STABILITY OF RECTANGULAR SANDWICH PLATES WITH CONSTRUCTIONALLY ORTHOTROPIC HARD LAYERS

PART II

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Abstract

The stability of asymmetrically built rectangular sandwich plate with constructionally orthotropic hard faces at Navier-type boundary conditions is studied. The governing equations derived in [5] from Trefftz—Bolotin variational principle are solved by Fourier method. The remarks are made on the consideration of the asymmetry of building and loading of the plate.

1. Introduction

According to the requirements of practical applications the hard layers (faces) of the sandwich plates are often "reinforced" by bending in one direction or otherwise (Fig. 1).

It was supposed that the layout of the hard layers permit to use the "effective" stiffnes theory and using the suppositions of the validity of the Kirchhoff—Love hypothesis the equivalent stiffness characteristics for these layers are determined by "smothing" of the stiffness characteristics of the reinforced layers. In the case of layers, stiffened by one direction bending the method and formulas for equivalent

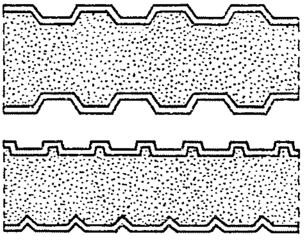

Fig. 1. The construction of the sandwich plate

stiffness characteristics depending on the geometric and material paramenters of the layers are shown in the Part I of this paper [5].

It was supposed that the material of all the layers is elastic and the stress-strain relations could be described by Hooke's law. Corresponding to the common assumptions of the theory of sandwich plates with transversally soft middle layer (core) it was supposed that the material of the hard layers is orthotropic, and the material of the core is transversally isotropic.

In the Part I the governing equations and natural boundary conditions of the stability problem are given, too.

So, using the common suppositions in the theory of sandwich plate [1—4] repeating the main statements — including the corrected governing equations — of Part I in this Part II we will investigate the stability of asymmetrically built and loaded rectangular sandwich plate with constructionally orthotropic hard and transversally isotropic soft layers. According to loading we suppose that each hard layer is loaded with normal and constant in plane forces on the edges, but this forces could be different — in particular case zero — for different layers and directions (Fig. 2). The algorithm of the investigation will be as usual.

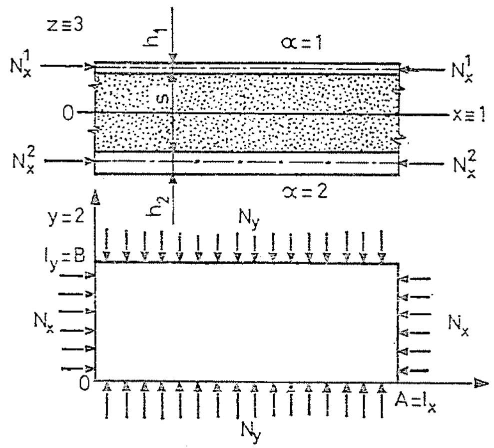

Fig. 2. The loading of the sandwich plate

2. Generalised constitutive equations

Using the common definitions and symbols for the internal membrane forces and internal moments, after integration of the Hooke's law or from the equivalence of deformation energy of the stiffened (bent) and flat plates with uniform (the same) thickness, we find the equivalent stiffness characteristics and the generalised Hooke's law connecting the internal forces and strains in the middle plane of the hard layer in form:

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \end{bmatrix} = \begin{bmatrix} C & K \\ K & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \varkappa \end{bmatrix} \tag{1}$$

Here

$$\mathbf{N} = \begin{bmatrix} N_x \\ N_y \\ T \end{bmatrix}; \ \mathbf{M} = \begin{bmatrix} M_x \\ M_y \\ H \end{bmatrix}; \ \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma \end{bmatrix}; \ \varkappa = \begin{bmatrix} \varkappa_x \\ \varkappa_y \\ 2\chi \end{bmatrix} \tag{2}$$

are the vector of membrane forces and internal moments, the strain vector and the curvature vector,

$$C_{ik}, \quad K_{ik}, \quad D_{ik}$$

are the stiffness characteristics, and the stiffness matrices have form:

$$C = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix}; \quad K = \begin{bmatrix} K_{11} & K_{12} & 0 \\ K_{12} & K_{22} & 0 \\ 0 & 0 & K_{66} \end{bmatrix} D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}. \tag{3}$$

The matrix K characterises the coupling effect between stretching and bending which is significant for the constructionally anisotropic plates.

