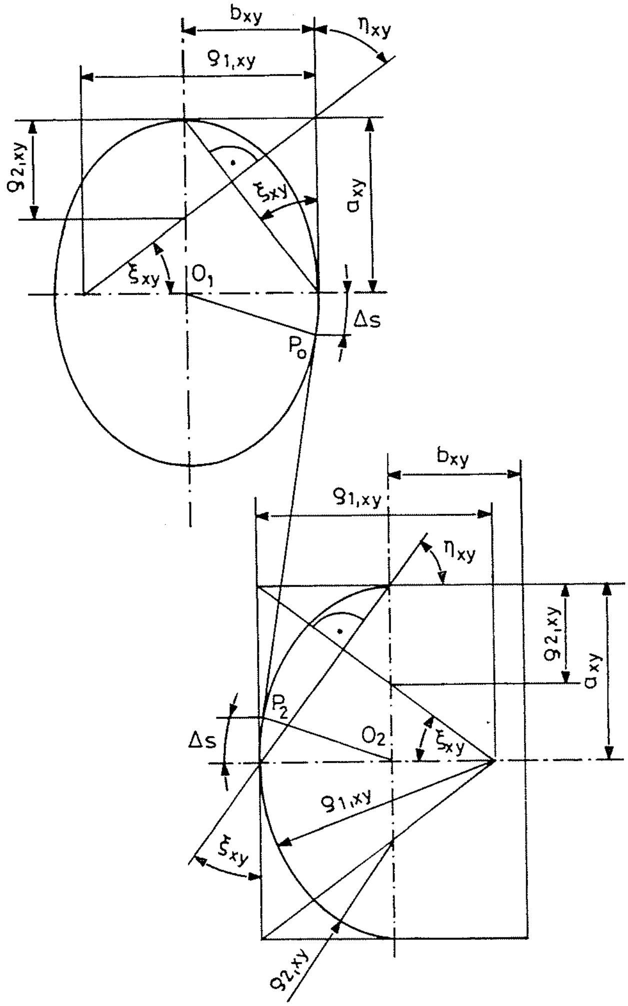

Fig. 10.

The angle η_{xy} belonging to the radius $\rho_{2.xy}$:

$$\eta_{xy} = 90° - \xi_{xy} = 50°10' = 0.8756\,\text{rad}\,.$$

So considering the increment Δs of arc length the component $B_{xy,t}$ is composed of 3 parts, which are:

$$B_{xy,t} = \frac{4\,\xi_{xy}}{\rho_{1,xy}} + \frac{4\,\eta_{xy}}{\rho_{2,xy}} + \frac{4\,\Delta s}{\rho^2_{1,xy}} = \frac{4\cdot 0.695}{1.855\,\delta} + \frac{4\cdot 0.8756}{1.0806\,\delta} + \frac{4\cdot 8.6824}{1.855^2\,\delta^2}\,.$$

The third term of that component can be made of the same kind if decomposing δ^2 and substituting the value $\delta = 20.83\cdot 10^{-2}$, that is:

$$\frac{4\cdot\Delta s}{\rho^2_{1,xy}} = \frac{4\cdot 8.6824\cdot 10^{-2}}{3.4\cdot 20.83\cdot 10^{-2}\delta} = 0.485\frac{1}{\delta}\,.$$

So

$$B_{xy,t} = (1.4986 + 3.2412 + 0.485)\frac{1}{\delta} = 5.2248\frac{1}{\delta}\,.$$

b. Calculating the Component $B_{zy,t}$

The component $B_{zy,t}$ can be computed on the basis of the side view in *Fig. 1*, as follows. Considering the mean line $E'FG$ of yarn as circle arc, so from the rectangular triangle $E'D'O_3$, we obtain:

$$R^2_{zy} = \left(\frac{H}{2}\right)^2 + (R_{zy} - w)^2\,,$$

from that:

$$R_{zy} = \frac{\left(\frac{H}{2}\right)^2 + w^2}{2w}\,.$$

Since

$$\frac{H}{2} = \frac{S + 2(\delta_a + x\delta) + \delta_b}{2} + \frac{3.8\,\delta + 3.099\,\delta}{2} = 3.5\,\delta$$

and

$$w = \frac{\delta_a}{2} + \frac{\delta_b}{2} + \Delta\delta = 1.1763\,\delta\,,$$

$$R_{zy} = \frac{\left(3.5^2 + 1.1763^2\right)\delta^2}{2\cdot 1.1763\,\delta} = 5.795\,\delta\,,$$

$$\alpha_{zy} = \arctan\frac{\dfrac{H}{2}}{R_{zy} - w} = \arctan 0.7578\,,$$

$$\alpha_{zy} = 43.41^\circ \qquad \text{and} \qquad \alpha_{zy} = 0.40206\,\text{rad}\,.$$

