

## SCIENTIFIC ACTIVITIES OF THE STAFF OF THE DEPARTMENT OF GEOMETRY 1977 TO 1992

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The field of interest of this Department's staff covers primarily various chapters of geometry, involving both classic and recent trends. There are four main scopes such as axiomatic geometry and theory of construction; discrete and combinatorial geometry; differential geometry; and computer graphics.

Research in the first scope is hallmarked by Gyula Strommer, who essentially endeavoured at a possibly elementary analysis of the construction of Gaussian regular polygons (STROMMER [4]–[13]). The construction of regular settimdecagon by a ruler and a gauge merits special interest. Problems of construction theory according to Bolyai's hyperbolic and non-Euclidean geometries (STROMMER [1], [2], [3], [7]) induced an axiomatic approach to special questions of foundations of geometry.

Results in the second scope are variegated, although three essential trends are manifest, such as /hyperbolic and Euclidean/ problems related to space tiling, investigations of the density of sphere packings, and lattice geometry related to geometric number theory. Imre Vermes pointed out an interesting property of asymptotic or regular  $\{p, 4\}$  mosaics (VERMES [1], [10]). Proving existence of the densest hypercycle packings in the hyperbolic plane (VERMES [2], [5]), the problem of the sparsest hypercycle covering is also solved (VERMES [8], [4], [6], [7]). An interesting problem is that of the density of congruent circles accommodable in the given hypercycle domain, assessed in (VERMES [8]).

Results by Emil Molnár in hyperbolic geometry are obtained from a method developed by him. Fundamental idea of this method and of the related theory is that: space tiling by polyhedra according to some combinatorially given group of transformations may be realized in some homogeneous space of constant curvature or other, depending on free parameters. A  $d$ -dimensional calculation procedure relying on projective metrics has

been developed (MOLNÁR [1], [2], [8]), with concrete, computerized applications.

These results, utilized in MOLNÁR [6], [7], [9], [14], are essentially interesting from hyperbolic geometry aspects. Overall frames of this theory have been developed in (MOLNÁR [8]), combined later with the method of D-symbols, and leading, — thanks to the cooperation of colleagues from Bielefeld, — to efficient computer algorithms. The obtained complete classifications are described in MOLNÁR [3], [6], [7], [12], [14], [15], while crystallographic applications motivated (MOLNÁR [11], [12]). MOLNÁR's papers written with co-authors from Belgrade (MOLNÁR [4], [5], [13]) conclude long-studied difficult classification problems by applications of discrete groups in the hyperbolic plane. Because of the high number of cases, in general, this topic cannot be handled without computer. To solve the arising problems, István Prok has developed a program to produce all the space groups for a polyhedron given with a fitting flag structure so that the polyhedron will be the fundamental domain of the corresponding space groups (PROK [1]). By this method all the crystal groups of cubic fundamental domain in the Euclidean space were determined (PROK [2], [3]). In a paper by István Prok and Emil MOLNÁR (PROK [4]) all the solid-transitive simplex tilings of the three-dimensional simply connected spaces are classified.

Emil Molnár, Ákos G. Horváth and Jenő Szirmai investigated problems of densest sphere packings in three-dimensional Euclidean space. Ákos G. Horváth demonstrated that the densest double-latticelike congruent sphere packings cannot be denser than the densest lattice-like ones (G. HORVÁTH [7]). In the joint paper with by E. Molnár the same is verified for all packings generated by the ten Euclidean space groups without fixed points (G. HORVÁTH [11], [12]). Jenő Szirmai is concerned with similar problems (SZIRMAI [1], [2]) for other discrete space groups /F23, F432, P432/.

Lattice geometry is the chapter of n-dimensional discrete geometry related to geometric number theory, a scope concerned with primarily by Ákos G. Horváth at this Department. Joining the Moscow school of this topic, problems of different lattice bases /or for primitive systems/ are studied in G. HORVÁTH [1], [2], [3], [6], [10], while those for minimum vectors in lattices in [4], [15]. Sections by a hyperplane of the Dirichlet cell system of particular point sets are investigated in G. HORVÁTH [8], while G. HORVÁTH [9] and [14], indicating lattices with many minimum vectors lead to the theory of finite-dimensional vector spaces over finite fields, that is, the coding theory. The finite geometries above have earlier been investigated by István Reiman. He studied certain relations between

finite geometries and graph theory. He succeeded in solving extremal graph problems by means of finite geometries.

At this Department, the convex geometry has been studied by Károly Böröczky, jr., examining the concept of 'rectangular convex' figures [1]. In discrete geometry, a densest configuration of finite spheres is described in BÖRÖCZKY, jr. [2], then some extremal properties of simplices are justified BÖRÖCZKY, jr. [3], [5]. We mention an interesting paper of VERMES [9], pointing to the relation between 'elementary' and 'differential' geometry by synthetic treatment of plane curves in hyperbolic geometry.

Investigations in differential geometry are of internationally comprehensive scopes, relied on a wide cooperation in the frames of a seminary headed by János Szenthe, and of the OTKA competition. One of its important events was the summer course 'Loop spaces' in June 1989, organized by János Szenthe, László Verhóczy and this Department. Leading professors were outstanding mathematicians such as M. F. Atiyah, R. Bott and G. B. Segal.

Differential geometry of curves in an  $n$ -dimensional Euclidean space was investigated by Gábor MOLNÁR-SÁSKA in [2] followed by investigations on isometry groups of Riemannian manifolds (MOLNÁR-SÁSKA [4], [5]). After having studied certain special group actions, János Szenthe generalized the concept of the Weyl group (SZENTHE [4]–[10]). In subsequent works of SZENTHE [11]–[19], certain comparison theorems of Riemannian geometry are extended and applied for isometric group actions. Then group actions on symplectic manifolds are investigated, applied in reducing mechanical systems. László Verhóczy has been concerned with special isometries, reflections of Riemannian geometries, then with submanifolds left fixed by isometries (VERHÓCZY [1], [3]). He studied isoparametric submanifolds in symmetric spaces, and planar mappings between manifolds endowed with affin connection (VERHÓCZY [4], [2], [5]).

Research in the theory of computer aided geometric design was initiated by Márta Szilvási-Nagy. Data systems were developed for geometric algorithms and spatial models, polyhedral data structures and intersection algorithms were developed with estimating the number of steps (SZILVÁSI [4]–[9]). Polyhedron algorithms rounding off edges and vertices by spline patches were given and mixed-type data structure was constructed from discrete and analytic models (SZILVÁSI [11], [12], [15]). Pál Ledneczki developed various spline curve and surface fitting methods (LEDNECZKI [2], [3], [4]), systemized motion transformations and mappings for the sake of updating the subjects from the aspects of computer methodology. The computer is likely to become the main tool also for research in geometry.

At last, let us have a look at the products of scientific activities in the past fifteen years. The Department staff has published 120 papers, 12

books, defended four Dr. Univ. Theses, one Ph. D. and one Candidate's Thesis, delivered 57 /unpublished/ scientific lectures.

*The papers and books written by the staff of the Department in the past fifteen years are listed in the following bibliography. In the cases of those papers which have been accepted but not published until now only the titles of the journals are presented.*

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