Corresponding to the assumptions for the stresses and deformations in the soft layer, which were assumed to be constant along the layer, for the generalised constitutive equation of the soft layer we have find:

$$\tilde{\mathbf{N}} = \begin{bmatrix} \tilde{T}_{xz} \\ \tilde{T}_{yz} \\ \tilde{N}_z \end{bmatrix} = \begin{bmatrix} \tilde{C}_{13} & 0 & 0 \\ 0 & \tilde{C}_{23} & 0 \\ 0 & 0 & \tilde{C}_{33} \end{bmatrix} \begin{bmatrix} \tilde{\gamma}_{xz} \\ \tilde{\gamma}_{yz} \\ \tilde{\varepsilon}_z \end{bmatrix} = \tilde{C} \cdot \tilde{\varepsilon}, \tilde{N}_x = \tilde{N}_y = 0, \tilde{T}_{xy} = 0 \tag{4}$$

where the stiffness characteristics are:

$$\tilde{C}_{13} = \tilde{C}_{23} = s^2 B; \ \tilde{C}_{33} = s^2 R. \tag{5}$$

Here

$$B = \frac{\tilde{G}}{s}, \quad R = \frac{\tilde{E}_z}{s}$$

where G and $\tilde{E}_z$ are the average of the shear moduli and the Young moduli in normal direction to the plate of the soft layer with thickness s.

3. Governing equations and natural boundary conditions

Using the Trefftz—Bolotin variational principle [3] in the I. Part of this paper the governing equations and corresponding natural boudary conditions were derived and we got:

$$\nabla_{c1}^{\alpha} u_{\alpha} + \nabla_{c3}^{\alpha} v_{\alpha} - \frac{\partial}{\partial x} \nabla_{k1}^{\alpha} w_{\alpha} + (-1)^{\alpha} sB\tilde{\gamma}_{xz} = 0,$$

$$\nabla_{c2}^{\alpha} v_{\alpha} + \nabla_{c3}^{\alpha} u_{\alpha} - \frac{\partial}{\partial y} \nabla_{k2}^{\alpha} w_{\alpha} + (-1)^{\alpha} sB\tilde{\gamma}_{yz} = 0, \tag{6}$$

$$\nabla_{D}^{\alpha} w_{\alpha} - \frac{\partial}{\partial x} \nabla_{k1}^{\alpha} u_{\alpha} - \frac{\partial}{\partial y} \nabla_{k2}^{\alpha} v_{z} - r_{\alpha} sB\left(\frac{\partial}{\partial x}\tilde{\gamma}_{xy} + \frac{\partial}{\partial y}\tilde{\gamma}_{yz}\right) - (-1)^{\alpha} R(w_1 - w_2) + \nabla_{q}^{\alpha} w_{\alpha} = 0$$

$$(\alpha = 1, 2)$$

where the operators are as follows:

$$\nabla_{c1}^{\alpha} = C_{11}^{\alpha} \frac{\partial^2}{\partial x^2} + C_{66}^{\alpha} \frac{\partial^2}{\partial y^2},$$

$$\nabla_{c2}^{\alpha} = C_{22}^{\alpha} \frac{\partial^2}{\partial y^2} + C_{66}^{\alpha} \frac{\partial^2}{\partial x^2},$$

$$\nabla_{c3}^{\alpha} = (C_{12}^{\alpha} + C_{66}^{\alpha}) \frac{\partial^2}{\partial x\, \partial y},$$

$$\nabla_{k1}^{\alpha} = K_{11}^{\alpha} \frac{\partial^2}{\partial x^2} + (K_{12}^{\alpha} + 2K_{66}^{\alpha}) \frac{\partial^2}{\partial y^2}, \tag{7}$$

