STUDY OF A QUASI-RESONANT CONVERTER WITH SYMMETRICAL COMPONENTS

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Abstract

A symmetrical DC—DC thyristor converter supplied by asymmetrical input voltages is investigated with symmetrical components. The converter needs no forced commutation and operates at high frequency at full load. The operational states of the so-called symmetrical, asymmetrical and general energy supply are discussed. The converter can be operated as an active filter. An approximate description of the transient response is given.

1. Introduction

The DC—DC thyristor converter shown in Fig. 1 supplied by asymmetrical input voltages ($u_{ip} \neq |u_{in}|$) is analysed with two phase symmetrical components. Its mode of action and its simplified mathematical analysis have been described in a previous paper [1], where condensers C and C_o were supposed to be in the same order of magnitude. Now for the sake of simplicity $C_o \gg C$ is assumed. As in the previous paper, discontinuous choke currents are supposed. This assumption is verified later in the paper.

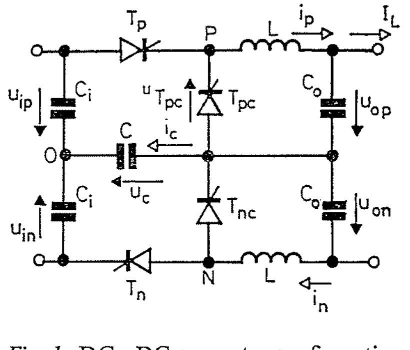

Fig. 1. DC—DC converter configuration

The converter offers some favourable features. It needs no forced commutation, operates at high frequency at full load.

Although the thyristors are conducting current pulses, there is only one sudden current variation in each current pulse. It occurs right after turning on the clamping thyristors (T_{pc}, T_{nc}). Even these sudden current changes are not generated at maximum output power. The electromagnetic interference problem is reduced.

The source of asymmetry in the supply voltages is discussed in Section 2.

Symmetrical firing of the clamping thyristors is assumed in most sections of the paper. Under this assumption three kinds of operational states are described.

In the state of symmetrical energy supply (SES) the current pulses flowing in the upper and in the lower branches of the circuit deliver the same output energy, although, in general, $u_{ip} \neq |u_{in}|$.

In the state of asymmetrical energy supply (AES) the ratio of the output energies W_{op}/W_{on} delivered by the current flowing in the upper and in the lower branches of the circuit is equal to the ratio of the supply voltages $u_{ip}/(-u_{in})$.

In the state of general energy supply (GES) the ratio of W_{op}/W_{on} can be arbitrary.

The effect of the asymmetrical firing of the clamping thyristors is mentioned as well.

The basic concept of the converter application as an active filter is described.

The fundamental statements needed for the current and voltage ratings of the thyristors are summarized.

An approximate investigation of the transient response in start-up is presented.

2. Configuration

The circuit has two main (T_p, T_n) and two clamping (T_{pc}, T_{nc}) switches, e.g. thyristors, the switched condenser C, two input (C_i) and output (C_o) condensers and two output chokes (L). The set-up is completely symmetrical (Fig. 1).

The source of asymmetry in the supply voltages can be either unequal battery voltages or unequal ratio of number of turns in the center tapped transformer supplying the rectifier. However, there are two more reasons for discussing the asymmetrical supply.

One of them is that the converter can be applied as an active filter. Even if the instantaneous values of the two input voltages u_{ip} and u_{in} are different, the output energy pulses and the output voltage can be kept approximately constant.

The other reason is the prospective application of the circuit given in Fig. 2 for three phase rectification or similar application.

The circuits to the right from terminals P—O—N in Figs. 1 and 2 are the same. The basic difference in the rest of the circuits is that the main thyristors operated by firing pulses and the supply voltages are cyclically changed in Fig. 2. During one

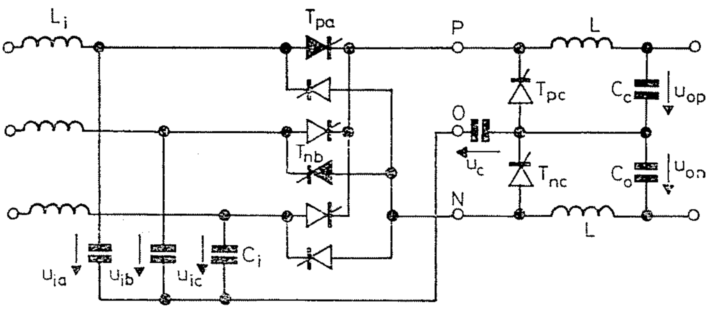

Fig. 2. Three phase thyristor rectifier with switched condenser

sixth of the period of the three phase supply voltages only one positive and one negative, for instance T_{pa} and T_{nb} thyristors are alternately gated. At the same time only one thyristor conducts current pulse transferring energy from the three phase supply to the output circuit and changes the polarity of the switched condenser C. A number of current pulses are flowing in one sixth of the supply period. The supply voltages u_{ia}, u_{ib} and u_{ic} change sinusoidally and they are not equal. The description of the circuit and its analysis is the subject of an other paper [1].

3. Clamping thyristors are not fired

In order to acquire a simple insight into the behaviour of the configuration in Fig. 1 constant and smooth positive $U_{ip}>0$ and negative $U_{in}<0$ input DC voltages and $U_{ip}\neq|U_{in}|$ are assumed. The "positive" T_p and the "negative" T_n thyristors will be fired alternately. $C_i\gg C$ and $C_o\gg C$ are assumed. Only the steady-state behaviour is treated.

After turning on the main thyristor T_p, the sinusoidal current pulse i_c flows in the circuit $C_i-T_p-L-C_o-C$ with an angular frequency $\omega=1/\sqrt{LC}$. It makes the voltage of the switched condenser u_c change from its negative maximum to its positive one (Fig. 3). Turning on the main thyristor T_n at angle $\gamma/2$, a negative sinusoidal current pulse flows across condenser C and changes the condenser voltage u_c back to its negative maximum value.

Energy considerations

The energy delivered by one positive and one negative current pulse from the input source is

$$W_{ip} = U_{ip}\int_0^{T_1} i_c \mathrm{d}t = U_{ip}C(U_{cp}-U_{cn}) \tag{1}$$

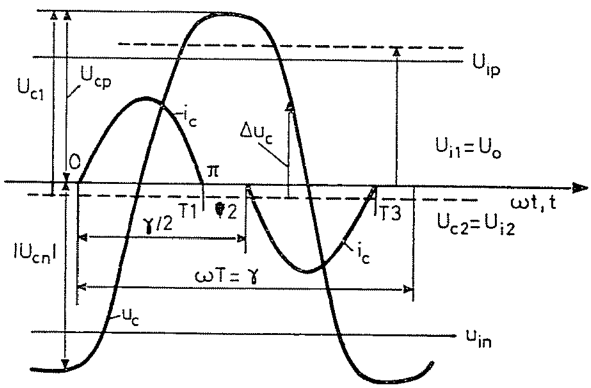

Fig. 3. Time functions with symmetrical components. Symmetrical Energy Supply. ($W_{op} = W_{on}$)

and

$$W_{in} = U_{in} \int_{T_2}^{T_3} i_c \mathrm{d}t = -U_{in} C (U_{cp} - U_{cn}), \tag{2}$$

respectively. Here U_{cp} and $(-U_{cn})$ are the positive and negative peak values of the condenser voltage swing u_c, respectively.

The corresponding energy change in the switched condenser is

$$\Delta W_{cp} = C(U_{cp}^2 - U_{cn}^2)/2, \tag{3}$$

$$\Delta W_{cn} = C(U_{cn}^2 - U_{cp}^2)/2. \tag{4}$$

Assuming lossless circuit, the output energy is

$$W_{op} = W_{ip} - \Delta W_{cp}, \tag{5}$$

$$W_{on} = W_{in} - \Delta W_{cn}. \tag{6}$$

From the last six equations

$$W_{pc} = W_{op}/C = (U_{cp} - U_{cn})[U_{ip} - (U_{cp} + U_{cn})/2], \tag{7}$$

$$W_{nc} = W_{on}/C = (U_{cp} - U_{cn})[-U_{in} + (U_{cp} + U_{cn})/2]. \tag{8}$$

The energy is proportional to the condenser voltage change $(U_{cp} - U_{cn})$ and to the input voltage as the first term on the right-hand side of the last two equations indicates, provided that $U_{cp} + U_{cn} = 0$. The second term differs from zero only if the positive and negative condenser voltage peaks, that is U_{cp} and $-U_{cn}$, are not equal.

Symmetrical component

The introduction of the two phase symmetrical components in the investigation of the asymmetrical supply is highly rewarding.

