THE FOURTH COLOUR FILTER OF TRISTIMULUS COLORIMETERS

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Abstract

We have proved that the application of the tristimulus value X_2 seems to be unnecessary. It can be suggested that the measuring of the tristimulus value X_2 should be left and the sign X_2 should be defined from the measured value Z on the base of the following connection:

$$X_2 \approx \frac{Z}{5{,}982}$$

It would be more proper to neglect the use of the tristimulus value X_2 but this would need the total reconstructing of the CIE colour-system.

Broad-band colour filters are applied in tristimulus colorimetrical instruments to determine the tristimulus values. Their spectral transmission factor is determined in accord with the CIE spectral functions of tristimulus values. According to relationships (1) and (2) [1]:

$$\begin{aligned} X &= k \int_{\lambda_1}^{\lambda_2} S(\lambda)\, \varrho(\lambda)\, \bar{x}(\lambda)\, d\lambda \\ Y &= k \int_{\lambda_1}^{\lambda_2} S(\lambda)\, \varrho(\lambda)\, \bar{y}(\lambda)\, d\lambda \\ Z &= k \int_{\lambda_1}^{\lambda_2} S(\lambda)\, \varrho(\lambda)\, \bar{z}(\lambda)\, d\lambda \end{aligned} \tag{1}$$

where X, Y, Z the tristimulus values,
$S(\lambda)$ the relative spectral power distribution of one of the standard CIE light sources,
$\varrho(\lambda)$ the spectral reflectancy of the examined surface,
$\bar{x}(\lambda)$, $\bar{y}(\lambda)$, $\bar{z}(\lambda)$ the CIE spectral colour-matching functions.

And

$$\tau_x(\lambda) = \bar{x}(\lambda)$$
$$\tau_y(\lambda) = \bar{y}(\lambda) \qquad (2)$$
$$\tau_z(\lambda) = \bar{z}(\lambda)$$

where $\tau_x(\lambda)$, $\tau_y(\lambda)$, $\tau_z(\lambda)$ the spectral transmittance of colour filters.

However, in principle three colour filters are needed to determine the three tristimulus values, in practice four colour filters are applied. The reason for this is the fact that $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are single-lobe functions and $\bar{x}(\lambda)$ is a double-lobe one. (Figure 1) This latter one can be realized by means of two colour filters. The range marked

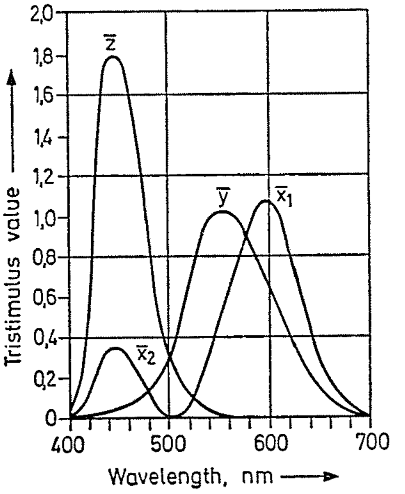

Fig. 1

$\bar{x}_1(\lambda)$ in Fig. 1 will need a colour filter of $\tau_{x1}(\lambda)$ spectral transmittance, while the range $\bar{x}_2(\lambda)$ will need a colour filter of $\tau_{x2}(\lambda)$ spectral transmittance. The values measured by means of them should be added to each other:

$$X_1 = k \int_{\lambda_1}^{\lambda_2} S(\lambda)\, \varrho(\lambda)\, \bar{x}_1(\lambda)\, d\lambda$$

$$X_2 = k \int_{\lambda_1}^{\lambda_2} S(\lambda)\, \varrho(\lambda)\, \bar{x}_2(\lambda)\, d\lambda \qquad (3)$$

and

$$X = X_1 + X_2. \qquad (4)$$

On the base of Figure 1 an interesting circumstance can be observed: while the spectral range of functions $\bar{x}_1(\lambda)$, $\bar{y}(\lambda)$ and $\bar{z}(\lambda)$ are well separated from one another, the range of $\bar{x}_2(\lambda)$ almost totally overlaps the range of $\bar{z}(\lambda)$, furthermore the shape of the functions is very similar to one another, they do differ only in height.

On the base of this observation two questions may arise:

1. Is it necessary to apply a filter of $\tau_{x2}(\lambda)$ transmittance; is it not possible to form sign X_2 from sign Z measured by a filter of $\tau_z(\lambda)$ transmittance?