$$\nabla_{k2}^{\alpha} = K_{22}^{\alpha} \frac{\partial^2}{\partial y^2} + (K_{12}^{\alpha} + 2K_{66}^{\alpha}) \frac{\partial^2}{\partial x^2},$$

$$\nabla_{D}^{\alpha} = D_{11}^{\alpha} \frac{\partial^4}{\partial x^4} + 2(D_{12}^{\alpha} + 2D_{66}^{\alpha}) \frac{\partial^4}{\partial x^2\, \partial y^2} + D_{22}^{\alpha} \frac{\partial^4}{\partial y^4},$$

$$\nabla_{q}^{\alpha} = \frac{\partial}{\partial x}\left[N_{x}^{\alpha} \frac{\partial}{\partial x} + N_{xy}^{\alpha} \frac{\partial}{\partial y}\right] + \frac{\partial}{\partial y}\left[N_{y}^{\alpha} \frac{\partial}{\partial y} + N_{yx}^{\alpha} \frac{\partial}{\partial x}\right]$$

and $\tilde{\gamma}_{xz}$, $\tilde{\gamma}_{yz}$ have forms

$$\tilde{\gamma}_{xz} = \frac{1}{s}\left[u_1 - u_2 + \frac{\partial}{\partial x}(r_1 w_1 + r_2 w_2)\right],$$

$$\tilde{\gamma}_{yz} = \frac{1}{s}\left[v_1 - v_2 + \frac{\partial}{\partial y}(r_1 w_1 + r_2 w_2)\right], \tag{8}$$

$$\tilde{\varepsilon}_z = \frac{1}{s}[w_1 - w_2],$$

where

$$r_\alpha = \frac{1}{2}(s+h_\alpha), \quad (\alpha = 1,2).$$

For the natural boundary conditions — after adequate transformations — on the boundary x=constant we have:

$$N_x^\alpha = 0,\ N_{xy}^\alpha = 0,\ M_x^\alpha = 0,$$

$$r_\alpha \tilde{\tau}_{xz} + N_x^\alpha \frac{\partial w_\alpha}{\partial x} + N_{xy}^\alpha \frac{\partial w_\alpha}{\partial y} - \left(\frac{\partial M_x^\alpha}{\partial x} + 2\frac{\partial M_{xy}^\alpha}{\partial y}\right) = 0\ (\alpha = 1,2). \tag{9}$$

Here $u_\alpha(x, y)$, $v_\alpha(x, y)$, $w_\alpha(x, y)$ $(\alpha=1, 2)$ are the displacements functions of the points belonging to the middle surfaces of the hard layers in the directions of the coordinates, $\tilde{\tau}_{xz}$ — are the shear stresses in the soft layer.

The meaning of the first three conditions is obvious and well known: the value of membrane edge forces acting in the plane of hard layers as well as the bending edge moment on the free boundary must be zero.

The fourth expression — analysed in Part I [5] — means, that the generalized transversal shear edge forces on the boundary must be zero.

The actual solution of the governing equations (6) shall be obtained with the prevailing boundary conditions, can be described accordingly being taken into consideration. As is well-known from the stability theory of plates, Fourier's method shall reasonably be applied to solve the problem of eigenvalue so arisen, provided at least two opposite boundaries are "simply supported" and the plates are only in compression i.e. $N_{xy}^\alpha=0$, $(\alpha=1, 2)$. In cases other than this, whether in case of more sophisticated boundary conditions or in case $N_{xy}^\alpha \neq 0$ (that means if the plate is subjected also to shear), other approximations shall be applied. In such cases to the solution of equations it is reasonable to prefer some direct method of variation calculation, based on expressions of the functional of the stability problem.