The positive and negative sequence components are: for the condenser voltage ($U_{cn}<0$):

$$U_{c1} = (U_{cp} - U_{cn})/2, \tag{9}$$

$$U_{c2} = (U_{cp} + U_{cn})/2, \tag{10}$$

for the input voltages ($U_{in}<0$):

$$U_{i1} = (U_{ip} - U_{in})/2, \tag{11}$$

$$U_{i2} = (U_{ip} + U_{in})/2, \tag{12}$$

and for the output energy:

$$W_{1c} = (W_{pc} + W_{nc})/2, \tag{13}$$

$$W_{2c} = (W_{pc} - W_{nc})/2, \tag{14}$$

where suffixes 1 and 2 denote the positive and negative sequence components, respectively.

Knowing the symmetrical components, the physical quantities can be calculated from the relations as follows

$$U_{cp} = U_{c1} + U_{c2}, \tag{15}$$

$$U_{cn} = -U_{c1} + U_{c2}. \tag{16}$$

Similarly

$$U_{ip} = U_{i1} + U_{i2}, \tag{17}$$

$$U_{in} = -U_{i1} + U_{i2} \tag{18}$$

and

$$W_{pc} = W_{1c} + W_{2c}, \tag{19}$$

$$W_{nc} = W_{1c} - W_{2c}. \tag{20}$$

Substituting eqs. (9) and (10) into eqs. (7) and (8) we end up with

$$W_{pc} = 2U_{c1}(U_{ip} - U_{c2}), \tag{21}$$

$$W_{nc} = 2U_{c1}(-U_{in} + U_{c2}). \tag{22}$$

Adding first and in the second step subtracting the last two equations and substituting eqs. (11)...(14) into the results the following relations are obtained

$$W_{1c} = 2U_{c1}U_{i1}, \tag{23}$$

$$W_{2c} = 2U_{c1}(U_{i2} - U_{c2}). \tag{24}$$

The interpretation of eqs. (23) and (24) is as follows: The positive sequence voltage components U_{c1} and U_{i1} determine the energy transport from the input to the output while the negative sequence components U_{c2} and U_{i2} are irrelevant to that.

Knowing the input voltages and prescribing the energies W_{pc} and W_{nc}, the positive and negative sequence components of the condenser voltage can be calculated from eqs. (23) and (24).

Symmetrical Energy Supply (SES)

One of the important cases, from a practical point of view, is the SES when

$$W_{op} = W_{on} \tag{25}$$

although $U_{ip} \neq |U_{in}|$. Eq. (25) ensures $U_{op} = U_{on} = U_o$.

The negative sequence component $W_{2c} = 0$ and from eq. (24)

$$U_{c2} = U_{i2}. \tag{26}$$

On the basis of eqs. (17), (18) and (26), Fig. 1 can be redrawn in the form shown in Fig. 4. The extension of the supply circuit drawn by a heavy line is based on eqs. (17) and (18). According to eq. (26), terminals $K1$ and $K2$ are equipotential. Both the upper and the lower side of the circuit are supplied by the positive sequence voltage U_{i1}.

After turning on either thyristor T_p or T_n, the current pulse i_c and the energy transport are not influenced by voltage U_{i2} since it is balanced by voltage U_{c2}.

The instantaneous value of the condenser voltage (Fig. 3)

$$u_c = U_{c2} + \Delta u_c. \tag{27}$$

When $u_c = U_{cp}$ then $\Delta u_c = U_{c1}$ [eq. (15)]

and

when $u_c = U_{cn}$ then $\Delta u_c = -U_{c1}$ [eq. (16)].

The condenser voltage u_c is symmetrical in respect to $U_{c2} = U_{i2}$ and it swings around U_{c2} with an amplitude U_{c1} (Fig. 3).

The positive and the negative current pulses must be the same since they generate the same condenser voltage change Δu_c in steady-state. The amplitude of the current pulses is $(U_{i1} + U_{c1} - U_o)/\sqrt{L/C}$.

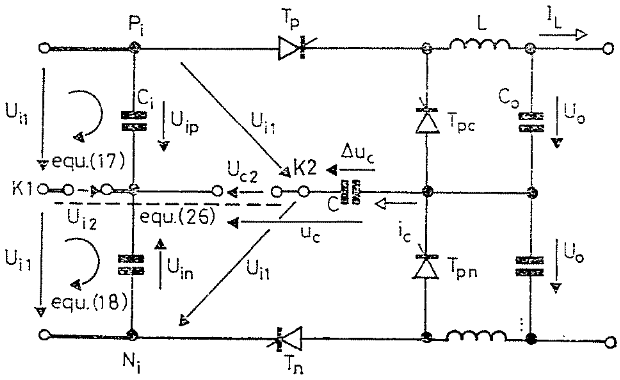

Fig. 4. Equivalent circuit for Symmetrical Energy Supply: $(W_{op} = W_{on})$

The condenser voltage amplitudes U_{cp} and U_{cn} after positive and negative current pulses in steady-state are

$$U_{cp} = -U_{cn} + 2(U_{ip} - U_o), \tag{28}$$

$$U_{cn} = -U_{cp} + 2(U_{in} + U_o), \tag{29}$$

respectively. The first term on the right-hand side corresponds to the initial condition of the condenser voltage which is reversed by the current pulse. The second term is the result of the resultant "input" voltage $(U_{ip} - U_o)$ or $(U_{in} + U_o)$ of the $L-C$ ringing circuit. Its double value develops across the condenser at the end of the current pulse.

From the last two equations

$$U_o = U_{i1}. \tag{30}$$

It means that after turning on thyristor T_p, voltages U_{i1} and U_o are balancing each other and the condenser voltage reverses itself around U_{c2} with an amplitude U_{c1} as the result of the current pulse. There are no means to change U_o in steady-state at constant supply voltages.

Another way to look at the circuit is to notice that the voltages between terminals P_i and $K2$ as well as between terminals $K2$ and N_i are U_{i1} since $K1$ and $K2$ are equipotential (Fig. 4).

It should be pointed out that though current pulses are the same both in the positive and in the negative sides of the circuit in steady-state, the energies delivered by them out of the input power source are still in ratio

$$W_{ip}/W_{in} = U_{ip}/(-U_{in}). \tag{31}$$

Assuming that $-U_{in} > U_{ip}$, at the end of the positive current pulse i_p the condenser energy is reduced and this condenser energy change ΔW_{cp} furthermore the energy W_{ip} taken by i_p from power source U_{ip} provide the output energy W_{op}. On the other hand, at the end of the negative current pulse i_n the condenser energy is increased and now the energy W_{in} taken by i_n from power source U_{in} partly supports the output energy W_{on} and partly the condenser energy change ΔW_{cn}. The condenser plays the role of a buffer. One of the power sources having higher voltage supplies more power than the other. The average output power from eq. (23) is

$$P_o = 2W_{o1}/T = 4CU_{c1}U_{i1}/T \tag{32}$$

and the output voltage:

$$U_o = 2CU_{c1}U_{i1}/(TI_L), \tag{33}$$

where I_L is the load current. Being $U_o = U_{i1}$ [eq. (30)]

$$U_{c1} = TI_L/2C. \tag{34}$$

In order to commutate thyristors T_p and T_n inequalities to be satisfied are $U_{cp} > U_{ip}$

and $-U_{cn} > -U_{in}$, respectively or with symmetrical components

$$U_{c1} = (TI_L/2C) > U_{i1} \tag{35}$$

must be satisfied. It means that the product TI_L must be kept above $2CU_{i1}$.

In the *symmetrical case* $U_{ip} = U_{in}$ and $U_{i2} = U_{c2} = 0$ as well as $U_{cp} = -U_{cn} = U_{c1}$. There is no energy exchange with the help of condenser C between the upper and lower sides of the circuit.

Asymmetrical Energy Supply (AES)

Another reasonable choice for the output energies is

$$W_{op} = [U_{ip}/(-U_{in})]W_{on}. \tag{36}$$

It means that the higher the voltage of the power source is, the higher the output energy delivered by its current pulse will be.

