# MOVING A ROBOT HAND TRANSPORTING FILLED GLASS 

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#### Abstract

A gripper orientation concept for a robot carrying filled glass along a prescribed path without spilling is presented. The application of the method is shown by a 5 -degree freedom high-speed laboratory robot with significant elasticities.


Keywords: elastic robot, optimum path planning, gripper orientation, filled glass transportation.

## Introduction

The optimum path planning for mechanical manipulators is an important problem in robot dynamics. Several methods have been developed and applied in the practice which serve for obtaining a minimum travelling time. For instance, Pfeiffer and Johanni (1987) presented a concept for arbitrary manipulators constrained by angle and torque or force limits in the joints and by the geometry of the prescribed trajectory. It provides the minimum time for the manipulator dynamics and kinematics.

However, the physical implementation needs a control concept which permits the realization of the prescribed motion of the robot. This problem has been also widely researched and published in recent years.

Beside the general kinematic and control questions, special tasks can arise. Moving functions can be carried out by the end effector or gripper of the robot, e.g. the transportation of liquid in open containers. The present paper shows a method for the gripper orientation assuring that the liquid does not spill out of the glass carried by the gripper along a prescribed path. The necessary kinematical data have been obtained by using the optimum path planning presented in Pfeiffer and Johanni, (1987). The method was attached to the control concept of RIchter (1991) and then implemented on a five degree freedom elastic laboratory robot.

## Gripper Position

The three degrees of freedom (DOF) for the position of the filled glass considered as a rigid body are reduced to one in the case of a motion along a given space curve. This motion can be formulated by means of a path parameter $s$. The position of the robot end point is represented by the vector of the generalized co-ordinates

$$
\begin{equation*}
\mathbf{G}_{p}=\mathbf{q}_{p}(s)=\left[q_{1}(s), \ldots, q_{\mathbf{n}}(s)\right]^{T} \tag{1}
\end{equation*}
$$

$n$ means the number of DOF of the robot without the end effector.
The trajectory in the work space is given by a space curve, either by a set of analytical equations or by a series of points, as a function of the path parameter $s$ :

$$
\begin{gather*}
\mathbf{r}=\mathbf{r}(s(t)), \quad \mathbf{r} \in \mathbf{R}^{3}, s \in\left[s_{A}, s_{B}\right] \\
\mathbf{r}(s)=\sum_{j=1}^{3} x_{j}(s) \mathbf{i}_{j}, x_{j}(s) \in C_{\left[s_{A}, s_{B}\right]}^{2}, \quad j=1,2,3 \tag{2}
\end{gather*}
$$

where to $s_{A}$ belongs the beginning, to $s_{B}$ the end point of the path and $t$ is the time. $\mathbf{i}_{j}, j=1,2,3$ are the basis unit vectors of the inertial frames of reference in the work space.

If the path is given pointwise, $x_{j}(s)$ are composed of third order splines and therefore this interpolation assures the satisfaction of the necessary continuity criteria. The path parameter $s$ can be mapped naturally or onto the $[0,1]$ interval. Using the method presented in (Pfeiffer and Johanni, 1987), we obtain the following functions:

- $\dot{s}$ path velocity;
- $\ddot{s}$ path acceleration;
- $t(s)$ time function;
$-s(t)$ path function;
and the joint angles $\mathbf{q}_{p}$ and the joint torques $\mathbf{M}_{p}$, which are necessary to realize the motion.