4. Solution of the stability problem of rectangular plate with Navier-type boundary conditions

Let us investigate the stability of rectangular sandwich plate with side lengths $l_x=A$, $l_y=B$, compressed by resultant edge forces N_x, N_y only $(N_{xy}=0)$, not depending on the coordinates. Let us suppose that the hard layers are "simply supported" on the edges and compressed by constant edge forces N_x^α, N_y^α for which we have (Fig. 2):

$$N_x^1 + N_x^2 = N_x$$

$$N_y^1 + N_y^2 = N_y.$$

According to the basic theory we also suppose that the variation of the tangential displacements normal to the edges and the rotation of the normal to the plate on the edges not equal to zero and — as well-known from the thin plates theory — the vanishing condition for the shear membrane forces on the edges can be satisfied in integral sense only.

In this case for the hard layers edges at x=const:

$$\delta u_\alpha \neq 0, \ \delta\left(\frac{\delta w_\alpha}{\partial x}\right) \neq 0$$

$$N_{xy} = \int_0^{ly} [C_{66}\gamma + K_{66} 2\chi]_\alpha \, dx = 0$$

and the boundary conditions are:

$$\begin{aligned} v_\alpha = 0, \quad w_\alpha = 0 \\ M_x^\alpha = [K_{11}\varepsilon_x + K_{12}\varepsilon_y + D_{11}\varkappa_x + D_{12}\varkappa_y]_\alpha = 0, \\ N_x^\alpha + [C_{11}\varepsilon_x + C_{12}\varepsilon_y + K_{11}\varkappa_x + K_{12}\varkappa_y]_\alpha = 0, \end{aligned} \tag{10}$$

or because of constant external edge forces N_x the last condition is valid in integral sense:

$$N_x^\alpha = -\frac{1}{l_y}\int_0^{ly} [C_{11}\varepsilon_x + C_{12}\varepsilon_y + K_{11}\varkappa_x + K_{12}\varkappa_y]_\alpha dy. \tag{11}$$

These boundary conditions correspond to the Navier type boundary conditions for the simply supported plate but here we have conditions for the tangential displacements, too and the condition $\delta u_\alpha \neq 0$ means that the edge of the hard layers can move in the plane of the layer so we have boundary conditions with moving edges.

These boundary conditions will be satisfied if — using the Fourier method — we take the solution in form:

$$\begin{aligned} u_\alpha &= \sum_i \sum_j U_\alpha \cos(k_1 x) \sin(k_2 y), \\ v_\alpha &= \sum_i \sum_j V_\alpha \sin(k_1 x) \cos(k_2 y), \\ w_\alpha &= \sum_i \sum_j W_\alpha \sin(k_1 x) \sin(k_2 y). \end{aligned} \tag{12}$$

Here

$$k_1 = \frac{i\pi}{A} = \frac{\pi}{\lambda_x}, \quad k_2 = \frac{j\pi}{D} = \frac{\pi}{\lambda_y}$$

are the wave numbers λ_x, λ_y — the half wave length in the x, y directions and i, j — natural whole numbers which run from 1 to ∞. Before the solution of the differential equation we should make some remarks to the asymmetry and loading of the plate.

4.1. *Choosing of the "basic surface" of the layers*

At optional chosen basic (middle) surface from the generalised constitutive equation (1) for the strain and curvature vector we obtain:

$$\begin{bmatrix}\varepsilon\\ \varkappa\end{bmatrix} = [I_3-\mathbf{C}^{-1}\mathbf{K}\mathbf{D}^{-1}\mathbf{K}]^{-1}\otimes I_2\begin{bmatrix}\mathbf{C}^{-1} & -C^{-1}\mathbf{K}D^{-1}\\ -D^{-1}\mathbf{K}C^{-1} & \mathbf{D}^{-1}\end{bmatrix}\begin{bmatrix}\mathbf{N}\\ \mathbf{M}\end{bmatrix} \tag{13}$$

where I_3 and I_2 are the 3×3 and 2×2 idem-matrices and $\otimes$ the sign of direct multiplication. At the stability problem before the buckling the plate should be in momentless (in plane) deformation state and so for the hard layers we should have $\varkappa=0$. With this conditions from the Eq. (13) we have