The positive sequence component of the two output energies is

$$W_{1c} = (W_{pc} + W_{nc})/2 = W_{nc}\, U_{i1}/(-U_{in}). \tag{37}$$

The negative sequence component of the output energies is

$$W_{2c} = (W_{pc} - W_{nc})/2 = W_{nc}\, U_{i2}/(-U_{in}). \tag{38}$$

From the last two equations

$$W_{o1}/W_{o2} = U_{i1}/U_{i2}. \tag{39}$$

On the basis of eqs. (23), (24) and (39)

$$U_{c2} = 0. \tag{40}$$

Now the condenser voltage swing is symmetrical to the zero axis (Fig. 5). Its amplitude $U_{c1} = U_{cp} = -U_{cn}$ is determined by the output power. The equivalent circuit is shown in Fig. 6. The negative sequence component of the condenser voltage U_{c2} does not balance U_{i2}. For this reason and because the current pulses in the two sides must be the same, the output voltages are different. The current pulses

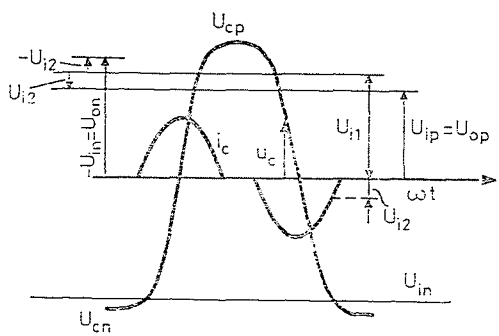

Fig. 5. Asymmetrical Energy Supply. Time functions. $W_{op} = [U_{ip}/(-U_{in})]\, W_{on}$

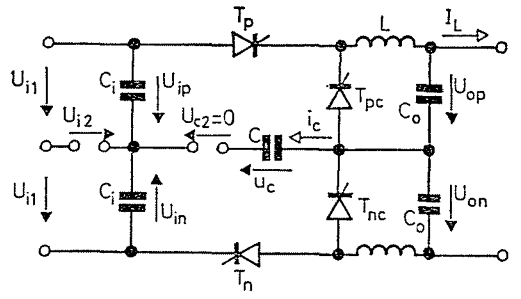

Fig. 6. Equivalent circuit for Asymmetrical Energy Supply: $W_{op}=[U_{ip}/(-U_{in})]\,W_{on}$

are the same if

$$U_{op} = U_{ip}, \tag{41}$$

$$U_{on} = -U_{in}. \tag{42}$$

The condenser energy is the same after each current pulse. The output energies W_{op} and W_{on} are not modified by the condenser energy changes

$$\Delta W_{cp} = 0, \quad \Delta W_{cn} = 0, \tag{43}$$

$$W_{op} = W_{ip}, \tag{44}$$

$$W_{on} = W_{in}. \tag{45}$$

The upper and lower sides of the circuit operate independent of each other. The average output power is

$$P_o = 2W_{o1}/T = 4CU_{c1}\,U_{i1}/T \tag{32}$$

and the resultant output voltage

$$U_{op}+U_{on} = 2U_{o1} = 4CU_{c1}\,U_{i1}/(TI_L). \tag{46}$$

Taking into account eqs. (41) and (42), relation

$$U_{o1} = U_{i1} \tag{47}$$

and

$$U_{c1} = TI_L/(2C) \tag{48}$$

are obtained again. Inequality (35) must be satisfied as well. The output voltage cannot be changed again in the case of fixed input voltages.

4. Clamping thyristors are fired

Now the output voltage becomes changeable in the range

$$0 \leqq 2U_{o1} \leqq 2U_{i1}. \tag{49}$$

By suitable selection of the firing angle of the clamping thyristors T_{pc} and T_{nc} either a symmetrical or an asymmetrical energy supply can be set.

In the next study discontinuous choke currents will be assumed, that is, the positive main thyristor is gated only after the positive choke current has reached zero. The same statement holds for the negative side of the circuit.

4.1. Symmetrical Energy Supply ($W_{op}=W_{on}$)

On the basis of Figs. 3 and 4 the time functions of i_c and u_c are shown in Fig. 7. Here and later the $*$ denotes the clamped value of the condenser voltage. The procedure of the calculation for given U_{ip}, U_{in}, U_o, P_o and $U_{c1}^{*\prime}=U_{c1}^*/U_{i1}>1$ is as follows. $U_{c1}^{*\prime}$ is the value in p.u.

Inequality $U_{c1}^*>U_i$ must be honoured [see eq. (35)]. It is a precondition for turning on thyristors T_{pc} and T_{nc} and for turning off thyristors T_p and T_n, respectively.

First U_{c2}^* is determined from eq. (24)

$$U_{c2}^* = U_{i2}. \tag{26}$$

Knowing U_{c2}^* and U_{c1}^*, the condenser voltage peak values U_{cp}^* and U_{cn}^* are calculated from eqs. (15) and (16). The condenser current pulse

$$i_c = \frac{U_{i1}+U_{c1}^*-U_o}{\sqrt{L/C}} \sin \omega t \tag{50}$$

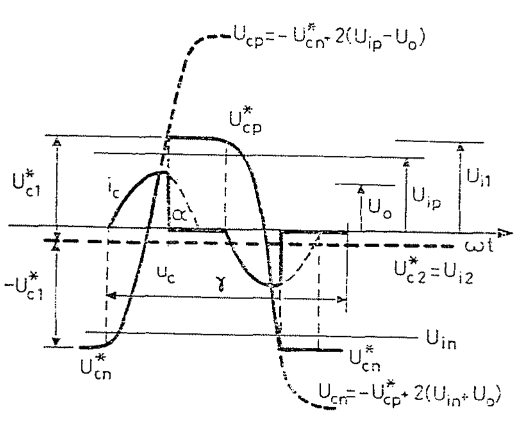

Fig. 7. Clamping thyristors are fired ($W_{op}=W_{on}$)

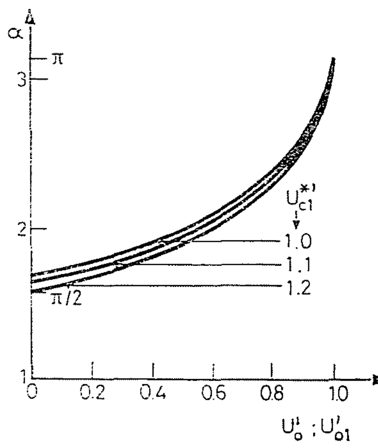

Fig. 8. Firing angle α versus U_o/U_{i1} or U_{o1}/U_{i1}

and the condenser voltage change between 0 and α is

$$U_{cp}^{*}-U_{cn}^{*}=\frac{1}{C}\int_{0}^{\alpha} i_c\,d\omega t=(U_{i1}+U_{c_1}^{*}-U_o)(1-\cos\alpha). \tag{51}$$

$\cos\alpha$ can be calculated

$$\cos\alpha=\frac{1-U_o'-U_{c1}^{*\prime}}{1-U_o'+U_{c1}^{*\prime}}, \tag{52}$$

where $U_o'=U_o/U_{i1}$. When $U_o=U_{i1}$ then $\cos\alpha=-1$, $\alpha=\pi$, independently of value $U_{c1}^{*\prime}$. Fig. 8 shows the variation of α as a function of U_o/U_{i1}.

Finally, the period T is calculated from eq. (32)

$$T=4CU_{c1}^{*\prime}\,U_{i1}/P_o. \tag{53}$$

The most important unknowns belonging to the variables given at the beginning have been determined.

Had the clamping thyristors not been fired eq. (51) would have been $(\alpha=\pi)$

$$U_{cp}-U_{cn}^{*}=2(U_{i1}+U_{c1}^{*}-U_o).$$

Taking into account $U_{c2}^{*}=U_{i2}$, the result is (Fig. 7)

$$U_{cp}=-U_{cn}^{*}+2(U_{ip}-U_o). \tag{54}$$

It can be easily interpreted on the basis of the superposition theorem. The condenser voltage U_{cp} at the end of the total current pulse (α) is equal with the reverse value of the initial condenser voltage $-U_{cn}^{*}$ plus the double value of the resultant "input" voltage $2(U_{ip}-U_o)$. The condenser voltage increase over the clamped value U_{cp}^{*} would be

$$U_{cp}-U_{cp}^{*}=2(U_{i1}-U_o). \tag{55}$$

It can be shown in a similar way that (Fig. 7)

$$U_{cn}-U_{cn}^{*}=-2(U_{i1}-U_o). \tag{56}$$

By firing the clamping thyristors at an angle α given in eq. (52), the continuous increase in the peak values of the condenser voltage after each current pulse can be avoided at $U_{i1}>U_o=\text{const.}$, or in other words peak values U_{cp}^{*} and U_{cn}^{*} can be kept constant.

Assuming $U_{ip}>0$ and $U_{in}=0$ as the highest asymmetry in the supply voltage, SES can be maintained.

The output voltage

$$U_o=\frac{2C}{T}\frac{U_{c1}^{*}U_{i1}}{I_L}=\frac{2C}{T}U_{c1}^{*\prime}\frac{U_{i1}^{2}}{I_L} \tag{33}$$

can be modified by changing period T when $U_{c1}^{*\prime}$, U_{i1} and I_L are constant.

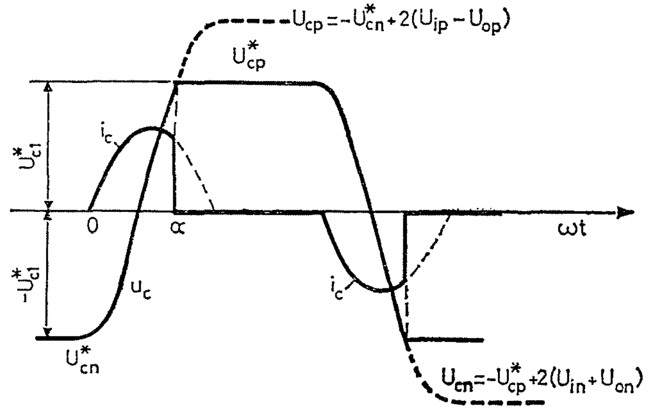

Fig. 9. Clamping thyristors are fired. Asymmetrical Energy Supply

4.2. *Asymmetrical Energy Supply* $[W_{op}=W_{on}U_{ip}/(-U_{in})]$

On the basis of Figs. 5 and 6 the time functions of i_c and u_c are shown in Fig. 9.

