HARDENING EFFECTS ON THE STRESS DISTRIBUTION IN A SHRINK FIT UNDER CYCLIC THERMAL LOADING

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Received December 4, 1991.

Abstract

The variation of the stress distribution during the thermoelastic-plastic deformation in an assembled shrink fit due to a steady-state, homogeneous temperature cycle is studied. The use of the Tresca yield condition and its associated flow rule with a linear isotropic hardening rule makes a semi-analytical presentation possible. Numerical results are shown and compared with those of a non-hardening model.

Keywords: shrink fit, thermoelastic-plastic deformations.

Notation

σ_r radial stress
σ_θ hoop stress
σ_z axial stress
$\bar{\sigma}$ Tresca-type equivalent stress
ϵ_r radial strain
ϵ_θ circumferential strain
ϵ_z axial strain
$\bar{\epsilon}^p$ equivalent plastic strain
u displacement
ϑ temperature [°C]
T absolute temperature [K]
Y yield stress
η hardening parameter
p_b joint pressure
i_0 initial interference.

Introduction

A shrink fit is composed from an inner and an outer cylindrical part often modelled as thick-walled tubes or rings. Thermal loading is necessary during the assemblage in order to vanish the initial interference, i.e. the overlapping of the rings. The effect of this initial thermal loading on the stress distribution has been studied in many papers in the last decade, e.g. by RASCHKE (1983), MACK (1986) and CORDTS (1990).

However, a thermal loading can occur after the assemblage, as well. For instance, when a warm liquid or gas is conveyed in a tube enforced by an outer ring or during an intermediate heating process in order to remove unwanted residual stresses. In shrink fits this heating can be dangerous from the point of view of the correct functioning, because with increasing the temperature the actual yield limit lowers and therefore plastification can occur. Large thermal loading can lead even to the full plastification of one or both parts of the fit which would mean the loss of stability, i.e. the load transmissibility of the device. On the other hand, the residual stress distribution is caused by the joint pressure which is proportional to the maximum transmissible external load. Therefore, the calculation of its variation under a temperature cycle is also important.

The first approach dealing with the consequences of a temperature cycle on the stress distribution after the assemblage was made by LIPPMANN (1990). An almost analytical method was presented with the assumption of plane stress state and thermoelastic – perfectly plastic – materials. The stresses and strains in the plastic domains have been calculated with the use of the Tresca yield condition and its associated flow rule. The present paper is a development of the above method including a linear, isotropic hardening rule. We assume that the initial stress state is elastic, the thermal loading is quasi-static in time and homogeneous in space, i.e. each part of the device is heated or cooled with the same temperature. The materials of the rings are homogeneous, isotropic and only infinitesimal strains occur. Rate dependence is disregarded. The thermal unloading is assumed to be purely elastic. Upon the above assumptions a semi-analytical method is derived. The numerical treatment of the problem by finite elements has been studied by KOVÁCS (1991).

Governing Equations

The equilibrium equation:

$$\frac{d\sigma_r}{dr} = \frac{\sigma_\theta - \sigma_r}{r}, \tag{1a}$$

or

$$\frac{d(r\sigma_r)}{dr} = \sigma_\theta, \tag{1b}$$

The geometric equations:

$$\epsilon_r = \frac{du}{dr}, \tag{2a}$$

$$\epsilon_\theta = \frac{u}{r}, \tag{2b}$$

or

$$\epsilon_r = \frac{d(r\epsilon_\theta)}{dr}. \tag{2c}$$

The total strains are decomposed into an elastic and a plastic part:

$$\epsilon_r = \epsilon_r^e + \epsilon_r^p,$$

$$\epsilon_\theta = \epsilon_\theta^e + \epsilon_\theta^p.$$

The Hooke's law:

$$\epsilon_r^e = \frac{1}{E}(\sigma_r - \nu\sigma_\theta) + \alpha T, eqno(3a)$$

$$\epsilon_\theta^e = \frac{1}{E}(\sigma_\theta - \nu\sigma_r) + \alpha T. \tag{3b}$$

The Tresca yield condition:

$$\bar{\sigma} \equiv \sigma_1 - \sigma_3 = Y, \tag{4a}$$

where

$$Y = Y_{\vartheta 0}(1 + \eta\bar{\epsilon}^p). \tag{4b}$$

The value of $Y_{\vartheta 0}$ is obtained from a simple approximation (LIPPMANN, 1990),

$$Y_{\vartheta 0} = Y_0 + m(\vartheta - \vartheta_0), \tag{4c}$$

$$\vartheta_0 = 20[^\circ C],\ \vartheta - \vartheta_0 < 150[^\circ C]$$

, m is a material parameter.