$$\varkappa = \mathbf{D}^{-1}[\mathbf{M}-\mathbf{K}\mathbf{C}^{-1}\mathbf{N}] = 0,$$

which means that for $\varkappa=\mathbf{0}$ we should choose a new "basic surface" on the "distance"

$$\mathbf{Z}_0 = \mathbf{K}\mathbf{C}^{-1}$$

from the previous. For this new "basic surface" $\mathbf{M}^*=\mathbf{0}$ (Fig. 3) and from the expression (13) we can see, that $\mathbf{K}^*=\mathbf{0}$, too. For the stiffness characteristic by bending in this case with the previous values we have:

$$\mathbf{D}^* = \mathbf{D}-\mathbf{K}\mathbf{K}\mathbf{C}^{-1}.$$

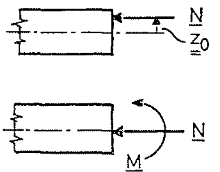

Fig. 3. Choosing of the "basic surface"

4.2. *Remarks to the loading of the plate*

Depending on the loading of the plate and on passing this loading to the hard layers we can distinguish some characteristic case of loading of the hard layers:

a) both hard layers are loaded
   - α) in both directions
   - β) in one and the same direction
   - γ) in different directions

b) one of the hard layers is loaded only
   - α) in both directions
   - β) in one and the same direction.

In these cases the in-plane deformation of the layers will be different in the post-buckling state but we can suppose — as usual in the thin plate theory — that before the buckling the hard layers have negligibly small in-plane deformation, and for the disturbation-less state of the plate the pressed curvature-less state is taken. In this bending-less state of the plate the strain vector $\boldsymbol{\varepsilon}^{\alpha}$ is:

$$\boldsymbol{\varepsilon}^{\alpha} = \mathbf{A}^{\alpha} \cdot \mathbf{N}^{\alpha}, \quad (\mathbf{A} = \mathbf{C}^{-1})$$

and for the in-plane deformation and membrane forces we have (Fig. 4):

$$\begin{aligned} \boldsymbol{\varepsilon}^1 &= \boldsymbol{\varepsilon}^2, \\ \mathbf{N}^1 + \mathbf{N}^2 &= \mathbf{N}. \end{aligned} \tag{14}$$

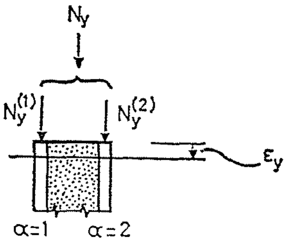

Fig. 4. Deformation in bending-less state

This is a set of four equations to determine the ratio of the loading membrane forces on the edges of the hard layers. Depending on the characteristic cases of loading we get different values for these ratios.

For example in the case a), α) we have:

$$\lambda_1 = \frac{N_x^1}{N_x^2} = \frac{n\beta_x^1 + \beta_y^1}{n\beta_x^2 - \beta_y^1}, \quad \lambda_2 = \frac{N_y^1}{N_y^2} = \frac{\alpha_y^1 - n\alpha_x^1}{\alpha_y^2 + n\alpha_x^2}, \tag{15}$$

where

$$n = \frac{N_x}{N_y} \neq 0,$$

and

$$\begin{aligned} \alpha_x^1 &= A_{11}^1 A_{21}^2 - A_{21}^1 A_{11}^2, \\ \alpha_y^1 &= A_{11}^2 A_{22}^2 - A_{12}^2 A_{21}^2 + A_{11}^1 A_{22}^2 - A_{21}^1 A_{12}^2, \\ \alpha_y^2 &= A_{11}^1 A_{22}^1 - A_{12}^1 A_{21}^1 + A_{11}^2 A_{22}^1 - A_{12}^1 A_{21}^2, \\ \beta_x^1 &= A_{11}^2 A_{22}^2 - A_{12}^2 A_{21}^2 + A_{11}^2 A_{22}^1 - A_{12}^1 A_{21}^2, \\ \beta_x^2 &= A_{11}^1 A_{22}^1 - A_{12}^1 A_{21}^1 + A_{11}^1 A_{22}^2 - A_{12}^2 A_{21}^1, \\ \beta_y^1 &= A_{11}^1 A_{22}^2 - A_{12}^2 A_{22}^1 \end{aligned}$$

are paramenters of stiffness (rigidity) characteristics of the layers.