Now the procedure to calculate a given U_{ip}, U_{in}, $(U_{op}+U_{on})$, P_o and U_{c1}^* is as follows. Given $U_{c2}^*=0$ [eq. (40)], the clamped peak values of the condenser voltages are the same

$$U_{cp}^* = -U_{cn}^* = U_{c1}^*. \tag{57}$$

The condenser voltage change generated by the positive i_c current pulse between 0 and α is

$$2U_{c1}^* = \frac{1}{\omega C}\int_0^\alpha i_c \,\mathrm{d}\omega t = (U_{ip}+U_{c1}^*-U_{op})(1-\cos\alpha). \tag{58}$$

The condenser voltage change caused by the negative i_c current pulse is

$$2U_{c1}^* = (-U_{in}+U_{c1}^*-U_{on})(1-\cos\alpha). \tag{59}$$

Equal clamping angle α has been chosen. It means that the positive and negative current pulses are identical, their peak values are the same (Fig. 6)

$$U_{i1}+U_{i2}+U_{c1}^*-U_{op} = U_{i1}-U_{i2}+U_{c1}^*-U_{on},$$

and from here:

$$U_{o2} = (U_{op}-U_{on})/2 = U_{i2}. \tag{60}$$

The peak value of the current with symmetrical components is:

$$I_{cp} = \frac{1}{\sqrt{L/C}}(U_{i1}+U_{c1}^*-U_{o1}) \tag{61}$$

where $U_{o1}=(U_{op}+U_{on})/2$.

Knowing U_{o1} and U_{o2} voltages U_{op} and U_{on} can be calculated.

Both eqs. (58) and (59) with symmetrical components take the form as follows

$$2U_{c1}^* = (U_{i1} + U_{c1}^* - U_{o1})(1 - \cos\alpha)$$

and from here

$$\cos\alpha = \frac{1 - U_{o1}' - U_{c1}^*}{1 - U_{o1}' + U_{c1}^*}. \tag{62}$$

The expression is the same as the one given by eq. (52) except that U_{o1}' replaces U_o'. The curves in Fig. 8 are valid for the asymmetrical energy supply as well.

The period is determined from the relation [see eq. (23)]

$$T = \frac{2W_{o1}}{P_o} = \frac{2C}{I_L}\frac{U_{c1}^* U_{i1}}{U_{o1}}. \tag{63}$$

In fact, both in symmetrical and in asymmetrical energy supply the configuration is a constant average output power delivering device at constant input voltage U_{i1}, condenser voltage swing U_{c1}^* and current pulse frequency $1/T$, since from eq. (32): $P_o = 4CU_{c1}^* U_{i1}/T$.

4.3. General Energy Supply (GES)

The ratio of the output energies in a general case is

$$W_{op}/W_{on} = \beta. \tag{64}$$

In SES $\beta = 1$ and in AES $\beta = U_{op}/|-U_{on}|$. The symmetrical components of the output energies are

$$W_{o1} = (\beta + 1)W_{on}/2, \tag{65}$$

$$W_{o2} = (\beta - 1)W_{on}/2. \tag{66}$$

The ratio of these energies from eqs. (23) and (24) is

$$\frac{W_{o2}}{W_{o1}} = \frac{\beta - 1}{\beta + 1} = \delta = \frac{U_{i2} - U_{c2}^*}{U_{i1}} \tag{67}$$

that is

$$U_{c2}^* = U_{i2} - \delta U_{i1}. \tag{68}$$

The procedure of the calculation for given U_{ip}, U_{in}, $(U_{op} + U_{on})$, P_o, U_{c1}^* and β is very similar to the case of AES.

U_{c2}^* can be calculated from eq. (68).

The condenser voltage change generated by the positive condenser current pulse is [eq. (58)]

$$2U_{c1}^* = (U_{ip} - U_{cn}^* - U_{op})(1 - \cos\alpha_p) \tag{69}$$

and that generated by the negative condenser current pulse is

$$2U_{c1}^* = (-U_{in} + U_{cp}^* - U_{on})(1 - \cos \alpha_n). \tag{70}$$

In eqs. (69) and (70) U_{op}, U_{on}, α_p and α_n are unknowns but $(U_{op}+U_{on})$ is given. One unknown variable out of the four can be arbitrarily chosen.

Symmetrical Clamping Angle (SCA)

Supposing again $\alpha=\alpha_p=\alpha_n$, the current peak values must be the same. Expressing the voltage sum in eqs. (69) and (70) by symmetrical components, the results are

$$U_{ip} - U_{cn}^* - U_{op} = \sum U + (\delta U_{i1} - U_{o2}) \tag{71}$$

and

$$-U_{in} + U_{cp}^* - U_{on} = \sum U - (\delta U_{i1} - U_{o2}) \tag{72}$$

where

$$\sum U = U_{i1} + U_{c1}^* - U_{o1}. \tag{73}$$

The current peak values are the same only if

$$U_{o2} = \delta U_{i1}. \tag{74}$$

U_{op} and U_{on} are now determined from U_{o1} and U_{o2}. The condenser current peak value I_{cp} from eq. (61)

$$(I_{cp})_{GES} = \frac{1}{\sqrt{L/C}}(U_{i1} + U_{c1}^* - U_{o1}). \tag{74a}$$

$\cos \alpha$ and T are calculated from eqs. (62) and (63), respectively.

The equivalent circuit for GES is shown in Fig. 10. It includes both the SES (Fig. 4) and the AES (Fig. 6).

In GES the condenser voltage $u_c = U_{c2}^* + \Delta u_c$ oscillates around a constant *DC* voltage $U_{c2}^* = U_{i2} - U_{o2}$ with amplitude U_{c1}^*, that is, around the asymmetry between the input voltages and that of the output voltages.

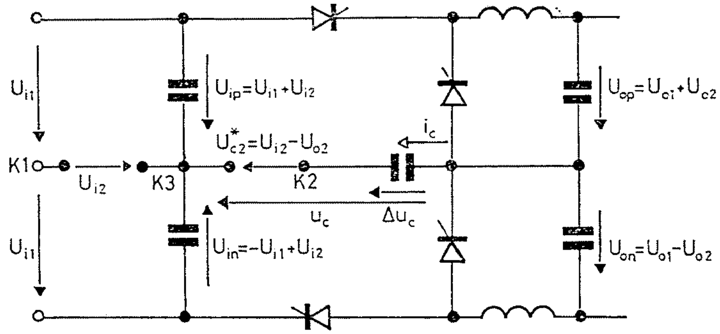

Fig. 10. Equivalent circuit for General Energy Supply

In SES $\beta = W_{op}/W_{on} = 1$ and $\delta = 0$ as well as $U_{o2} = 0$, that is, $U_{op} = U_{on} = U_{o1}$. Terminals $K1$ and $K2$ are short circuited. There being no asymmetry in the output voltages, u_c oscillates around U_{i2}.

In AES $\beta = U_{ip}/(-U_{in})$ and $\delta = U_{i2}/U_{i1}$ as well as $U_{o2} = U_{i2}$, that is, $U_{c2}^* = 0$. Terminals $K2$ and $K3$ are short circuited. As the asymmetry is the same in the input voltages and in the output voltages, u_c oscillates around zero.

The various operational states can be set by choosing the clamped peak values of the condenser voltage U_{cn}^* and U_{cp}^*, that is, by the firing angle of the clamping thyristors α.

5. Asymmetrical Clamping Angle (ACA)

In all previous cases $\alpha_p = \alpha_n = \alpha$ were set. One of the consequences was that the peak values of the positive $(I_{cp})_p$ and that of the negative $(I_{cp})_n$ condenser current pulses were the same. Another consequence was that

$$\text{in SES} \quad U_{op} = U_{on} = U_{o1} = U_o, \quad U_{o2} = 0$$

and

$$\text{in AES} \quad U_{op} \neq U_{on}, \quad U_{o2} = U_{i2}$$

were obtained, respectively. None of these hold any longer in ACA even in SES or in AES, as we will see.

Setting α_p and α_n at different values, the peak currents $(I_{cp})_p$ and $(I_{cp})_n$ are not the same, since the average values of the positive and of the negative condenser current pulses must be equal, their peak values must be different. On the basis of eqs. (71)...(73)

$$\frac{(I_{cp})_p}{(I_{cp})_n} = \frac{1 - U'_{o1} + U_{c1}^{*\prime} + U'_\delta}{1 - U'_{o1} + U_{c1}^{*\prime} - U'_\delta} \tag{75}$$

where $U'_\delta = U_\delta / U_{i1}$ and

$$U_\delta = \delta U_{i1} - U_{o2}. \tag{76}$$

$\cos\alpha_p$ and $\cos\alpha_n$ from eqs. (69)...(73) are

$$\cos\alpha_p = \frac{1 - U'_{o1} - U_{c1}^{*\prime} + U'_\delta}{1 - U'_{o1} + U_{c1}^{*\prime} + U'_\delta} \tag{77}$$

and

$$\cos\alpha_n = \frac{1 - U'_{o1} - U_{c1}^{*\prime} - U'_\delta}{1 - U'_{o1} + U_{c1}^{*\prime} - U'_\delta}, \tag{78}$$

respectively.