The associated flow rule:

$$\dot{\epsilon}_1^p + \dot{\epsilon}_2^p + \dot{\epsilon}_3^p = 0, \tag{5a}$$

$$\dot{\epsilon}_1^p \leq 0, \dot{\epsilon}_2^p = 0, \dot{\epsilon}_3^p \geq 0. \tag{5b}$$

We use the integrated form of the flow rule. Since the structure is loaded statically and the initial state is elastic

$$\epsilon_1^p + \epsilon_2^p + \epsilon_3^p = 0, \tag{6a}$$

$$\epsilon_1^p \leq 0,\ \epsilon_2^p = 0,\ \epsilon_3^p \geq 0. \tag{6b}$$

Formulation

The shrink fit consists of two rings. In the following the subscript i denotes the inner ring (shaft) and a denotes the outer ring (hub). Since the materials of the rings do not have to be the same, all material parameters are subscribed.

Plastification starts at the inner surface of the rings, therefore two plastic radii can be defined, x in the shaft and y in the hub.

Elastic-plastic Deformations in the Shaft

The principal stresses are

$$\sigma_1 = \sigma_z = 0, \ \sigma_2 = \sigma_r, \ \sigma_3 = \sigma_\theta.$$

Because of the isotropy of the material

$$\epsilon_1 = \epsilon_z, \ \epsilon_2 = \epsilon_r, \ \epsilon_3 = \epsilon_\theta.$$

Plastic zone: $a \leq r \leq x$

From (4a) we obtain

$$\sigma_\theta = -Y_i. \tag{7}$$

Solving the *Eq.*(1b) with the boundary condition

$$\sigma_r(a) = 0,$$

the radial stress is

$$\sigma_r = -\frac{1}{r}\int_a^r Y_i dr. \tag{8}$$

From *Eq.*2c)

$$\epsilon_\theta = \frac{1}{r}\int_a^r \epsilon_r dr + \frac{a}{r}\epsilon_\theta(a). \tag{9}$$

Let $C_1 = a\epsilon_\theta(a)$.

Since $\epsilon_2 = \epsilon_r$, thus from *Eq. (6b)*

$$\epsilon_r = \epsilon_r^e, \tag{10}$$

and we can apply Hooke's law

$$\epsilon_r^e = \frac{1}{E_i}(\sigma_r - \nu_i\sigma_\theta) + \alpha_i T. \tag{11}$$

Substituting (7) and (8) into (11), we obtain

$$\epsilon_r = \frac{1}{E_i}(-\frac{1}{r}\int_a^r Y_i dr + \nu_i Y_i) + \alpha_i T, \tag{12}$$

and from (9)

$$\epsilon_\theta = \frac{1}{E_i}\left[\frac{1}{r}\int_a^r \left(-\frac{1}{r}\int_a^r Y_i dr + \nu_i Y_i + \alpha_i T E_i\right) dr\right] + \frac{C_1}{r}. \tag{13}$$

The plastic circumferential strain is

$$\epsilon_\theta^p = \epsilon_\theta - \epsilon_\theta^e. \tag{14}$$

Since ν_i, α_i, E_i and T are independent of r, thus with the use of *Eqs.* (3b), (7), (8) and (14)

$$\epsilon_\theta^p = \frac{1}{E_i}\left[Y_i - \frac{1}{r}\int_a^r \left(\frac{1}{r}\int_a^r Y_i dr\right) dr\right] - \alpha_i T\frac{a}{r} + \frac{C_1}{r}. \tag{15}$$