In the case a), β), $N_x=0$, which can be in two different ways:

1) $N_x^2 = -N_x^1$

2) $N_x^1 = N_x^2 = 0$.

In the first case we can use Eq. (14) from which with $n=0$ we get:

$$\lambda_1 = -1, \quad \lambda_2 = \frac{\alpha_y^1}{\alpha_y^2},$$

but in the second case Eq. (14) leads to contradictions because the number of equations more than the number of unknown. In this case $\varepsilon_x^1=\varepsilon_x^2$ and $\varepsilon_y^1=\varepsilon_y^2$ cannot exist simultaneously except when

$$\Delta_0 = A_{12}^1 A_{22}^2 - A_{22}^1 A_{12}^2 = 0$$

and so practical to prescribe the realisation one of these equality of the deformations, e.g.: $\varepsilon_y^1=\varepsilon_y^2$ corresponding to $N_y \neq 0$, in this case we have not value for λ_1, but

$$\lambda_2 = \frac{A_{22}^2}{A_{22}^1}.$$

In the case b), α) all the external forces are acting on one of the layers and so we do not need (and they do not exist) the ratios λ_1 and λ_2. This is the situation in the case b), β), too.

Using these expressions we can determine the loading parameters as functions of the chosen unknown membrane edge forces, e.g. as functions N_y.

4.3. *Critical values of external loads*

Substituting the solution (12) into the differential equations (6) we obtain a second order algebraic equation for the loading parameters

$$N_1 = k_1^2 N_x^1 + k_2^2 N_y^2 = \lambda_3 N_y^1, \tag{16}$$

$$N_2 = k_1^2 N_x^2 + k_2^2 N_y^2 = \lambda_4 N_y^1$$

in form:

$$N_1 N_2 - bN_1 - cN_2 + d = 0 \tag{17}$$

where b, c, d, are the parameters of the system and parameters λ_3, λ_4 depend/correspond to the characteristic cases of loading.

In the case a), α):

$$\lambda_3 = k_2^2 + k_1^2 n \frac{\lambda_1(1+\lambda_2)}{\lambda_2(1+\lambda_1)},$$

$$\lambda_4 = \frac{1}{\lambda_2}\left[k_2^2 + k_1^2 n \frac{1+\lambda_2}{1+\lambda_1}\right].$$

In the case a), β) 1):

$$\lambda_3 = k_2^2 + \frac{k_1^2}{\Delta} \beta_y^1 \frac{1+\lambda_2}{\lambda_2},$$

$$\lambda_4 = \frac{1}{\lambda_2} k_2^2 - \frac{k_1^2}{\Delta} \beta_y^1 (1+\lambda_2),$$

where:

$$\Delta = \beta_x^1 + \beta_x^2, \quad \lambda_2 = \frac{\alpha_y^1}{\alpha_y^2}.$$

In the case a), β) 2):

$$\lambda_3 = k_2^2, \quad \lambda_4 = \frac{k_2^2}{\lambda_2},$$

where

$$\lambda_2 = \frac{A_{22}^2}{A_{22}^1}.$$

In the case b), α):

When the $\alpha = 1$ layer is loaded $(Q=1)$: $\lambda_3 = k_2^2 + nk_1^2$, $\lambda_4 = 0$.

When the $\alpha = 2$ layer is loaded $(Q=2)$: $\lambda_3 = 0$, $\lambda_4 = k_2^2 + nk_1^2$.