GES includes SES and AES. $\alpha_p \neq \alpha_n$ can be set in each case. As a consequence U_{op} will not be the same as U_{on} even in SES. For example, let α_p and α_n be π and $\pi/2$, respectively and $U_{c1}^{*\prime} = 1$ in SES when $\beta = 1$, $\delta = 0$ and $U_\delta = -U_{o2}$. From eq. (78)

$$U_{o2} = U_{o1}.$$

It means that $U_{on}=0$, $U_{op}=2U_{o1}$, that is, the output voltages are far from being equal. Now $U_{o1}=U_{i1}/2$ [eq. (77)] and $(I_{cp})_p/(I_{cp})_n=1/2$ [eq. (75)] are obtained, respectively. Theoretically it is a limit case. It has been assumed at the beginning that the choke currents extinct as a result of the output voltage before firing the next main thyristors. But now the output voltage $U_{on}=0$.

There is an interesting possibility in ACA. One of the angles is kept at value π and only the other one is changed. Be $\alpha_p=\pi$. By changing only α_n, the limitation of the condenser voltage amplitude U_{c1}^* is ensured.

From eq. (77) at $\alpha_p=\pi$

$$U_{op} = U_{o1}+U_{o2} = (1+\delta)\,U_{i1}. \tag{79}$$

In SES $\delta=0$ and $U_{op}=U_{i1}$.

In AES $\delta=U_{i2}/U_{i1}$ and $U_{op}=U_{i1}+U_{i2}=U_{ip}$.

U_{op} is constant and the output voltage $2U_{o1}$ can be changed only by the variation of U_{on}.

From eq. (78)

$$U_{on} = U_{o1}-U_{o2} = U_{i1}\left(1-\delta+\frac{\cos\alpha_n+1}{\cos\alpha_n-1}\,U_{c1}^{*\prime}\right). \tag{80}$$

$U_{on}=U_{in}(1-\delta)$ when $\alpha_n=\pi$. Reducing α_n, U_{on} can be decreased to zero reaching thus the limit case. When the output voltage has to be changed only in the range:

$$U_{i1} < U_{op}+U_{on} = 2U_{o1} < 2U_{i1}$$

one of the clamping thyristors can be omitted.

6. Discontinuous choke current

It has been assumed that the choke currents are discontinuous. In other words it means that for the extinction angles

$$\alpha_{ep} < \omega T \tag{81}$$

and

$$\alpha_{en} < \omega T \tag{82}$$

must be satisfied.

Now the realization of conditions (81) and (82) is investigated. The study is restricted to the case of $\alpha_p=\alpha_n=\alpha$ and to SES and AES.

The time functions of the choke current are

$$i_p(\omega t) = -\frac{U_{op}}{\omega L}(\omega t-\alpha)+i_c(\alpha) \tag{83}$$

and

$$i_n(\omega t) = -\frac{U_{on}}{\omega L}(\omega t-\alpha)+i_c(\alpha), \tag{84}$$

respectively. At the extinction angles $i_p(\alpha_{ep})=0$ and $i_n(\alpha_{en})=0$. The extinction angles are

$$\alpha_{ep} = \alpha + X_L i_c(\alpha)/U_{op}, \tag{85}$$

$$\alpha_{en} = \alpha + X_L i_c(\alpha)/U_{on}, \tag{86}$$

where $X_L=\omega L$. The choke current is discontinuous if both

$$\alpha_{ep} \leqq \omega T \tag{87}$$

and

$$\alpha_{en} \leqq \omega T. \tag{88}$$

Substituting into (87) and (88)

$$i_c(\alpha) = I_{cp} \sin \alpha, \tag{89}$$

$$I_{cp} = \frac{1}{\sqrt{L/C}}(U_{i1} + U_{c1}^{*} - U_{o1}) \tag{61}$$

as well as eqs. (62) and (63), the results are

$$\alpha_{ep} = \cos^{-1} H + N\sqrt{1-H^2}\frac{1}{U'_{o1}+U'_{o2}} < \frac{\pi}{I'_L}\frac{U_{c1}^{*\prime}}{U'_{o1}}, \tag{90}$$

$$\alpha_{en} = \cos^{-1} H + N\sqrt{1-H^2}\frac{1}{U'_{o1}-U'_{o2}} < \frac{\pi}{I'_L}\frac{U_{c1}^{*\prime}}{U'_{o1}}, \tag{91}$$

where

$$U'_{o1} = U_{o1}/U_{i1};\; U'_{o2} = U_{o2}/U_{i1},$$

$$N = 1 + U_{c1}^{*\prime} - U'_{o1};\; H = (1 - U_{c1}^{*\prime} - U'_{o1})/N$$

and

$$I'_L = \frac{I_L}{\frac{2}{\pi}\omega C U_{i1}}. \tag{92}$$

In *SES* $U'_{o2}=0$ and $\alpha_{ep}=\alpha_{en}=\alpha_e$. As long as the load current is smaller than the limit value of the load current $(I'_L)_{\text{limit}}$ the choke current is discontinuous. The limit value of the load current from eq. (90)

$$(I'_L)_{\text{limit}} = \pi \frac{U_{c1}^{*\prime}}{U'_{o1}} \frac{1}{\alpha_e}. \tag{93}$$

Fig. 11 shows $(I'_L)_{\text{limit}}$ as a function of U'_{o1} for different peak values of condenser voltage u_c. The choke current is discontinuous in the full range of U'_{o1}, provided that the load current $I'_L \leqq 1$.

In *AES* $U'_{o2} \neq 0$ and $\alpha_{ep} \neq \alpha_{en}$. To calculate the limit value of the load current the more severe condition between (90) and (91) must be used. For example, when $U'_{o2}>0$

$$(I'_L)_{\text{limit}} = \pi \frac{U_{c1}^{*\prime}}{U'_{o1}} \frac{U'_{o1}-U'_{o2}}{(U'_{o1}-U'_{o2})\cos^{-1} H + N\sqrt{1-H^2}}. \tag{94}$$

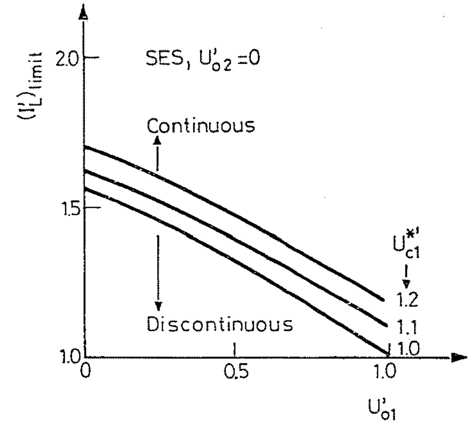

Fig. 11. The border between the continuous and the discontinuous current conduction region in SES

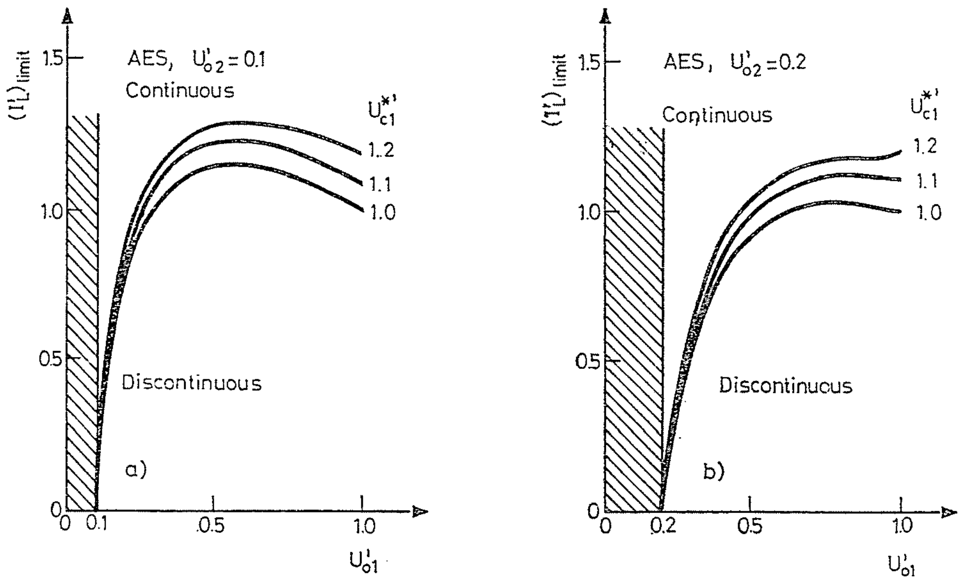

Fig. 12. The border between the continuous and the discontinuous current conduction regions in AES

Fig. 12 presents $(I'_L)_{\text{limit}}$ as a function of U'_{o1}. The parameter is $U^{*\prime}_{c1}$ again. The discontinuous current region is smaller in AES than in SES and the larger the asymmetry, that is, U'_{o2} is, the smaller this region will be. The hatched area belonging to region $0 \leqq U'_{o1} \leqq U'_{o2}$ is excluded since U_{on} cannot be negative.