2. Is the sign X_2 itself necessary; does X_2 give further information as compared to signs X_1, Y, Z when the spectral reflectance (colour) of the $\varrho(\lambda)$ surface is concerned; and if it does mean some further information, is its application advantageous or rather disadvantageous?

In order to answer the questions raised, first let us examine the similarity of functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$ thoroughly.

The comparison of functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$

The numerical values of functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$ on the base of [2] are listed in Table 1. Function $\bar{x}_2(\lambda)$ is considered to be a part of function $\bar{x}(\lambda)$ within the range of 360...504 nm. For the minimum of function $\bar{x}(\lambda)$ can be found at $\lambda_k = 504$ nm, from which (in the range of $\lambda = 505...830$ nm) the function $\bar{x}_1(\lambda)$ can be defined.

In order to reveal the similarity the relative values of functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$ are defined in Table I. as related to their own maxima as 100% and these values are shown in Fig. 2.

On the base of Table 1. and Fig. 2 it can be pointed out that the similarity of the functions is outstanding mainly in the neighbourhood of maximum places occurring at almost the same wavelength and it is valid in the full range of the function $\bar{x}_2(\lambda)$ (between $\lambda = 505...650$ nm). Deviation can only be observed at further parts of the spectrum (between $\lambda = 505...650$ nm) because $\bar{z}(\lambda)$ has quite significant values here while $\bar{x}_2(\lambda)$ is already considered to be zero.

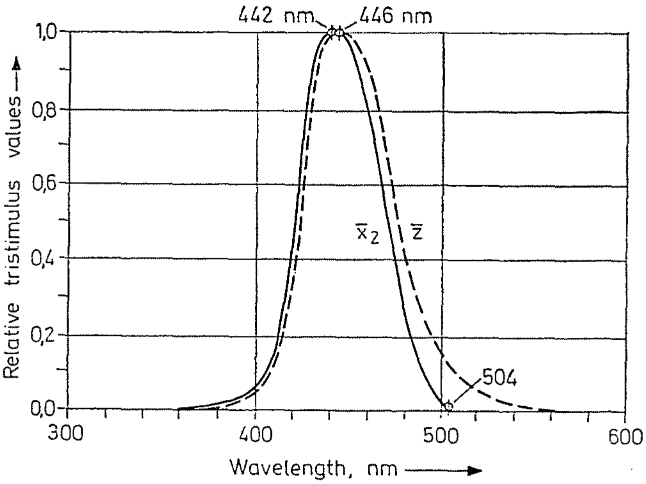

Fig. 2

Table 1

	$\bar{x}$	$\bar{z}$	$\frac{\bar{x}}{(\bar{x})_{max}}100\%$	$\frac{\bar{z}}{(\bar{z})_{max}}100\%$
350				
360	0.000 129 90	0.000 606 10	0.037 1	0.340 0
370	0.000 414 90	0.001 946 00	0.1185	0.109 1
380	0.001 368 00	0.006 450 00	0.3907	0,361 8
390	0.004 243 00	0.020 050 01	1.211 8	1.124 5
400	0.014 310 00	0.067 850 01	4.0869	3.805 5
410	0.043 510 00	0.207 400 0	12.426 2	11.632 3
420	0.134 380 0	0.645 600 0	38.378 1	36.209 3
430	0.283 900 0	1.385 600 0	81.0801	77.713 1
440	0.348 280 0	1.747 060 0	99.4667	97.986 0
442	0.350 147 4	1.769 623 3	100.000 0	99.251 5
446	0.346 373 3	1.782 968 2	98.922 1	100.000 0
450	0.336 200 0	1.772 110 0	96.0167	99.391 0
460	0.290 800 0	1.669 200 0	83.050 7	93.619 2
470	0.195 360 0	1.287 640 0	55.793 6	72.218 9
480	0.095 640 0	0.812 950 1	27.314 2	45.595 3
490	0.032 010 0	0.465 180 0	9.141 9	26.090 2
500	0.004 900 0	0.272 000 0	1.399 4	15.255 5
510	—	0.158 200 0	—	8.872 8
520	—	0.078 249 9	—	4.388 7
530	—	0.042 160 0	—	2.364 6
540	—	0.020 300 0	—	1.138 6
550	—	0.008 749 9	—	0.490 7
560	—	0.003 900 0	—	0.218 7
570	—	0.002 100 0	—	0.117 8
580	—	0.001 650 0	—	0.092 5
590	—	0.001 100 0	—	0.061 7
600	—	0.000 800 0	—	0.044 9
610	—	0.000 340 0	—	0.019 1
620	—	0.000 190 0	—	0.010 7
630	—	0.000 049 9	—	0.002 7
640	—	0.000 020 0	—	0.000 1
650	—	0.000 000 0	—	0.000 0