Let $C_2 = C_1 - \alpha_i T a$, thus

$$\epsilon_\theta^p = \frac{1}{E_i}\left[Y_i - \frac{1}{r}\int_a^r \left(\frac{1}{r}\int_a^r Y_i dr\right) dr\right] + \frac{C_2}{r}. \tag{16}$$

The equivalence of the plastic work gives (GAMER, 1983):

$$\sigma_1 \epsilon_1^p + \sigma_2 \epsilon_2^p + \sigma_3 \epsilon_3^p = \bar{\sigma}\bar{\epsilon}^p.$$

Since $\sigma_1 = 0$, $\epsilon_2^p = 0$ and $\bar{\sigma} = -\sigma_3$, thus

$$\bar{\epsilon}^p = -\epsilon_3^p \equiv -\epsilon_\theta^p. \tag{17}$$

Substituting (16) into (17), then into (4b), we obtain

$$Y_i = Y_{i\vartheta o}\left[1 - \frac{\eta_i}{E_i}\left(Y_i - \frac{1}{r}\int_a^r \left(\frac{1}{r}\int_a^r Y_i dr\right) dr + \frac{EC_2}{r}\right)\right]. \tag{18}$$

Assuming that $\eta_i \neq 0$, we arrange (18) in the following form

$$Y_i \frac{1}{\delta^2} - \frac{1}{r} \int_a^r \Big(\frac{1}{r} \int_a^r Y_i dr \Big) dr = \frac{1-\delta^2}{\delta^2} Y_{i\vartheta 0} - \frac{C_2}{r}, \tag{19}$$

where

$$\delta^2 \equiv \frac{\eta_i \frac{Y_{i\vartheta 0}}{E_i}}{1 + \eta_i \frac{Y_{i\vartheta o}}{E_i}}.$$

If $\eta_i = 0$, then $\sigma_\theta = -Y_{i\vartheta 0}$=const., $\sigma_r = -Y_{i\vartheta 0}(1 - a/r)$, see in (LIPPMANN, 1990).
We multiply *Eq.(19)* by r and derive it with respect to r twice. Finally, we obtain the following second order ODE

$$r^2 \frac{d^2 Y_i}{dr^2} + 3r \frac{dY_i}{dr} + (1-\delta^2) Y_i = (1-\delta^2) Y_{i\vartheta 0}. \tag{20}$$

The general solution of (20) can be written in the following form

$$Y_i = Y_{i\vartheta 0} + C_3 r^{-1+\delta} + C_4 r^{-1-\delta}, \tag{21}$$

C_3 and C_4 are integration constants. Their values can be determined from the following boundary conditions

$$\bar{\epsilon}^p(x) = 0 \tag{22}$$

and

$$F(a) = 0, \tag{23}$$

where $F(r)$ is the primitive function of $Y_i(r)$. Using (22), we obtain

$$C_4 = -C_3 x^{2\delta}, \tag{24a}$$

while from (23)

$$\frac{C_3}{\delta} = -Y_{i\vartheta 0} \frac{a^{1+\delta}}{a^{2\delta} + x^{2\delta}}. \tag{24b}$$

Substituting these values into (21) and then into (8), we obtain

$$\sigma_r = -Y_{i\vartheta o} \Big[1 - \frac{a^{1+\delta}}{a^{2\delta} + x^{2\delta}} \Big(r^{-1+\delta} + x^{2\delta} r^{-1-\delta} \Big) \Big], \tag{25}$$

or, with the use of the dimensionless geometrical parameters $q_i = a/b$, $\xi_i = x/b$

$$\sigma_r = -Y_{i\vartheta o}\left[1 - \frac{q_i^{1+\delta}}{q_i^{2\delta} + \xi_i^{2\delta}}\left(\left(\frac{r}{b}\right)^{-1+\delta} + \xi_i^{2\delta}\left(\frac{r}{b}\right)^{-1-\delta}\right)\right]. \tag{26}$$

With *Eqs.*(24a) and (24b) (21) has the following form

$$Y_i = Y_{i\vartheta 0}\left[1 - \delta\frac{q_i^{1+\delta}}{q_i^{2\delta} + \xi_i^{2\delta}}\left(\left(\frac{r}{b}\right)^{-1+\delta} - \xi_i^{2\delta}\left(\frac{r}{b}\right)^{-1-\delta}\right)\right]. \tag{27}$$

Substituting (27) into (7), σ_θ can be determined.