7. Active filter

The configuration can be operated both as a DC—DC converter setting the output voltage $2U_{o1}$ in the range $0 \leqq 2U_{o1} \leqq 2U_{i1}$ and at the same time as an active filter. The concept is rather simple. Besides varying input voltages the output energies W_{op}, W_{on} and approximately the output voltage and power are kept constant. The converter is operated at a rather high frequency compared to the frequency of the fundamental component in the input voltages. The low frequency pulsation of the input voltages is hardly reflected by the converter to the output side. As a consequence of the high frequency operation the output voltage filtering can be achieved easier.

As an example to illustrate the concept, let us assume a sinusoidal variation in the input voltage

$$u'_{ip} = 1 - 2U'_{im} \cos \Omega t, \tag{95}$$

$$u'_{in} = -1 \tag{96}$$

where the $'$ denotes variables in per unit and the base quantity is $U_{i1} = \text{const.}$, as well as $\Omega \ll 2\pi/T$, that is, the converter is operated at a high frequency compared to the frequency of the input voltages. The instantaneous voltages of u_{ip} and u_{in} are different in all center tapped, multiphase rectifications. The same statement holds for the circuit shown in Fig. 2. A similar supply may arise in the rare case when a rectifier supplies one of the input voltages and the other input voltage is taken from a battery.

The positive and the negative symmetrical components are

$$u'_{i1} = 1 - U'_{im} \cos \Omega t, \tag{97}$$

$$u'_{i2} = \quad - U'_{im} \cos \Omega t, \tag{98}$$

respectively.

In the time span $T = \gamma/\omega$ the input voltage u_{ip} is approximated by a constant value (Fig. 13). The period of the input voltages is divided into $N\gamma$ equidistant intervals where N can be as high as 100, or so. The approximate input voltages in the k'th γ cycle are:

$$u'_{i1}(k) = 1 - U'_{im} \cos k\gamma, \tag{99}$$

$$u'_{i2}(k) = \quad - U_{im} \cos k\gamma, \tag{100}$$

where $k = 0, 1, 2, \ldots, (N-1)$. Better approximations can be easily devised.

The positive sequence component of the output energy W_{o1} has to be kept constant [eq. (23)]

$$W'_{o1} = W_{1c}/(2U_i^2) = u'_{i1}(k)\, u^{*\prime}_{c1}(k)$$

that is

$$u^{*\prime}_{c1}(k) = W'_{o1} \frac{1}{1 - U'_{im} \cos k\gamma} = \frac{U'}{1 - U'_{im} \cos k\gamma}. \tag{101}$$

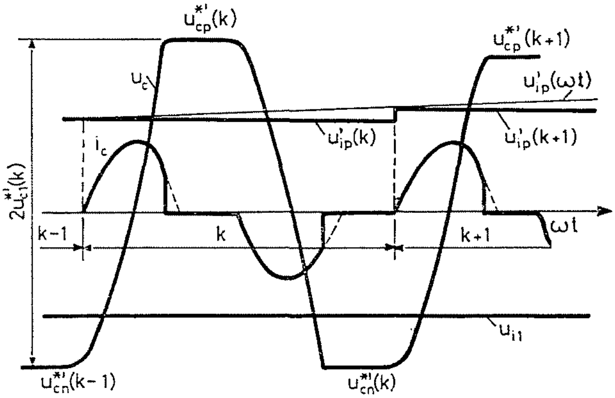

Fig. 13. The approximation of input voltage $u'_{ip}(\omega t)$

Now

$$\Delta = \frac{u^{*\prime}_{c1}}{u'_{i1}} = \frac{U'}{(1-U'_{im}\cos k\gamma)^2} \tag{102}$$

is not constant. U' must be chosen in such a way that $\Delta > 1$ be always satisfied [see eq. (35)], that is,

$$U' \geqq (1+U'_{im})^2. \tag{103}$$

Relation

$$u^{*\prime}_{cp}(k) \geqq u'_{ip}(k) \tag{104}$$

must be met as well. Substituting here eqs. (9) and (95):

$$u^{*\prime}_{cn}(k) \geqq 1-2U'_{im}\cos k\gamma - 2u^{*\prime}_{c1}(k). \tag{104a}$$

As $u'_{in}=-1$, viz. it is constant, it is reasonable to assume that $u^{*\prime}_{cn}(k)=U^{*\prime}_{cn}=\text{const.}<0$. Taking into account eqs. (101) and (104a),

$$U^{*\prime}_{cn} > 1-2U'_{im}\cos k\gamma - \frac{2U'}{1-U'_m\cos k\gamma} = M. \tag{105}$$

M takes its maximum value, that is, its smallest negative value at $\cos k\gamma = -1$. Assuming $U'=\varrho(1+U'_{im})^2$, the smallest possible value for $|U^{*\prime}_{cn}|$ from eq. (105) at a given ϱ is

$$U^{*\prime}_{cn} = 1+2U'_{im} - \frac{2\varrho(1+U'_{im})^2}{1+U'_{im}} = 1-2\varrho+2U'_{im}(1-\varrho) \tag{106}$$

where $\varrho \geqq 1$ positive number. $U^{*\prime}_{cn}=-1$ when $\varrho=1$.

The positive peak value of the condenser voltage from eq. (9)

$$u^{*\prime}_{cp}(k) = 2u^{*\prime}_{c1}(k)+U^{*\prime}_{cn}.$$

Substituting here eq. (101)

$$u_{cp}^{*\prime}(k) = \frac{2\varrho(1+U_{im}')^2}{1-U_{im}'\cos k\gamma} + U_{cn}^{*\prime}. \tag{107}$$

In general, the circuit is in the state of GES. The value $\delta(k)$ is changing from cycle to cycle and its value can be determined from eq. (68)

$$\delta(k) = \frac{u_{i2}'(k) - u_{c2}^{*\prime}(k)}{u_{i1}'(k)}.$$

The calculation results are given in Fig. 14. By changing $u_{cp}^{*\prime}(k)$ in the way given in eq. (107), the output energy $W_{o1}''(k)$ in each period is the same.

In most cases the *DC* and the alternating components are equal in u_{ip} and u_{in}. Assuming

$$u_{ip}' = -u_{in}' = 1 - U_{im}'\cos\Omega t,$$

the concept of active filtering described above can be applied in this case as well. For the sake of convenience let us use the symmetrical components again. Now

$$u_{i1}' = u_{ip}' = 1 - U_{im}'\cos\Omega t$$

and

$$u_{i2}' = 0.$$

The same expressions as above can be used for the calculation of $u_{c1}^{*\prime}$, Δ and U'.

Taking now the negative peak value of the condenser voltage

$$u_{cn}^{*\prime}(k) = u_{in}'(k) = -[1 - U_{im}'\cos k\gamma] \tag{106a}$$

rather than keeping it constant.

$u_{cp}^{*\prime}(k)$ has to be changed as follows:

$$u_{cp}^{*\prime}(k) = 2u_{c1}^{*\prime}(k) + u_{cn}^{*\prime}(k). \tag{107a}$$

The output energy $W_{o1}''(k)$ is kept constant again. The fundamental component with angular frequency Ω is approximately eliminated in the resultant output voltage $(u_{op}+u_{on})$.

Fig. 15 shows $u_{cp}^{*\prime}(k)$ and $u_{c2}^{*\prime}(k)$. The functions $u_{ip}'(k)$, $\Delta(k)$, $u_{i1}'(k)$, $u_{c1}^{*\prime}(k)$ and those plotted in Fig. 14 c. are the same in both cases. The large fluctuation of the peak value in the condenser voltage $u_{cp}^{*\prime}$ and $u_{cn}^{*\prime}$ is the price that has to be paid for keeping the output energy W_{o1}=const.

8. Thyristor ratings

The various time functions are shown in Fig. 16 for the symmetrical energy supply (SES). Similar time functions can be drawn for AES and GES. First the $\alpha_p=\alpha_n=\alpha$ case will be treated here and all three states, namely SES, AES and GES will be discussed.