In the case b) β):

$$\text{At } \; Q = 1: \quad \lambda_3 = k_2^2, \quad \lambda_4 = 0,$$

$$\text{At } \; Q = 2: \quad \lambda_3 = 0, \quad \lambda_4 = k_2^2.$$

Using these parameters and Eq. (16) for different cases of loading from Eq. (17) we can determine the critical value of external forces $(N_y^1)_{\text{crit}}$ and finally — with parameters n, λ_1, λ_2 — we find:

$$N_y^* = (N_y)_{\text{crit}} = (N_y^1)_{\text{crit}} + (N_y^2)_{\text{crit}},$$

$$N_x^* = (N_x)_{\text{crit}} = (N_x^1)_{\text{crit}} + (N_x^2)_{\text{crit}}.$$

Equation (17) gives two values for N_y^1 which are connected with the asymmetrical and symmetrical forms of instability. These forms can be separated analytically for the symmetrically built and loaded sandwich plate, only. In given case they can be determined by the calculation of the ratio of normal displacement amplitudes W_1/W_2 and when Sign $(W_1'/W_2')=1$ or Sign $(W_1'/W_2')=-1$ the form of instability asymmetric or symmetric, respectively.

The solution can slightly or hardly depend on the boundary conditions which determine the local or global character of instability, respectively. These characters can be determined by the ratio of half wave length to the side-length of the plate in given direction. For example if in the moment of instability $(\lambda_y)_{\text{crit}} = \lambda_* < B$, then the character of instability is local, if $\lambda_* \geqq B$, then it is global.

The elaborated computer program gives possibility to analyse the stability of asymmetrically built and loaded sandwich plate.

5. Stability analysis of some constructions

Two types of hard layer were chosen for building the asymmetric sandwich plate. The first one was a standard trapezoid plate (TR 13/63) with material and measurement characteristics:

$$\bar{E} = 6{,}8 \cdot 10^6 \,\mathrm{kN/cm^2}, \quad \bar{\nu} = 0{,}3, \quad h_1 = h_2 = 1\,\mathrm{mm},$$

and the second one was a flat plate for which $\bar{E}$, $\bar{\nu}$ was the same as for trapezoid plate but the thickness was $h_\alpha=1$ mm or $h_\alpha=2$ mm.

The soft layer was supposed to be made from polyurethane foam with material and measurement characteristics:

$$E_z = 180\,\mathrm{kN/cm^2}, \quad G = 80\,\mathrm{kN/cm^2}, \quad s = 80\,\mathrm{mm}.$$

With these layers three types of sandwich plate were built, so that the upper layer was stiffer than the lower one:

a) both of layers are trapezoid plate,
b) the $\alpha=1$ layer is trapezoid, but the $\alpha=2$ layer is flat plate ($h_2=1$ mm),
c) both of layers are flat plates, but their thickness is different: $h_1=2$ mm, $h_2=1$ mm.

The Fig. 5 shows the values of critical loads N_y^* versus side length B in y direction for different case of construction (a, b, c), loading $(Q=0, 1, 2)$ and side length A in x direction of the sandwich plate at $n=N_x/N_y=0$.

It can be seen that the plate is not very sensitive to the character of loading. The curves for different Q are quite close to each other — as in the case of short plate there is a sensitive difference between the cases $Q=0$ and $Q=1, 2$ only.

The smaller critical value of external load N_y^* corresponds to the asymmetrical form of instability and in the Fig. 5 the wave lengths λ_* are shown which in the given case divide the local and global forms of instability.

The Fig. 5 shows that the stability of the plate is sensitive to the construction and side-lengths. At $A=250$ cm it is found that if $B<\lambda_0$ then the construction c), but at $B>\lambda_0$ construction a) and b) have the lowest critical value of external forces at the same side-length B.

The plan of external loads could be different in different directions and the plan of loading must not coincide with the basic surface (plan) of the plate. On the Fig. 6 it is shown the effect of the variation of the loading planes in different directions for the construction c) of the plate. The ratios T_1/T in x direction, the ratio T_2/T in y direction show the place of the planes of loading, measured from the basic surface of $\alpha=1$ layer. The curves are close enough which means that the plate is not very sensitive to the different plane of loading in different directions. For example if the $\alpha=1$ layer is loaded in x direction, but $\alpha=2$ layer in y direction (see in the Fig. 5 case Δ: $T_1/T=0$, $T_2/T=1$) the Δ-curve of critical loads is quite close to the others, getting at different plane of loading.

