INSTABILITY CAUSED BY DELAY IN ROBOT SYSTEMS

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Abstract

It is well-known that delayed feedback in the control of robots may cause stability problems. This paper presents an analytical investigation of this effect by means of stability charts on the plane of the parameters of simple but typical robot systems.

1. Delays in robot systems

Delay may be one of the most important reasons of "unexpected" stability problems in robotics. Let us see the cases where this dead time cannot be left out of consideration.

Delay always occurs in the control system of the robot. It can clearly be seen in master-slave systems where the time lag is equal to the delay of reflexes of the human operator which is more than 0,1 second [1]. However, the situation is very similar in the case of an on-line control through a computer where the sample period may serve as a basis for the value of the time lag. It is only results gained by simulation or experiments that are known for the critical value of the delay when stability problems arise and the strategy of the control is strongly determined by this [2]. The dead time is about $0.01 \div 0.001$ second for computers even in the case of the best algorithms and best ways of the description of motion equations like those of Appel—Gibbs or Kane (see [3, 4]).

In another important group of problems the delay occurs in the information transmission. It may cause difficulties for example in undersea or space teleoperations. The value of the dead time may exceed $0.1 \div 1$ second (see [5]) which is equal to the time needed by the ultrasonic or electromagnetic wave to cover the distance between the master and the slave.

A delayed feedback can occur in the mechanical part of the robot as well if it works in a material forming process like milling, rolling, welding, etc. This delay is inversely proportional to the relative velocity of the surfaces of the tool and work-piece [6, 7]. However, the instability caused by this delay is usually not considered as a problem of robotics.

After this short summary of the typical delays in robotics the mathematical formulation will be given to investigate the stability (in Lyapunov sense) of end positions of robots.

2. Motion equations

The position of the robot manipulator can be described by the N-dimensional vector $\mathbf{q}$ containing the general coordinates q_k $(k=1, \ldots, N)$. This description takes into account the geometrical constraints in the system. Let the end position of the robot be at $\mathbf{q}=\mathbf{0}$.

However, in a lot of cases the robot is not a holonomic system because its control is often described by (stationary) kinematical constraints:

$$\mathbf{l}_j^*(\mathbf{q}_t)\,\dot{\mathbf{q}}+g_j(\mathbf{q}_t) = 0, \quad (j = 1, \ldots, h) \tag{1}$$

where h is the number of these constraints. In (1) $\mathbf{q}_t$ contains the influence of the "past" in the control according to chapter 1. Mathematically, it is defined as follows:

$$\mathbf{q}_t(\vartheta) = \mathbf{q}(t+\vartheta), \quad \vartheta\in[-r, 0],$$

where $r\geqq 0$ is the length of retardation and t stands for time. In this way the vector $\mathbf{l}_j$ and the scalar g_j depend on functions so (1) is a so-called first-order functional differential equation for any j.

The Appell—Gibbs equations will be used for this anholonomic robot system (see [3, 8]). Thus, the pseudovelocities v_k $(k=1, \ldots, N-h)$ have to be defined as linear combinations of the general velocities:

$$\mathbf{v} = \mathbf{K}(\mathbf{q})\,\dot{\mathbf{q}} \tag{2}$$

where the $(N-h)\times N$ dimensional matrix $\mathbf{K}$ can be chosen optionally where

$$\det\left(\begin{bmatrix}\mathbf{L}\\ \mathbf{K}\end{bmatrix}\right) \neq 0; \quad \mathbf{L}_{hxN} = \begin{bmatrix}\mathbf{l}_1^*\\ \vdots\\ \mathbf{l}_h^*\end{bmatrix}\cdot \tag{3}$$

Hence, (1)+(2) can be solved as a linear algebraic equation for $\dot{\mathbf{q}}$.

The determinations of the Appell function

$$S(\mathbf{q}, \mathbf{v}, \dot{\mathbf{v}})$$

and the pseudoforces

$$\Pi_k(\mathbf{q}_t, \mathbf{v}_t), \quad (k = 1, \dots, N-h) \tag{4}$$

can be based on the standard methods of the special literature [8]. Note that the active forces at the actuators of the robots may depend on the delayed values of the coordinates and velocities which are expressed by (4).