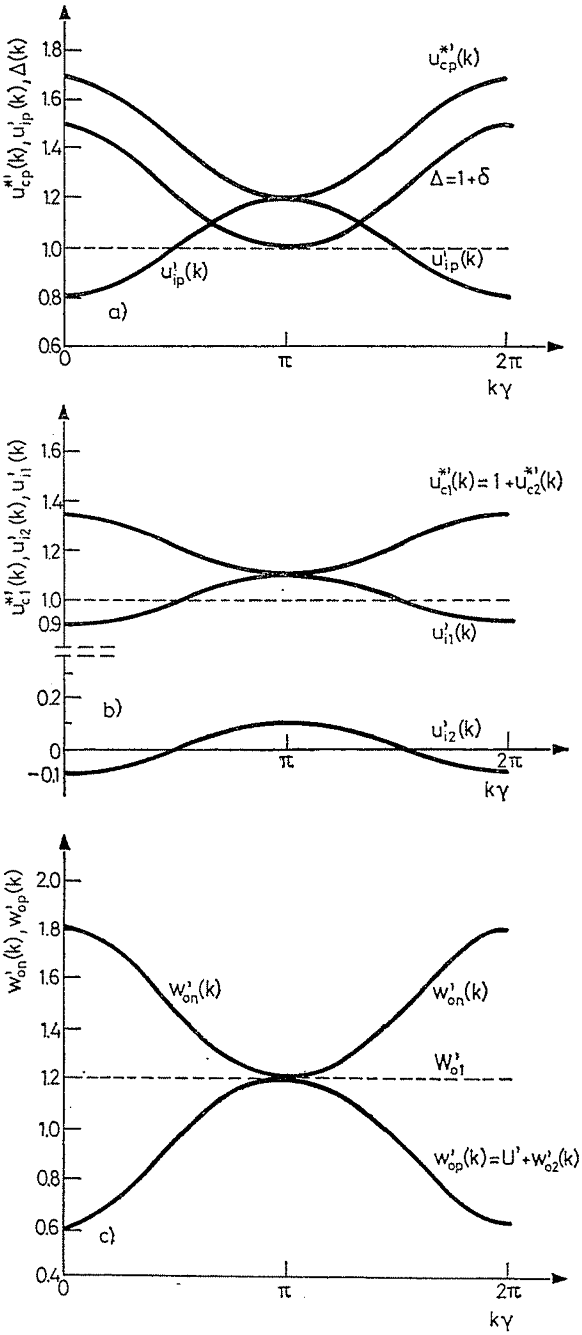

Fig. 14. $u'_{ip}=1-2U'_{im}\cos\Omega t$ and $u'_{in}=-1$. The output energy W_{o1} is kept constant in each period by changing only the condenser voltage amplitude $u^{*\prime}_{cp}(k)$. Data: $U'_{im}=0.1$, $\gamma=3.6$ and $\varrho=1$

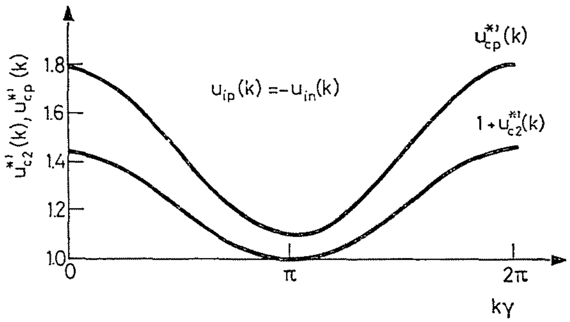

Fig. 15. $u'_{ip} = -u'_{in} = 1 - U'_{im} \cos \Omega t$. The output energy W_{o1} is kept constant in each period by changing both condenser voltage amplitudes $u^{*\prime}_{cp}(k)$ and $u^{*\prime}_{cn}(k)$. Data: $U'_{im} = 0.1$, $\gamma = 3.6$ and $\varrho = 1$

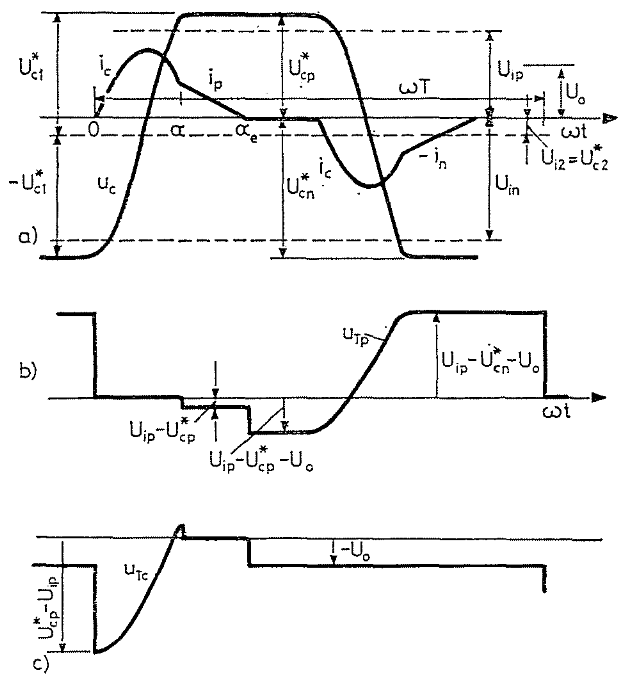

Fig. 16. Time functions for Symmetrical Energy Supply.

Fig. a) Condenser current and voltage.
Fig. b) Main thyristor voltage u_{Tp}.
Fig. c) Clamping thyristor voltage u_{Tc}.
Scale is different in c) and b) compared to a)

Current rating

Main thyristors

Assuming equal output power P_o, output frequency and output voltage U_{o1} for the SES, AES and GES cases, the main thyristor currents are identical though the supply voltages are different. In fact, the peak value of the current is (Fig. 4)

$$(I_{cp})_{\text{SES}} = \frac{1}{\sqrt{L/C}}(U_{i1} + U_{c1}^{*} - U_o) \tag{108a}$$

and [eqs. (61), (74a)]

$$(I_{cp})_{\text{AES}} = (I_{cp})_{\text{GES}} = \frac{1}{\sqrt{L/C}}(U_{i1} + U_{c1}^{*} - U_{o1}). \tag{108b}$$

Since the output power and frequency are the same the condenser voltage swing U_{c1}^{*} is equal for the three operation states [eq. (63)]. Furthermore as $U_o = U_{o1}$, the condenser current peaks are the same

$$(I_{cp})_{\text{SES}} = (I_{cp})_{\text{AES}} = (I_{cp})_{\text{GES}}. \tag{108c}$$

Conclusion: The current ratings for the main thyristors are the same on the positive and on the negative sides for all three states.

Clamping thyristors

The thyristor currents are identical in the case of SES. On the other hand, they are different in the case of AES and GES. The currents start towards zero from the same value but with different rates. The ratio of the rates for the current reduction is U_{op}/U_{on}. Assuming equal output voltages for SES and AES, that is, $U_o = U_{o1}$, the average value of the current in one of the thyristors is higher in the case of AES and GES than in the case of SES.

Conclusion: The current load in one of the clamping thyristors is higher in AES and GES than in SES.

Voltage rating

Main thyristors

In SES the peak value of the voltage (Fig. 16) is either

$$(U_{Tp})_{\text{peak}} = U_{ip} - U_{cn}^{*} - U_o \tag{109}$$

or

$$(U_{Tn})_{\text{peak}} = -U_{in} + U_{cp}^{*} - U_o. \tag{110}$$

Introducing here the symmetrical components and taking into account $U_{c2}^{*} = U_{i2}$, the result is

$$(U_{Tp})_{\text{peak}} = (U_{Tn})_{\text{peak}} = U_{i1} + U_{c1}^{*} - U_o. \tag{111}$$

On the other hand, in AES and GES the peak value of the voltage is either

$$(U_{Tp})_{\text{peak}} = U_{ip} - U_{cn}^* - U_{op} \tag{112}$$

or

$$(U_{Tn})_{\text{peak}} = -U_{in} + U_{cp}^* - U_{on}. \tag{113}$$

Introducing again the symmetrical components and taking into account $U_{c2}^*=0$ and $U_{o2}=U_{i2}$ in AES, and $U_{c2}^*=U_{i2}-\delta U_{i1}$ as well as $U_{o2}=\delta U_{i1}$ in GES, the result is

$$(U_{Tp})_{\text{peak}} = (U_{Tn})_{\text{peak}} = U_{i1} + U_{c1}^* - U_{o1}. \tag{114}$$

Conclusion: The voltage ratings of the main thyristors are the same in all three operational states.

Clamping thyristors

The peak voltage is (Fig. 16) either

$$(U_{Tpc})_{\text{peak}} = U_{cn}^* - U_{ip} \tag{115}$$

or

$$(U_{Tnc})_{\text{peak}} = -U_{cp}^* + U_{in}. \tag{116}$$

By introducing the symmetrical components comparable results are obtained

$$(U_{Tpc})_{\text{peak}} = -U_{c1}^{*\prime} - U_{i1} - \delta U_{i1}, \tag{117}$$

$$(U_{Tnc})_{\text{peak}} = -U_{c1}^{*\prime} - U_{i1} + \delta U_{i1} \tag{118}$$

where $U_{c2}^*=U_{i2}-\delta U_{i1}$ has been substituted here [eq. (68)]. In SES $\delta=0$, while in AES $\delta=U_{i2}/U_{i1}$ and in GES δ is arbitrary.

Conclusion: The voltage ratings of the clamping thyristors in the two sides are the same in SES but they are different in the other two operational states and higher by δU_{i1} than in SES on one side.