In the range of $\lambda = 360...504$ nm the similarity between the functions $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$ is so outstanding that we may suppose that it can be defined by the functions of λ of the same type; consequently we may also suppose that nearly linear connection exists between $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$. We supposed that there is the following relationship between $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$:

$$[\bar{z}(\lambda)]_{\lambda=360}^{504} = a_0 + a_1 [\bar{x}(\lambda)]_{\lambda=360}^{504} \tag{5}$$

We have defined the values of a_0 and a_1 from the data in Table 1 by linear regression calculation:

$$a_0 = 0{,}0927\,199\,7$$

$$a_1 = 4.970\,814\,2$$

The correlation factor is

$$r = 0.983\,600\,26$$

Thus there exists a really definite, close, almost linear relationship between the two variables.

Can sign X_2 be formed from sign Z?

Let us examine how incorrect the calculation would be if we left the filter measuring sign X_2 and formed X_2 from sign Z by means of dividing with a suitably chosen constant C.

First let us chose the value of the constant. There are several possibilities:

$$c_0 = a_1 = 4.970\,814\,2$$

$$c_1 = \frac{\bar{z}(\lambda)\max}{\bar{x}_2(\lambda)\max} = \frac{1.782\,968\,2}{0.350\,147\,4} = 5.092$$

$$c_2 = \frac{\int_{420}^{470} \bar{z}(\lambda)\,d\lambda}{\int_{420}^{470} \bar{x}_2(\lambda)\,d\lambda} = \frac{76.205\,885}{14.446\,529} = 5.275$$

$$c_3 = \frac{\int_{360}^{504} \bar{z}(\lambda)\,d\lambda}{\int_{360}^{504} \bar{x}_2(\lambda)\,d\lambda} = \frac{103.559\,240}{17.868\,906} = 5.795$$

$$c_4 = \frac{\int_{360}^{650} \bar{z}(\lambda)\,d\lambda}{\int_{360}^{650} \bar{x}_2(\lambda)\,d\lambda} = \frac{106.892\,25}{17.868\,906} = 5.982$$

$$c_5 = \frac{\sum_{i=1}^{15} Z_i\,OMH}{\sum_{i=1}^{15} X_{2i}\,OMH} = \frac{346.06}{55.47} = 6.239$$

Among them C_0 is equal to the constant a_1 defined by means of linear regression calculation, that is the relation of the average values of $\bar{z}(\lambda)$ and $\bar{x}_2(\lambda)$; C_1 is the quotient of the maximum values of functions $\bar{z}(\lambda)$ and $\bar{x}_2(\lambda)$ in the maximum places' neighbourhood of ± 25 nm breadth; C_3 is the quotient of the integral of the functions $\bar{z}(\lambda)$ and $\bar{x}_2(\lambda)$ in the full wavelength-range; C_4 is the quotient of the integral of the functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$ in the full wavelength-range of $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)$ in total, while C_5 is an empirical value — the quotient of the tristimulus values Z and X_2 of all the members of one of the National Office for Measurement's coloured enamel-standard set.

It is expedient to chose that value of constant C among the six possible values which in the course of the measurements averagely ensures the best correspondence between the tristimulus value X_2 of the different colours and the Z/C value. On the other hand, this deviation completely depends on the fact what colour, what spectral reflectance does have the sample examined. For example in the case of an ideally white or grey colour-sample the deviation of the functions $\bar{x}_2(\lambda)$ and $\bar{z}(\lambda)/C$ cannot be realized at all; when constant C_4 is applied there should be full correspondence that is $X_2 = Z/C_4$. The constant should be chosen on the base of the expectable tristimulus values X_2 and Z of the different colours to be measured.

Let us consider the coloured enamel-standard set of the National Office for Measurement to be the representative statistical sample for all the possible colours and let us choose the most suitable C constant value on the base of the data of the standard set and the error analysis of the measurements done by means of the standard set.