The motion equations have the form:

$$\left.\begin{aligned} \mathbf{grad}_{\dot{\mathbf{v}}}\, S &= \mathbf{\Pi} \\ \dot{\mathbf{q}} &= \mathbf{H}\mathbf{v}+\mathbf{h} \end{aligned}\right\} \tag{5}$$

where the first $N-h$ equations are the Appell—Gibbs equations ($\mathbf{\Pi}$ contains the pseudoforces) and the remaining N equations come from the solution of (1)+(2) with respect to $\dot{\mathbf{q}}$, where $\mathbf{H}(\mathbf{q}_t)$ is a matrix of $N \times N$ and $\mathbf{h}(\mathbf{q}_t)$ is an N dimensional vector. Taking into account that S is a quadratic expression of the pseudo-accelerations $\dot{v}_k$ it is easy to see that (5) is equivalent to the $2N-h$ dimensional system of first order functional differential equations

$$\dot{\mathbf{x}} = f(\mathbf{x}_t) \tag{6}$$

where

$$\mathbf{x} = \begin{bmatrix} \mathbf{v} \\ \mathbf{q} \end{bmatrix}.$$

The stability investigation of the end position of the robot is equivalent to the stability analysis of the trivial solution of (6).

3. Stability investigation

The linearized form of (6) at $\mathbf{x}=\mathbf{0}$ is

$$\dot{\mathbf{x}} = \int_{-r}^{0} \mathrm{d}\boldsymbol{\eta}(\vartheta)\, \mathbf{x}(t+\vartheta) \tag{7}$$

where the $(2N-h)\times(2N-h)$ dimensional matrix $\boldsymbol{\eta}$ is a function of bounded variation in the interval $[-r, 0]$ (see [9]). The stability investigation is based on the determination of the loci of the complex zeroes λ of the characteristic function of (7):

$$D(\lambda) = \det\Big(\lambda\mathbf{I} - \int_{-r}^{0} e^{\lambda\vartheta}\, \mathrm{d}\boldsymbol{\eta}(\vartheta)\Big), \tag{8}$$

where $\mathbf{I}$ stands for the unit matrix.

Let the functions M and S be defined as follows:

$$M(y) = \operatorname{Re} D(iy), \quad i = \sqrt{-1} \tag{9}$$

$$S(y) = \operatorname{Im} D(iy). \tag{10}$$

$\mu_1 \geqq \ldots \geqq \mu_m \geqq 0$ are the real zeroes of M and $\sigma_1 \geqq \ldots \geqq \sigma_s = 0$ are those of S. The stability criterion is presented in (10) for two cases.

The end position of the robot is asymptotically stable if and only if

a) $2N-h=2n$ (n integer) and

$S(\mu_k) \neq 0, \quad k = 1, \ldots, m$ and

$$\sum_{k=1}^{m} (-1)^k \operatorname{sign} S(\mu_k) = (-1)^n n; \tag{11}$$

b) $2N-h = 2n+1$ and

$M(\sigma_k) \neq 0, \quad k \quad 1, \ldots, s$ and $M(0) > 0$ and

$$\sum_{k=1}^{s-1} (-1)^k \operatorname{sign} M(\sigma_k) + \frac{1}{2}((-1)^s + (-1)^n) + (-1)^n n = 0 \tag{12}$$

Note that n is just the number of the degrees of freedom of holonomic systems in case a).

4. Basic stability charts

Figure 1 shows the stability chart on the plane of the parameters a_0 and b for the scalar system

$$\ddot{q}(t) + a_0 q(t) = bq(t-1), \tag{13}$$

where the length of retardation is assumed to be 1. In this case $n=1$ and

$$D(\lambda) = \lambda^2 + a_0 - be^{-\lambda};$$

$$M(y) = -y^2 + a_0 - b \cos y;$$

$$S(y) = b \sin y. \tag{14}$$

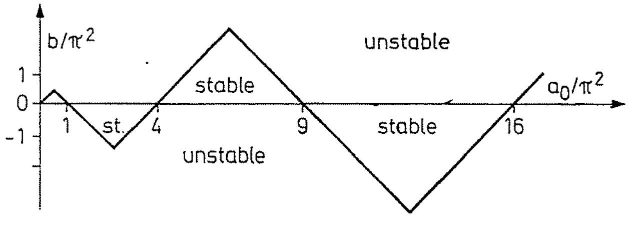

Fig. 1

It is easy to give the regions of stability on the plane (a_0, b) because sign S can simply be given in (11) (see (14)).