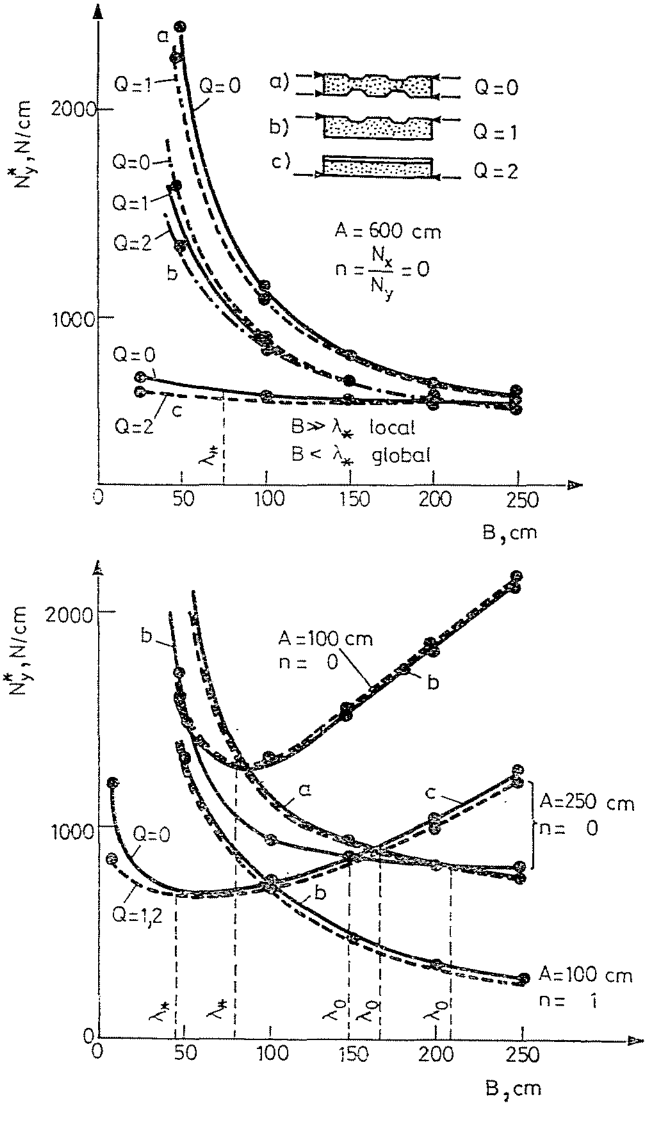

Fig. 5. Critical loads vs. side length B for different case of construction (a, b, c) loading ($Q=0, 1, 2$) and side length A

We should remark that in our investigations for the disturbation-less state of the plate the in-plane strain-stress state of plate was taken. If we want to take into consideration the in-plane deformation of the plate we should drawing up the nonlinear (postbuckling) problem of the sandwich plate [6, 7].

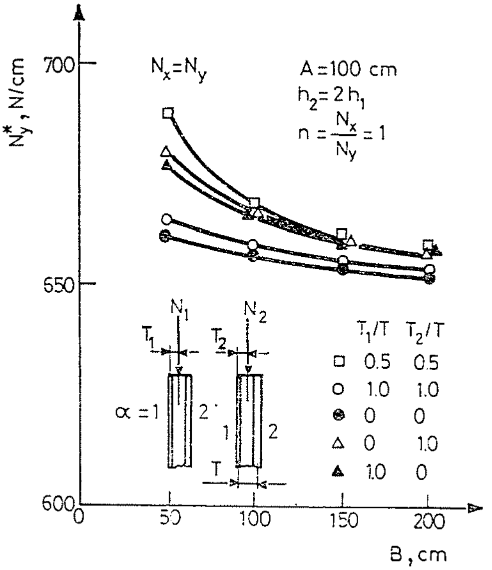

Fig. 6. Effect of the variation of the planes of external forces for the critical loads

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