The tristimulus values X_1, X_2 and Z of the coloured enamel-standard set No. 7453 are listed in Table 2. In the table those deviations (ΔX_2) are calculated which are formed between the Z/C values calculated by means of the tristimulus value X_2 and the different C constants:

$$(\Delta x_2)_c = X_2 - \frac{Z}{C} \tag{6}$$

It is obvious from Table 2 that the smallest deviations are formed when applying the constant C_4: here the average and the standard average as well as the deviations' absolute value are the smallest.

In the last three columns of Table 2 we have given the measuring errors ΔX_1, ΔX_2, ΔZ actually made when measuring the tristimulus values X_1, X_2, Z so that it may be decided whether these errors are of permissible degree.

The measurements were made by examination student János Harkay in 1985 by means of the MOMCOLOR D tristimulus colour-measuring device No. 198893 of the BUDALAKK RESEARCH INSTITUTE. The data given in the tables were formed out of the average of 6-6 measurements. The average and the variance of the measuring data were also defined.

Statistical checks can be made to decide whether the substitution of the sign x_2 with the value Z/C_4 compared with the actual measuring error-data ΔX_2 will result in a bigger or in an identical measuring error $(\Delta X_2)_{c4}$. The errors can be expected to follow a normal distribution, since as already mentioned the volume of errors

Table 2

Serial number in the standard set	Tristimulus values			Calculated errors resulting from the substitution of the datas X_2 with Z/C						Real measuring errors (The average of 6—6 measurements)		
	X_1	X_2	Z	$(\Delta X_2)_{C0}$	$(\Delta X_2)_{C1}$	$(\Delta X_2)_{C2}$	$(\Delta X_2)_{C3}$	$(\Delta X_2)_{C4}$	$(\Delta X_2)_{C5}$	ΔX_1	ΔX_2	ΔZ
01	4.88	1.44	8.61	−0.29	−0.25	−0.19	−0.05	0.00	+0.06	+0.08	−0.03	+0.21
02	8.88	0.13	0.80	−0.03	−0.03	−0.02	−0.01	0.00	0.00	+1.97	−0.01	+0.30
03	3.88	0.18	1.11	−0.04	−0.04	−0.03	−0.01	−0.01	0.00	+1.09	−0.02	+0.24
04	25.31	2.57	15.45	−0.54	−0.46	−0.36	−0.10	−0.01	+0.09	+1.49	−0.02	+0.25
05	49.43	10.83	64.23	−2.09	−1.78	−1.35	−0.25	+0.09	+0.54	+1.09	−0.06	−0.37
06	51.33	0.23	2.02	−0.18	−0.17	−0.15	−0.12	−0.11	−0.09	+0.26	−0.03	+0.74
07	8.72	0.32	2.17	−0.12	−0.11	−0.09	−0.05	−0.04	−0.03	+0.13	−0.03	+0.29
08	24.40	0.75	5.03	−0.26	−0.24	−0.20	−0.12	−0.09	−0.06	+0.32	−0.02	+0.45
09	49.11	5.26	32.39	−1.26	−0.10	−0.88	−0.33	−0.15	+0.07	+0.41	−0.01	+0.56
10	7.90	1.61	10.21	−0.44	−0.40	−0.33	−0.15	−0.10	−0.03	+0.17	−0.06	+0.01
11	2.09	0.68	4.18	−0.16	−0.14	−0.11	−0.04	−0.02	+0.01	+0.20	−0.02	+0.30
12	41.46	9.76	60.50	−2.41	−2.12	−1.70	−0.68	−0.35	+0.06	−0.42	+0.07	+0.88
13	1.35	2.86	17.32	−0.62	−0.54	−0.42	−0.13	−0.04	+0.08	+0.60	+0.01	+0.42
14	12.67	8.09	48.62	−1.68	−1.46	−1.13	−0.30	−0.04	+0.30	+0.58	+0.02	+0.27
15	28.29	10.76	64.56	−2.23	−1.92	−1.48	−0.38	−0.03	+0.41	+0.06	−0.01	−0.38
The average of the absolute values				0.8244	0.7168	0.5636	0.1812	0.0720	0.1220	0.6473	0.0280	0.3780
variance				0.6998	0.5228	0.3159	0.0308	0.0093	0.0301	0.3026	0.0004	0.0436