In the case of asymmetrical clamping angles $(\alpha_p \neq \alpha_n)$ our remarks are restricted to SES. Here the symmetry is completely destroyed. The output voltages U_{op} and U_{on} are not the same any longer since $U_{o2}=-U_\delta$. The peak values of the currents in the main thyristors are different [eq. (75)]. The currents in the clamping thyristors become different. In fact, their initial values and their declining rates are not the same.

The maximum voltages of the main thyristors are

$$(U_{Tp})_{\text{peak}} = U_{i1} - U_{o1} + U_{c1}^* + U_\delta$$

and

$$(U_{Tn})_{\text{peak}} = U_{i1} - U_{o1} + U_{c1}^* - U_\delta,$$

respectively. Only the voltage stress across the clamping thyristors remains symmetrical.

9. Transient process

The energies stored in the circuit shown in Fig. 1 are zero at the very beginning of the process but in the input condensers. The *DC* input voltages U_{ip} and U_{in} are constant. Firing the thyristors alternately from $t=0$ on with a constant frequency, the output voltages increase from zero up to their steady-state value. The investigation is confined to the symmetrical energy supply $(W_{op}=W_{on})$ and to the pure resistive load R_L. γ cycles pursue each other (see Fig. 7). In each cycle the condenser voltage is clamped by the clamping thyristors at values

$$U_{cp} = U_{ip}+\Delta U, \quad (119)$$

$$U_{cn} = U_{in}-\Delta U \quad (120)$$

where $\Delta U>0$. The $*$ is omitted here and later for the sake of simplicity. Satisfying equation $U_{c2}=U_{i2}$, the output energy carried by each current pulse is constant [see eq. (23)]

$$W_o = W_{o1} = 2CU_{i1}(U_{i1}+\Delta U). \quad (121)$$

The energy balance equation for the output circuit in the k'th γ cycle $(k=1, 2, \ldots)$ is (Fig. 13)

$$\left.\begin{matrix}\text{Output energy delivered}\\ \text{by two current pulses}\end{matrix}\right\} = \left.\begin{matrix}\text{Energy changes in}\\ \text{output condensers}\end{matrix}\right\} + \begin{matrix}\text{Energy delivered}\\ \text{to load}\end{matrix}$$

$$2W_o = C_o[u_{op}^2(k)-u_{op}^2(k-1)+u_{on}^2(k)-u_{on}^2(k-1)]/2+W_L(k). \quad (122)$$

Here for example $u_{op}(k-1)$ is the u_{op} voltage at the end of the $(k-1)'$th γ cycle.

The energy $W_L(k)$ delivered to the load in the k'th γ cycle can be approximately calculated by supposing constant output voltage during the whole k'th cycle. As an extreme assumption the value of the output voltage at the end of the previous cycle $(k-1)$ is assumed constant in the whole k'th cycle

$$[u_{op}(k-1)+u_{on}(k-1)] = \text{const.} \quad (123)$$

As another extreme assumption the value of the output voltage at the end of the k'th cycle is assumed constant in the whole k'th cycle

$$[u_{op}(k)+u_{on}(k)] = \text{const.} \quad (124)$$

Furthermore $u_{op}(k-1)=u_{on}(k-1)=u_o(k-1)$ and $u_{op}(k)=u_{on}(k)=u_0(k)$ are assumed in eq. (123) and eq. (124), respectively.

Substituting first eq. (123) into eq. (122) the relation

$$x(k)-a_1x(k-1) = b_1 \quad (125a)$$

is obtained. Substituting now eq. (124) into eq. (122), the relation

$$x(k)-a_2x(k-1) = b_2 \quad (125b)$$

is the result where

$$x(k) = u_o^2(k), \tag{126}$$

$$a_1 = 1 - \frac{4T}{C_o R_L}, \qquad b_1 = \frac{2}{C_o} W_o, \tag{127}$$

$$a_2 = \frac{1}{1 + \frac{4T}{C_o R_L}}, \qquad b_2 = \frac{2}{C_o} \frac{W_o}{1 + \frac{4T}{C_o R_L}}. \tag{128}$$

The Z transform of difference equation (125)

$$x(z) = b \frac{z}{z-1} \frac{1}{z-a} = \frac{b}{1-b} \left(\frac{z}{z-1} - \frac{z}{z-a} \right), \tag{129}$$

where a and b are either a_1 or a_2 and b_1 or b_2, respectively.

The inverse Z transform is

$$x(k) = \frac{b}{1-a} (1 - a^k) \tag{130}$$

and the output voltage at the end of the k'th period is

$$u_o(k) = \sqrt{\frac{b}{1-a}} \sqrt{1-a^k}. \tag{131}$$

The steady state value of voltage u_o

$$u_o(\infty) = \sqrt{\frac{b}{1-a}} = \sqrt{\frac{R_L W_o}{2T}}. \tag{132}$$

The last relation states that the output power $2W_o/T$ delivered by the circuit covers the load power $[2u_o(\infty)]^2/R_L$. Dividing eq. (131) by (132)

$$r = \frac{u_o(k)}{u_o(\infty)} = \sqrt{1-a^k}. \tag{133}$$

Expressing k

$$k = \frac{\ln(1-r^2)}{\ln a}.$$

As $\ln a = \ln(1-q)$ where $q = \frac{4T}{C_o R_L} \ll 1$,

$$\ln(1-q) = -\left(q + \frac{q^2}{2} + \frac{q^3}{3} + \dots \right) \cong -q.$$

Therefore

$$k \cong [-\ln(1-r^2)] \frac{C_o R_L}{4T_1} \frac{T_1}{T} \tag{134}$$

where T_1 is the base value for the cycle time. $u_o(k)/u_o(\infty)$ is plotted as a function of

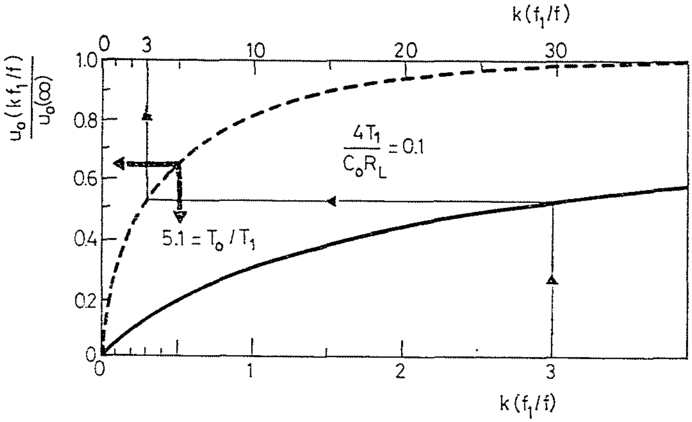

Fig. 17. Transient process

$(T/T_1)k = (f_1/f)k$ at a given $C_o R_L/4T_1$ in Fig. 17. The following two conclusions can be drawn from eqs. (133) and (134):

1. The same time functions can be used at a different pulse frequency f.
2. In order to obtain some figure on the rate of voltage change u_o, time $T_o = kT$ needed to reach 63% of its final value is determined from eq. (134)

$$T_o \cong 0.1275 C_o R_L. \tag{135}$$

The speed of the response depends of course on time constant $C_o R_L$.

$u_o(\infty)$ increases with f. On the other hand, keeping voltage change Δu_o constant and increasing f, the time needed for the voltage change can be reduced.

10. Conclusion

The behaviour of a DC—DC thyristor converter has been described by two-phase symmetrical components in the case of asymmetrical input voltages. Three kinds of operational states were investigated; the symmetrical, the asymmetrical and the general energy supply.

The operation state can be set by a suitable gating of the clamping thyristors.

The gating of the clamping thyristors prevents an unlimited increase with time in the voltage of the switched condenser amplitude. No forced commutation is needed.

With identical clamping angles $(\alpha_p = \alpha_n)$ and under identical input and output conditions the current load and the voltage stress of the main thyristors are the same in all three operational states. Both the current load and the voltage stress of the clamping thyristors are the same in SES. They are different in the other two operational states and on one side of the circuit both the current load and the voltage stress are higher than in SES. The most favourable operational state from the viewpoint of thyristor rating is SES. On the other hand, though the current load is the same in all three operational states in the switched condenser, the voltage stress is the smallest in AES since here the *DC* component U_{c2}^* is zero.

It has to be emphasized that the various operational states have no effect what so ever on the power supply and on the output circuit. It is an internal affair of the converter. The switched condenser equalizes the output energies and voltages of the two halves of the circuit and the voltage stress in the two gating thyristors but at the same time it takes a higher voltage stress in SES compared to the AES.

The asymmetry in the clamping angles destroys the symmetry even in SES.

The converter can be operated as an active filter. Besides varying input voltages the output energies and approximately the output voltages are kept constant since the converter frequency is much higher than the fundamental frequency of the input voltages. The low frequency pulsation of the input voltages is not reflected to the output side by the converter.

The most effective way to influence the converter response time is to vary the switching frequency.

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