If the third order scalar system

$$\dddot{q}(t)+a_1\dot{q}(t) = bq(t-1) \tag{15}$$

is investigated then $n=1$ and

$$D(\lambda) = \lambda^3+a_1\lambda-be^{-\lambda}; \tag{16}$$

$$M(y) = -b\cos y;$$

$$S(y) = -y^3+a_1 y+b\sin y.$$

Now, sign M can be determined simply in (12) and we get the stability chart of Fig. 2.

As we shall see in the examples, equations (13) and (15) are typical in undamped robot systems.

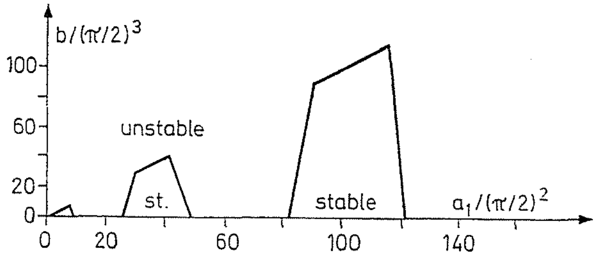

Fig. 2

5. Examples

Figure 3 shows the first example. The $q_1=0$ position has to be found with the help of a camera fixed at the end of the elastic arm (f is the coefficient of viscous friction, s is the coefficient of stiffness and J is the mass). There is a (linearized) kinematical constraint with delay in the control:

$$\dot{q}_2(t)+kq_1(t-\tau) = 0,$$

where k is the gain, τ is the delay.

The equations of motion are:

$$\left.\begin{aligned} \dot{v}(t) &= -2\varkappa\alpha v(t)-(2\varkappa\alpha k+\alpha^2)q_1(t)+\alpha^2 q_2(t) \\ \dot{q}_1(t) &= v(t) \\ \dot{q}_2(t) &= -kq_1(t-\tau) \end{aligned}\right\} \tag{17}$$

where

$$\alpha = \sqrt{\frac{s}{J}}, \quad \varkappa = \frac{f}{2J\alpha}$$

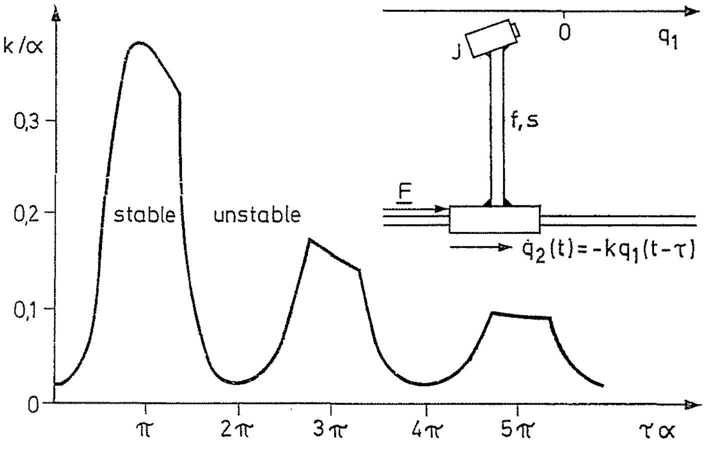

Fig. 3

are the eigenfrequency and the relative damping, respectively. In (17) the first equation is the Appell—Gibbs equation, the second is the definition of the pseudovelocity ν and the third comes from the kinematical constraint.

The characteristic function has the form:

$$D(\lambda) = \det\left(\begin{bmatrix} \lambda+2\varkappa\alpha & 2\varkappa\alpha k+\alpha^2 & -\alpha^2 \\ -1 & \lambda & 0 \\ 0 & ke^{-\lambda\tau} & \lambda \end{bmatrix}\right) = \lambda^3+2\varkappa\alpha\lambda^2+(2\varkappa\alpha k+\alpha^2)\,\lambda+\alpha^2ke^{-\lambda\tau}.$$

It is easy to see that for the undamped case ($\varkappa=0$) this function can be transformed into (16) and the stability results of Fig. 2 can be used here. The stability chart of Fig. 3 was based on these results when $\varkappa=0{,}01$. Note that there are quite great stable regions for great delays.