## Gripper Orientation

The description of the gripper orientation needs further three DOF,

$$
\begin{equation*}
\mathbf{q}_{0}=\mathbf{q}_{0}(s)=\left[q_{n+1}(s), \ldots, q_{n+3}(s)\right]^{T} \tag{3}
\end{equation*}
$$

so that we have the generalized co-ordinate vector

$$
\begin{equation*}
\mathbf{q}=\mathbf{q}_{p}+\mathbf{q}_{0}, \quad \mathbf{q} \in \mathbf{R}^{n+3} \tag{4}
\end{equation*}
$$

for the full motion representation in the configuration space.
The glass can be rotated about its centre of gravity, the distance between it and the end point of the last arm is neglected. In order to assure the completion of the request for the liquid in the glass, the gripper holding the glass has always to be rotated in the appropriate direction at each point of the trajectory. This adequate attitude behaviour of the gripper depends on the local velocity and acceleration/deceleration of the robot wrist tracking its path. During the motion on the space curve, the glass represents for the liquid a carrying system with known acceleration

$$
\begin{equation*}
\ddot{\mathbf{r}}=\ddot{\mathbf{r}}(s, \dot{s}, \ddot{s})=\sum_{j=1}^{3}\left[\ddot{s} x_{j}^{\prime}(s)+\dot{s}^{2} x_{j}^{\prime \prime}(s)\right] \mathbf{i}_{j}, \tag{5}
\end{equation*}
$$

where $x_{j}^{\prime}$ and $x_{j}$ " are the first and second derivatives with respect to the path parameter.
In the carrying system the acceleration of the centre of gravity of the liquid has two components: one of them is due to the gravity, the other is directed to - $\ddot{r}$. The latter acceleration component is due to the inertia of the liquid. The spilling can be avoided if at each stage of the motion the bottom of the glass is oriented perpendicular to the resultant acceleration, i.e. the normal vector of the liquid surface is always parallel to $\ddot{\mathbf{r}}-\mathbf{g}$. Thus,

$$
\begin{equation*}
\mathbf{n} \| \ddot{\mathbf{r}}-g \tag{6}
\end{equation*}
$$

$\mathrm{g}=-g \mathbf{i}_{3}, g=9.81\left[\mathrm{~m} / \mathrm{s}^{2}\right]$ is the gravitational acceleration (See Fig. 1).
The direction vector $\mathbf{n}$ described in world co-ordinates has to be transformed into one that is attached to the last arm, because the Cardan angles of the direction vector in this reference system are identical with the angles $q_{n+1}, q_{n+2}, q_{n+3}$ :

$$
\begin{align*}
& q_{n+1}=-\arctan \frac{\tilde{u}_{2}}{\tilde{u}_{3}} \\
& q_{n+2}=\arctan \frac{\tilde{u}_{1}}{\sqrt{\tilde{u}_{2}^{2}+\tilde{u}_{3}^{2}}} \tag{7}
\end{align*}
$$

where $\tilde{u}_{1}, \tilde{u}_{2}, \tilde{u}_{3}$ are the co-ordinates of $\mathbf{n}$ with respect to the system of the last arm. These co-ordinates are to be obtained by

$$
\tilde{\mathbf{n}}=T \cdot \mathbf{n}
$$



Fig. 1. Prevention of the running out


Fig. 2. Robot configuration
in the case of $n=$ three DOF where

$$
\mathbf{T}=\left(\begin{array}{ccc}
\cos q_{1} & \sin q_{1} & 0  \tag{8}\\
-\sin q_{1} \cos \left(q_{2}+q_{3}\right) & \cos q_{1} \cos \left(q_{2}+q_{3}\right) & \sin \left(q_{2}+q_{3}\right) \\
\sin q_{1} \sin \left(q_{2}+q_{3}\right) & -\cos q_{1} \sin \left(q_{2}+q_{3}\right) & \cos \left(q_{2}+q_{3}\right)
\end{array}\right),
$$

in other cases the modified Newton Iteration Method (Pfeiffer and Reithmeier, 1987) has to be applied.The third Cardan angle $q_{n+3}$ defining the rotation around the own axis of symmetry of the glass is not taken into account.
The path cannot be prescribed fully arbitrarily within the reach of the hand: $\ddot{\mathbf{r}}$ must not be equal to $\mathbf{g}$ at any points, otherwise $\mathbf{n}$ would be indefinite, according to Eq. (6). The actuator torques realizing the rotation of the hand are calculated by solving the equations of motion (PFEIFFER and Johanni, 1987).