The radial strain is given by *Eq.*(3a), the elastic part of the circumferential strain by *Eq.*(3b). From the hardening equation (4b) we obtain

$$\bar{\epsilon}^p = \frac{1}{\eta_i}\left(\frac{Y_i}{Y_{i\vartheta 0}} - 1\right), \tag{28}$$

thus with *Eqs.*(27) and (17) ϵ_θ^p can also be calculated. Finally, u can be determined from (2b).

Elastic zone: $x \le r \le b$

It is well-known from the elasticity theory applied to thick-walled cylinders that the radial and hoop stresses are

$$\sigma_r = A_1 - \frac{A_2}{r^2}, \quad \sigma_\theta = A_1 + \frac{A_2}{r^2},$$

where A_1 and A_2 can be determined from

$$\sigma_r(b) = -p_b, \tag{29a}$$

$$\sigma_\theta(-x) = \sigma_\theta(+x). \tag{29b}$$

The latter condition means the continuity of the hoop stress at the limit of the plastic zone. With the use of these equations we obtain

$$\sigma_r = -\frac{1}{1+\xi_i^2}\left[Y_{i\vartheta 0}\xi_i^2\left(1 - \frac{1}{(r/b)^2}\right) + p_b\left(1 + \frac{\xi_i^2}{(r/b)^2}\right)\right], \tag{30}$$

$$\sigma_\theta = -\frac{1}{1+\xi_i^2}\left[Y_{i\vartheta 0}\xi_i^2\left(1 + \frac{1}{(r/b)^2}\right) + p_b\left(1 - \frac{\xi_i^2}{(r/b)^2}\right)\right]. \tag{31}$$

The strains and the displacement can be determined from *Eqs.*(3a), (3b) and (2b), respectively.

Elastic-Plastic Deformations in the Hub

The principal stresses and strains are the following:

$$\sigma_1 = \sigma_\theta,\ \sigma_2 = \sigma_z = 0,\ \sigma_3 = \sigma_r,$$

$$\epsilon_1 = \epsilon_\theta,\ \epsilon_2 = \epsilon_z,\ \epsilon_3 = \epsilon_r.$$

Plastic zone: $b \le r \le y$

From *Eq.*(4a) we have

$$\sigma_\theta - \sigma_r = Y_a, \tag{32}$$

thus from *Eq.*(1a)

$$\frac{d\sigma_r}{dr} = \frac{Y_a}{r}. \tag{33}$$

Solving (33), we obtain

$$\sigma_r = \int_b^r \frac{Y_a}{r} dr - p_b, \tag{34}$$

and from *Eq.*(32)

$$\sigma_\theta = \int_b^r \frac{Y_a}{r} dr - p_b + Y_a. \tag{35}$$

From *Eq.*(6b) we have $\epsilon_z^p = 0$, thus in eq.(6a)

$$\epsilon_\theta^p + \epsilon_r^p = 0, \tag{36}$$

which means that

$$\epsilon_\theta + \epsilon_r = \epsilon_\theta^e + \epsilon_r^e. \tag{37}$$

We substitute (34) and (35) into (3a) and (3b), respectively, then using (2a) and (2b), we obtain from (37)

$$\frac{du}{dr} + \frac{u}{r} = \frac{1-\nu_a}{E_a}\left(2\int_b^r \frac{Y_a}{r} dr - 2p_b + Y_a\right) + 2\alpha_a T. \tag{38}$$

The solution of this linear first order ODE is

$$\frac{u}{r} = \frac{1-\nu_a}{E_a} \int_b^r \frac{Y_a}{r} dr - (1-\nu_a) \frac{p_b}{E_a} + \alpha_a T + \frac{D_1}{r^2}, \tag{39}$$

where D_1 is an integration constant.