Let the second example be the experimental master-slave system based on the force-reflective manipulator MA23 of CNES in ARA project [4]. The linearized equations of motion of this holonomic system are:

$$\left.\begin{aligned} J_1\ddot{q}_1(t)+f_1\dot{q}_1(t)+k\big(q_1(t)-q_2(t-\tau)\big) &= Q_1 \\ J_2\ddot{q}_2(t)+f_2\dot{q}_2(t)+k\big(q_2(t)-q_1(t-\tau)\big) &= Q_2 \end{aligned}\right\} \tag{18}$$

with subscript 1 for the master and 2 for the slave, τ is the delay in information transmission. Let us suppose that an operator with delay r of reflexes tries to find the $q_1=q_2=0$ position. There is not any load on the slave ($Q_2=0$) and the operator gets information about the position q_2 of the slave only with delay τ (i.e. by means of a camera):

$$Q_1(t) = -bq_2(t-\tau-r).$$

If the inertia J_1 and the viscous friction f_1 are negligible in master we get the characteristic function

$$D(\lambda) = J_2\lambda^2 + f_2\lambda + k(1 - e^{-2\tau\lambda}) + be^{-(2\tau+r)\lambda}$$

which shows that there are two discrete delays in this system: 2τ and $2\tau + r$.

Let us use the stability condition (11) here as a sufficient condition only:

$$S(y) = \operatorname{Im} D(iy) = f_2 y + k \sin(2\tau y) - b \sin((2\tau + r)y) \geqq$$
$$\geqq (f_2 - 2\tau k - (2\tau + r)b)y > 0$$

implies

$$\tau(2k + 2b) + rb < f_2 \tag{19}$$

for $y > 0$. (11) fulfills because the number m of the positive zeroes μ_k of M is odd since

$$M(0) = b > 0 \quad \text{and} \quad \lim_{y\to\infty} M(y) = -\infty.$$

Figure 4 shows the stable region on the plane of the delays according to (19).

The necessary and sufficient regions of stability have been determined by computer when $f_2 = 0$ (see Fig. 5).

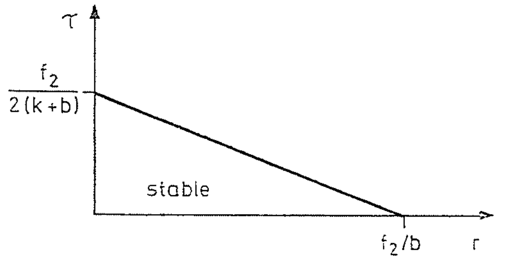

Fig. 4

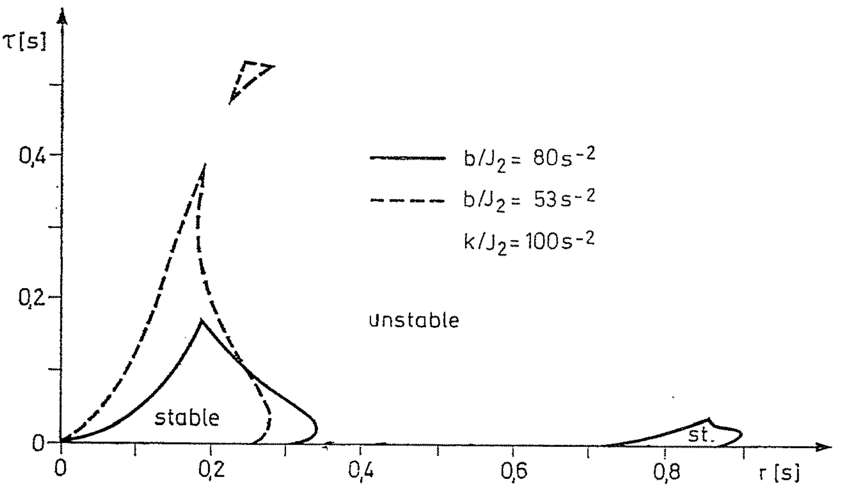

Fig. 5

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