## Implementation

The travelling of the filled glass by the above method has been tested in several cases. Now, one of them is presented with the following parameters: the path is a $1.5[\mathrm{~m}]$ long section of a spiral of $1.96[\mathrm{~m}]$ diameter and 3.7 [ m ] thread-slope given by the equations

$$
\begin{aligned}
& x_{1}(s)=0.98 \cos (1.405 s+0.67)+0.18, \\
& x_{2}(s)=0.6 s-0.3 \\
& x_{3}(s)=0.98 \sin (1.405 s+0.67)+0.49 .
\end{aligned}
$$

The laboratory robot has five DOF, its two elastic arms are 0.75 [ m ] and $0.732[\mathrm{~m}]$ long with a cross section of $20 \times 20\left[\mathrm{~mm}^{2}\right]$ and $15 \times 15\left[\mathrm{~mm}^{2}\right]$, respectively. The whole moving mass is about $12[\mathrm{~kg}]$. In Fig. 3. the laboratory robot, in Fig. 4. the hand can be seen. The exact technical data and the control concept can be found in (Richter, 1991). The filled glass was transported along the spiral section in 1.2 [s], while the angles $q_{4}$ and $q_{5}$ of the hand were being controlled by Eq. (7) and no drop of liquid was spilled. For the comparison, the test was repeated without the gripper orientation, i.e. the glass remained vertical during the motion. In this case about $1 / 3$ of the liquid was lost.
In Fig. 5. the passed time, the $\dot{s}$ path velocity and the $\ddot{s}$ path acceleration curves along the path can be seen. It has to be noted that for having the conventional units ( $[\mathrm{m} / \mathrm{s}]$ for the velocity, $\left[\mathrm{m} / \mathrm{s}^{2}\right]$ for the acceleration), the values on the vertical axis must be multiplied by a constant which corresponds to the length of the curve between $s=0$ and $s=1$. This constant


Fig. 3. Laboratory robot


Fig. 4. The gripper with the filled glass


Fig. 5. Time, path velocity and acceleration over the path parameter
has no importance, because it cancels in $E q$. (5) and therefore it does not modify Eq. (7). In Fig. 6. the changing of the angles (in degrees) along the path are shown. The first three angles have been determined using the method of PFEIFFER and Johanni (1987), $q_{4}$ and $q_{5}$ are prescribed by $E q$. (7). A further example has been presented in Pfeiffer, Richter and Kovács-Bende, (1991).





Fig. 6. Joint angles

## Conclusion

The time-minimal path planning algorithm with respect to a prescribed path geometry and manipulator constraints due to limitations of the joint torques and of joint kinematics has been extended by a criterion concerning the attitude behaviour of the gripper, satisfying the requirement of a stable liquid surface.
The theory tested on an institute-developed manipulator with five revolute joints and two elastic arms proved to be fairly good: no liquid was lost. Though the method has been put into practice for a tree-like structure robot with three DOF for the position and further two ones for the orientation of the end effector, it can be applied for arbitrary types of industrial robots or manipulators, supposing that the connected $s-\dot{s}-\ddot{s}$ value-triples along the path are known.

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## References

Pfeiffer, F. - Johanni, R. (1987): A Concept for Manipulator Trajectory Planning. IEEE Journal of Robotics and Automation, Vol. 3, No. 2, pp. 115-123.
Pfeiffer, F. - Reithmeier, E. (1987): Roboterdynamik. Teubner Verlag, Stuttgart.
Pfeiffer, F. - Richter, K. - Kovács-Bende, M. (1991): Augmented Flexible Link Manipulator Trajectory Control: Moving a filled glass. Proc. IEEE Int. Conf. Robotics and Automation (submitted).
Richter, K. (1991): Kraftregelung elastischer Roboter. Fortschr. -Ber. VDI Reihe 8, Nr. 259, : VDI Verlag, Düsseldorf.

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