Since $\epsilon_\theta^p = \epsilon_\theta - \epsilon_\theta^e$, thus from *Eqs.* (39), (2b) and (3b) we obtain

$$\epsilon_\theta^p = \frac{D_1}{r^2} - \frac{Y_a}{E_a}. \tag{40}$$

The equivalency of the plastic work gives

$$\sigma_\theta d\epsilon_\theta^p + \sigma_r d\epsilon_r^p = \bar{\sigma} d\bar{\epsilon}^p. \tag{41}$$

From *Eq.*(36) we have $\epsilon_r^p = -\epsilon_\theta^p$, thus $d\epsilon_r^p = -d\epsilon_\theta^p$ and

$$(\sigma_\theta - \sigma_r)\, d\epsilon_\theta^p = \bar{\sigma} d\bar{\epsilon}^p. \tag{42}$$

Comparing *Eqs.* (42) and (4a), one can say that

$$d\epsilon_\theta^p = d\bar{\epsilon}^p.$$

Considering elastic initial state, the integration of the latter equation gives

$$\bar{\epsilon}^p = \epsilon_\theta^p. \tag{43}$$

With the use of *Eqs.* (43) and (40) we get from eq.(4b)

$$Y_a = S\left(1 + \eta_a \frac{D_1}{r^2}\right), \tag{44}$$

where

$$S \equiv \frac{Y_{a\vartheta 0}}{1 + \eta_a \frac{Y_{a\vartheta 0}}{E_a}}.$$

Substituting (44) into (34) and (35), we obtain

$$\sigma_r = S\left(\ln\frac{r}{b} - \frac{\eta_a}{2}\frac{D_1}{r^2}\right) + D_2, \tag{45}$$

$$\sigma_\theta = S\left(1 + \ln\frac{r}{b} + \frac{\eta_a}{2}\frac{D_1}{r^2}\right) + D_2, \tag{46}$$

where D_2 is an integration constant.

Elastic zone: $y \leq r \leq c$

The elastic stresses are

$$\sigma_r = A_3 - \frac{A_4}{r^2}, \quad \sigma_\theta = A_3 + \frac{A_4}{r^2},$$

while from *Eqs.*(2b) and (3b) we have

$$\frac{u}{r} = \frac{1-\nu_a}{E_a} A_3 + \frac{1+\nu_a}{E_a}\frac{A_4}{r^2} + \alpha_a T.$$

For the calculation of D_1, D_2, A_3 and A_4 we use the following boundary and continuity conditions

$$\begin{aligned} \sigma_r(c) &= 0 \\ \sigma_r(-y) &= \sigma_r(+y) \\ \bar{\sigma}(+y) &= Y_{a\vartheta 0} \\ u(-y) &= u(+y). \end{aligned}$$

The solution of this set of equations gives

$$A_3 = \frac{Y_{a\vartheta 0}}{2}\left(\frac{y}{c}\right)^2, \quad A_4 = \frac{Y_{a\vartheta 0}}{2} y^2$$

$$D_1 = \frac{Y_{a\vartheta 0}}{E_a} y^2, \quad D_2 = \frac{Y_{a\vartheta 0}}{2}\left[-1 + \left(\frac{y}{c}\right)^2 - \frac{2S}{Y_{a\vartheta 0}} \ln \frac{y}{b} + S\frac{\eta_a}{E_a}\right].$$

Substituting these values into (45) and (46), the stresses in the plastic zone can be determined. With the use of the dimensionless geometrical parameters $q_a = c/b$ and $\xi_a = y/b$ the elastic stresses are

$$\sigma_r = -\frac{Y_{a\vartheta 0}}{2}\left(\frac{\xi_a}{q_a}\right)^2\left[\frac{q_a^2}{(r/b)^2} - 1\right], \tag{47}$$

$$\sigma_\theta = \frac{Y_{a\vartheta 0}}{2}\left(\frac{\xi_a}{q_a}\right)^2\left[\frac{q_a^2}{(r/b)^2} + 1\right]. \tag{48}$$

The strains and the displacement can be calculated from *Eqs.*(3a), (3b) and (2b), respectively.