So the, Bernoulli's component:

$$B_{zy,t} = \frac{4\alpha_{zy}}{R_{zy}} = \frac{1.60824}{5.795\,\delta} = 0.2775\frac{1}{\delta}\,.$$

c. *Calculating the Bernoulli's Component $B_{zx,t}$*

Since it is all about an elliptical figure but without the increment Δs of arc the calculation of this component takes place in the same way as in the case of component $B_{xy,t}$ but the axis a_{zx} and b_{zx} are different. On the basis of the top view in *Fig. 1* the axis of the half-ellipse[2] ABC:

$$a_{zx} = \frac{3\,\delta_a + 2x\delta}{2} = \frac{(3 \cdot 0.776 + 0.26)\delta}{2} = 1.294\,\delta\,,$$
$$b_{zx} = w = 1.1763\,\delta\,.$$

So the radii belonging to the substituting basket-curve are:

$$\rho_{1,zx} = \frac{a_{zx}^2}{b_{zx}} = 1.4234\,\delta \qquad \text{and} \qquad \rho_{2,zx} = \frac{b_{zx}^2}{a_{zx}} = 1.0693\,\delta$$

and the angles belonging to those:

$$\xi_{zx} = \arctan\frac{b_{zx}}{a_{zx}} = \arctan 0.912 = 42.35^\circ\,, \qquad \xi = 0.7391\,\text{rad}\,,$$

$$\eta_{zx} = 90^\circ - 42.35^\circ = 47.65^\circ\,, \qquad \eta = 0.8315\,\text{rad}\,.$$

So the Bernoulli's component:

$$B_{zx,t} = \frac{4\xi_{zx}}{\rho_{1,zx}} + \frac{4\eta_{zx}}{\rho_{2,zx}} = (2.076 + 3.1103)\frac{1}{\delta} = 5.1863\frac{1}{\delta}\,.$$

Consequently the sum of the three components that is the Bernoulli's index proportional to the potential energy of loop considering the deformation of yarn is as follows:

$$B_{1,t} = B_{xy,t} + B_{zy,t} + B_{zx,t} = (5.2248 + 0.2775 + 5.1863)\frac{1}{\delta} = 10.69\frac{1}{\delta}\,. \quad (14)$$

Bernoulli's Index of the Loop without Yarn Deformation

To judge the effect of yarn deformation is necessary to compare the result above to the Bernoulli's index of an element of the fabric with regular structure made of the same yarn with the same compactness if the yarn in

[2] On the lower part of the loop leg the quarter-ellipse $\overline{MN}$ and $\overline{KT}$ connect to the half-ellipse ABC.

loop does not deform. Such a connection of loops is visible in three views in *Fig. 5*.

On the basis of the front view in *Fig. 5* considering the remarks added to *Fig. 7*, $B_{xy,e}$ forms as follows, since $R_{xy,e} = 1.5\,\delta$:

$$B_{xy,e} = \frac{2\pi + 4\varepsilon}{R_{xy,e}} = \frac{6.2832 + 1.2985}{1.5\,\delta} = 5.0545\frac{1}{\delta}.$$

On the basis of the side view in *Fig. 5* the curvature radius needed by the component $B_{zy,e}$ can be obtained from the triangle EDO_4 using the analogy of the method applied for $B_{zy,t}$ and the relationship $S = 2\sqrt{3}\,\delta$ valid for the fabric of regular structure:

$$R_{zy,e} = \frac{[0.5(S + 3\delta)]^2 + \delta^2}{2\delta} = \frac{\left[(2\sqrt{3} + 3)^2 + 4\right]\delta}{8} = 5723\,\delta.$$

The angle belonging to the curvature radius mentioned above:

$$\alpha_{zy} = \arccos\frac{R_{zy,e} - \delta}{R_{zy,e}} = \arccos 0.82526,$$

$$\alpha_{zy} = 34.23^\circ, \qquad \alpha_{zy} = 0.6\,\mathrm{rad}.$$

So the component $B_{zy,e}$ is:

$$B_{zy,e} = \frac{4\alpha_{zy}}{R_{zy,e}} = \frac{2.4}{5.723} = 0.4194\frac{1}{\delta}.$$

At last the curvature radius and the angle belonging to that which are necessary for the third component of the Bernoulli's index can be computed on the basis of the top view of *Fig. 5* and considering the relationship $\frac{h}{2} = 1.5\,\delta$

$$R_{zx,e} = \frac{\left(\frac{h}{2}\right)^2 + \delta^2}{2\delta} = \frac{(2.25 + 1)\delta^2}{2\delta} = 1.625\,\delta$$

and

$$\alpha_{zx} = \arccos\frac{R_{zx,e} - \delta}{R_{zx,e}} = \arccos 0.38461,$$

$$\alpha_{zx} = 67^\circ 23', \quad \text{and}\alpha_{zx} = 1.1761\,\mathrm{rad}.$$