In order to calculate the three remaining unknowns, x, y and p_b we use the following conditions

$$\sigma_r(-x) = \sigma_r(+x),$$
$$\sigma_r(+b) = -p_b,$$
$$| u(+b) - u(-b) | = i_0.$$

After the substitution we obtain the following set of transcendent equations

$$1 - \frac{2q_i^{1+\delta}\xi_i^{-1+\delta}}{\xi_i^{2\delta} + q_i^{2\delta}} = -\frac{1}{1+\xi_i^2}\left(\xi_i^2 - 1 + \frac{2p_b}{Y_{i\vartheta 0}}\right),$$

$$\frac{p_b}{Y_{a\vartheta 0}} + \frac{1}{2}\left[-1 + \left(\frac{\xi_a}{q_a}\right)^2 - \frac{2}{1+Q}\ln\xi_a + \frac{Q}{1+Q}\left(1 - \xi_a^2\right)\right] = 0,$$

$$| \frac{Y_{a\vartheta 0}}{E_a}\left[\frac{1-\nu_a}{2}\left(-1 + \left(\frac{\xi_a}{q_a}\right)^2 - \frac{1}{1+Q}\left[2\ln\xi_a + Q\left(\xi_a^2 - 1\right)\right]\right) + \xi_a^2\right] -$$

$$-\frac{\xi_i^2}{1+\xi_i^2}\left[-2\frac{Y_i}{E_i} + \frac{p_b}{E_i}(1+\nu_i) - \frac{1}{\xi_i^2}\frac{p_b}{E_i}(1-\nu_i)\right] + (\alpha_a - \alpha_i)\Delta\vartheta \,|= \frac{i_0}{b},$$

where $Q \equiv \eta_a Y_{a\vartheta 0}/E_a$.

This set of equations can be solved numerically. We have to avoid the full plastification of one of the rings, therefore

$$\xi_i < 1, \ \xi_a < q_a$$

must be satisfied. If the first part of the thermal cycle is heating, an upper bound can be derived from these conditions for the joint pressure. Namely, from *Eq.*(26)

$$p_b = Y_{i\vartheta 0}\left(1 - 2\frac{q_i^{1+\delta}}{1 + q_i^{2\delta}}\right) \tag{49}$$

would cause the failure of the shaft and from *Eq.* (45)

$$p_b = Y_{a\vartheta 0}\left[\frac{1}{1+Q}\ln q_a + \frac{Q}{2(1+Q)}\left(q_a^2 - 1\right)\right] \tag{50}$$

would cause the failure of the hub. The lower bound at elastic-plastic deformation would be the pure elastic deformation, i.e. when $\xi_i = q_i$ and $\xi_a = 1$. From *Eqs.* (26), (30) and (45) we obtain

$$p_b = \frac{Y_{i\vartheta 0}}{2}\left(1 - q_i^2\right), \tag{51}$$

$$p_b = \frac{Y_{a\vartheta 0}}{2}\left(1 - \frac{1}{q_a^2}\right). \tag{52}$$

Thus

$$p_{bmin} \leq p_b < p_{bmax},$$

where p_{bmin} is the smaller p_b from eqs.(49) and (50), while p_{bmax} is the greater p_b from *Eqs.* (51) and (52).

Unloading

The unloading process is assumed to be completely elastic, therefore the unloading/reloading procedure presented in (LIPPMANN, 1990) can be applied in order to determine the final joint pressure. Since the material parameters E, ν and α can vary with the temperature, but this variation is elastic, this is treated as if the device were first thermally and mechanically unloaded under the old parameters and then reloaded elastically under the new ones.

The elastic stresses and displacement are given formally by

$$\sigma_r = -p_1 f_r\left(\frac{r_2}{r_1}, \frac{r_2}{r}\right), \quad \sigma_\theta = p_1 f_\theta\left(\frac{r_2}{r_1}, \frac{r_2}{r}\right), \tag{53a – b}$$

$$\frac{u}{r} = \frac{p_1}{E} f_u\left(\nu, \frac{r_2}{r_1}, \frac{r_2}{r}\right) + \alpha T, \tag{53c}$$

where

$$f_r(\gamma, \zeta) = \frac{\zeta^2 - 1}{\gamma^2 - 1}, \quad f_\theta(\gamma, \zeta) = \frac{\zeta^2 + 1}{\gamma^2 - 1},$$

$$f_u(\nu, \gamma, \zeta) = \frac{1 - \nu + (1 + \nu)\zeta^2}{\gamma^2 - 1}$$

and $p_1 = \sigma_r(r_1)$. Let the initial joint pressure be p_{b0}, the intermediate one (after the thermal loading) p_{b1} and the final one (after the thermal unloading) p_{b2}. The displacement of the joint, $r = b$ can be given as

$$\frac{u_i}{b} = -\frac{p_{b1}}{E_{i1}} f_u\left(\nu_{i1}, q_i, q_i\right) - \alpha_{i1}(T_1 - T^\star) + \frac{p_{b2}}{E_{i0}} f_u\left(\nu_{i1}, q_i, q_i\right) + \alpha_{i0}(T_0 - T^\star),$$

$$\frac{u_a}{b} = -\frac{p_{b1}}{E_{a1}} f_u(\nu_{a1}, q_a, q_a) - \alpha_{a1}(T_1 - T^*) + \frac{p_{b2}}{E_{a0}} f_u(\nu_{a1}, q_a, q_a) + \alpha_{a0}(T_0 - T^*)$$

in the shaft and in the hub, respectively.

For the sake of convenience, the reference temperature T^* is kept to be the room temperature, i.e.

$$T^* = T_0 = 293[\mathrm{K}].$$

Equating both displacements, the final joint pressure happens to be

$$p_{b2} = E_{i0} \frac{\frac{p_{b1}}{E_{i1}}\left[f_u(\nu_{i1}, q_i, q_i) - \frac{E_{i1}}{E_{a1}} f_u(\nu_{a1}, q_a, q_a)\right] + (\alpha_{i1} - \alpha_{a1})(T_1 - T_0)}{f_u(\nu_{i0}, q_i, q_i) - \frac{E_{i0}}{E_{a0}} f_u(\nu_{a1}, q_a, q_a)}. \tag{54}$$

The difference of the joint pressure is therefore

$$\Delta p_b = p_{b2} - p_{b1}.$$

The final stresses are the following:

$$\sigma_{r2} = \sigma_{r1} + \Delta\sigma_r, \quad \sigma_{\theta 2} = \sigma_{\theta 1} + \Delta\sigma_\theta,$$

where the stress increments $\Delta\sigma_r$ and $\Delta\sigma_\theta$ can be calculated after the substitution of Δp_b into the *Eqs.* (53a), (53b), respectively.

Numerical Example

We consider a shrink fit made from aluminium and copper with the geometrical and material parameters given in *Table 1* (LIPPMANN, 1990).

Table 1

$q_a = 0.25$, $q_b = 1.25$

	$Y_0[MPa]$	$m/Y_0[1/K]$	E[GPa]	ν	$\alpha[1/K]$
Shaft	50	$5 \cdot 10^{-3}$	68.67	0.3	$2.38 \cdot 10^{-5}$
Hub	130	$4.23 \cdot 10^{-3}$	113.8	0.35	$1.698 \cdot 10^{-5}$

The hardening parameters are (MEGAHED, 1991): $\eta_i = 2.5$, $\eta_a = 4.24$. The initial joint pressure is p_{b0}=17.5 [MPa]. The thermal cycle means a homogeneous temperature rise from $\vartheta_0 = 20$[°C] to $\vartheta_1 = 75$[°C] and then a cooling from $\vartheta_1 = 75$[°C] to $\vartheta_2 = \vartheta_0 = 20$[°C]. The comparison of the stress distributions with and without hardening can be seen in *Fig.1* and *Fig. 2* after the heating and at the end of the thermal cycle, respectively. The figures show that the hardening has practically no effect on the stress distribution. In both rings plastification occurred. In the shaft the plastic zone expands to that radius as far as the hoop stress is constant, while in the hub the constant difference of the hoop and the radial stresses shows the plastic zone. The joint pressures and the dimensionless plastic radii are shown in *Table 2*.