So the third component is:

$$B_{zy,e} = \frac{4\alpha_{zx}}{R_{zy,e}} = \frac{4.704}{1.625\,\delta} = 2.8948\frac{1}{\delta}.$$

Therefore the entire Bernoulli's index of loop without deformation:

$$B_{1,e} = B_{xy,e}+B_{zy,e}+B_{zx,e} = (5.0545+0.4194+2.8948)\frac{1}{\delta} = 8.3687\frac{1}{\delta}\,. \quad (15)$$

As the results show, in consequence of the changed arrangement of yarn parts there is a considerable variation in the Bernoulli's indices referred to only one loop, because:

$$B_{1,t} - B_{1,e} = 10.69\frac{1}{\delta} - 8.37\frac{1}{\delta} = 2.32\frac{1}{\delta}\,, \quad (16)$$

which means an increment of 27.71%. Outside this difference the effect of inertial moments of the yarn cross-sections changed influences the potential energy of the fabric, as well.

Effect of Deformation of the Yarn Cross-Section on the Bending Work

Assuming the unchanged cross-section of yarn the same inertial moment can be taken into consideration for all of the three Bernoulli's components, thus the bending work referred to only one loop:

$$L_{e,lm} = \frac{1}{2}B_{l,e}E\frac{\delta^4\pi}{64}\,.$$

In the case of yarn deformation the inertial moment however depends on the direction of bending and the position of the yarn cross-section, therefore:

$$L_{xy,t} = \frac{1}{2}B_{xy,t}\,E\frac{\delta_b^3\delta_a\pi}{64}\,,$$

$$L_{zy,t} = \frac{1}{2}B_{zy,t}\,E\frac{\delta_b^3\delta_a\pi}{64}\,,$$

$$L_{zx,t} = \frac{1}{2}B_{zx,t}\,E\frac{\delta_b\delta_a^3\pi}{64}\,.$$

So the bending work referred to only one loop:

$$L_{l,t} = \frac{1}{2}E\frac{\pi}{64}\delta_b\delta_a\left[(B_{xy,t} + B_{zy,t})\,\delta_b^2 + \delta_a^2 B_{zx,t}\right]\,.$$

Since in the case of unchanged cross-section of yarn the bending moment can be written also in the following form:

$$L_{l,e} = \frac{1}{2}E\delta^2\frac{\pi}{64}B_{1,e}\delta^2$$

and according to our assumption $\delta^2 = \delta_b\delta_a$ so the rate of the two kinds of bending work forms as follows:

$$\frac{L_{l,t}}{L_{l,e}} = \frac{\delta_b^2\left(B_{xy,t} + B_{zy,t}\right) + \delta_a^2 B_{zx,t}}{\delta^2 B_{1,e}}\,.$$

Using the substitutions applied for the *Eq.* (4)

$$\frac{L_{l,t}}{L_{l,e}} = \frac{\eta^2\left(B_{xy,t} + B_{zy,t}\right) + \varepsilon^2 B_{zy,t}}{B_{1,e}} = . \tag{17}$$

Substituting the values obtained for the yarn with count 36/1 according to those $\eta = 1.287$ and $\varepsilon = 0.776$ as well as the numerical values of the Bernoulli's indices so we get:

$$\frac{L_{l,t}}{L_{l,e}} = \frac{1.287^2(5.2248 + 0.2775) + 0.776^2 \cdot 5.1863}{8.3687} = 1.4622\,.$$

Consequently considering the bending moment, as well, the potential energy of loop rose almost by 50%.

On the basis of knowledge of the potential energy of loop the informing data worth practice that is the potential energy of fabric per m^2 can be calculated.

E. g. the potential energy of weft-knitted fabric of regular structure made of cotton yarn with a count of 36/1 ($\delta = 20.83 \cdot 10^{-2}$ mm) and an elastic modulus $E = 150\,\mathrm{N/mm^2}$ is on the basis of the bending work:

$$L_{1m^2,e} = \frac{10^6}{PS}\frac{1}{2}B_{1,e}E\frac{\delta^4\pi}{64}\,.$$

Since for the regular structure $P = 4\,\delta$ and $S = 2\sqrt{3}\,\delta$, so:

$$L_{1m^2,e} = \frac{10^6}{8\sqrt{3}\,\delta^2}\frac{1}{2}8.3687 \cdot 150\frac{0.2083^3\pi}{64} = 462\,929.6\,\mathrm{Nmm}\,,$$

$$L_{lm^2,e} = 462.93\,\mathrm{J} = 0.11057\,\mathrm{kcal}\,.$$

The calculations presented in this paper give a proof related to that in the introduction that on the basis of the geometry of fabric there can be found such a property of the fabric which has a large influence on the usage worth of the fabric because this data worked out above connects tightly with the inclination of the fabric for deformation.

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