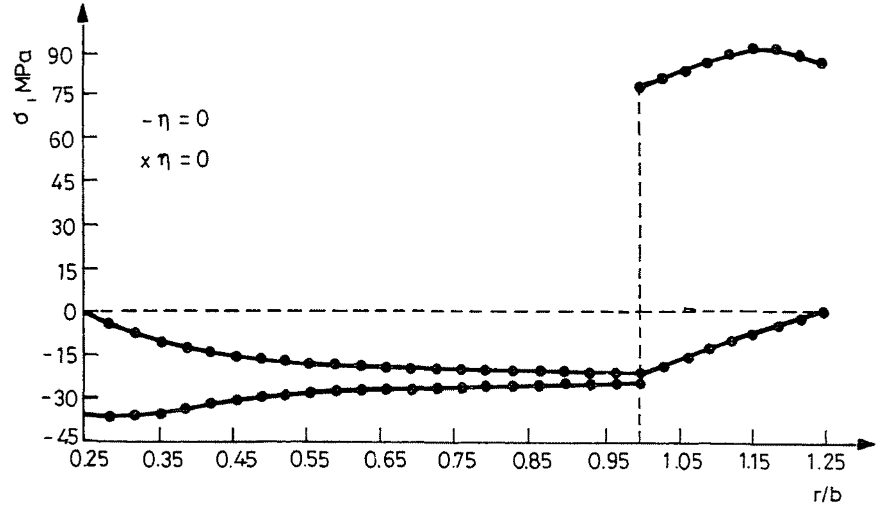

Fig. 1. Stress distribution after temperature increase of $\Delta\vartheta = 55$°C

Table 2

	ξ_i	ξ_a	p_{b1} [MPa]	p_{b2} [MPa]
$\eta = 0$	0.353	1.168	21.82	15.03
$\eta \neq 0$	0.344	1.169	21.83	15.04

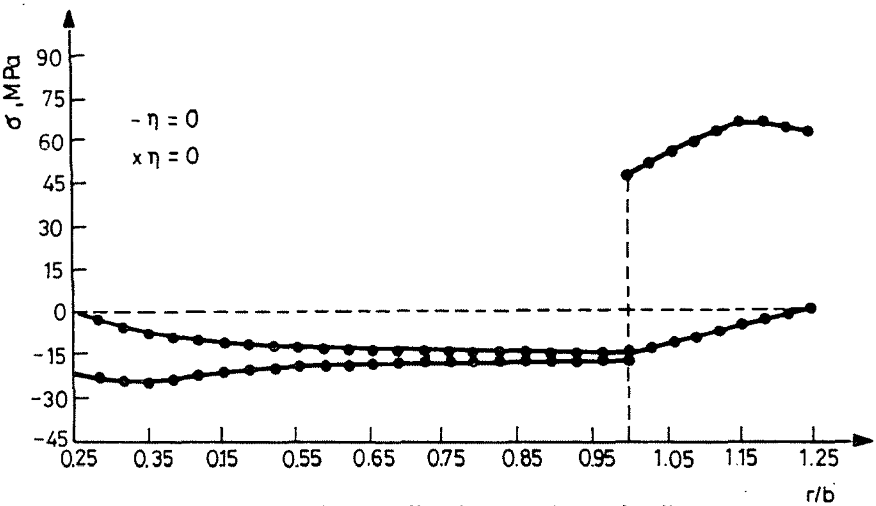

Fig. 2. Final stress distribution after unloading

Conclusion

In the above example, the maximum temperature difference was small enough, therefore, the equivalent plastic strain remained small compared to 1. However, the device must not be loaded thermally much more because of the fast full plastification (LIPPMANN, 1990). Therefore, one can say that the incorporation of complicated hardening rules into the model is not worth the trouble: the loss of stability ensues much sooner than large plastic strains could arise which are not negligible.

Acknowledgement

The above research has been carried out during a DAAD-fellowship in the Lehrstuhl A für Mechanik, TU München. I am highly indebted to Professor Dr H. Lippmann, head of the Department, for the subject of this paper and for the good research conditions.

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