depends on the spectral reflectance of randomly chosen colours, thus it is also random. The average of the errors calculated $(\Delta X_2)_{c4}$ and the errors measured ΔX_2 can be compared by means of the Welch-test. [3]. Let it be the hypothesis 0 that the averages correspond.
The term t_f is:

$$t_f = \frac{\bar{\xi}^2 - \bar{\eta}^2}{\sqrt{\frac{\delta_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{7}$$

where: $\xi = 0.072$ the average of the deviations' $(\Delta X_2)_{c4}$ absolute values,
$\bar{\eta} = 0.028$ the average of the measured deviations' ΔX_2 absolute values,
$\delta_1^2 = 0.0093$ the variance of the deviations $(\Delta X_2)_{c4}$
$\delta_2^2 = 0.0004$ the variance of the measured deviations ΔX_2,
$n = m = 15$ the number of data,
and with them
$t_f = 1.806$ the value of the Welch-test term.

On the other hand, if we wish to examine the question with 99% significance level, from the table of the Student-distribution $t = 2.977$. As $t_f < t$, we accept the hypothesis 0, that is the averages are considered to be identical. Thus we have proved that the tristimulus values X_2 which can be measured by means of colour filters of transmittance x_2 (λ) do not significantly deviate from the values 1/5.982 times as great as the values Z measured by means of filters of transmittance $\bar{z}(\lambda)$:

$$X_2 \approx \frac{Z}{5.982} \tag{8}$$

Thus in this case the sign X_2 can be formed out of the sign Z.

In case of colour-measuring of high accuracy it may eventually be reasonable to form signs X_2 and Z separately. It can be decided if necessary on the base of similar series of measurements and evaluation.

Measuring errors in colour difference ΔE using both the calculated and the measured tristimulus values

In the Table 3 we show the measuring errors in colour difference ΔE using both the measured tristimulus value of X_2 (these are ΔE-s) and the measured value of the X_2 (these are ΔE^*-s). It is shown also the colour difference of the colours using the measured X_2-s and the colours using the calculated X_2-s.

Comparing both the average of ΔE-s and the average of ΔE^*-s using the Welch-test and comparing both the variance of ΔE-s, and the variance of ΔE^*-s using the F-test the identity of ΔE and ΔE^* is proved.

Table 3

	Tristimulus values of set N° 7453 measured by OMH				Tristimulus values of set N° 7453 measured by MOMCOLOR N°198893				Calculated values	Measuring errors		
	X_{1E}	X_{2E}	Y_E	Z_E	X_1	X_2	Y	Z	$x_2^* = \frac{Z}{5.982}$	ΔE	ΔE^*	$\Delta(\Delta E)$
01	4.88	1.44	6.52	8.61	4.98	1.42	6.67	8.84	1.48	0.79	0.38	0.63
02	8.88	0.13	4.41	0.80	10.77	0.12	5.25	1.11	0.19	4.81	5.38	0.51
03	3.88	0.18	2.34	1.11	5.00	0.16	2.80	1.34	0.23	5.98	6.77	0.84
04	25.31	2.57	21.25	15.45	26.97	2.55	22.32	15.71	2.63	2.39	2.08	0.30
05	49.43	10.83	55.93	64.23	50.54	10.77	56.85	63.90	10.68	1.31	1.29	0.21
06	51.33	0.23	49.52	2.02	51.62	0.20	51.17	2.76	0.46	5.49	3.69	0.67
07	8.72	0.32	9.60	2.17	8.87	0.29	9.92	2.47	0.41	2.10	1.28	0.99
08	24.40	0.75	26.69	5.03	24.73	0.75	27.50	5.48	0.92	2.12	1.54	0.71
09	49.11	5.26	57.67	32.39	49.54	5.27	58.84	32.96	5.51	1.82	1.32	0.60
10	7.90	1.61	14.76	10.21	8.07	1.55	14.43	10.22	1.71	3.02	4.24	1.27
11	2.09	0.68	3.77	4.18	2.29	0.66	3.90	4.47	0.75	1.57	1.04	1.57
12	41.46	9.76	58.40	60.50	40.60	9.84	58.56	61.35	10.26	2.51	1.46	1.11
13	1.35	2.86	4.82	17.32	1.95	2.87	4.74	17.72	2.96	9.17	10.30	1.13
14	12.67	8.09	23.45	48.62	13.25	8.11	23.06	48.88	8.17	4.68	4.95	0.28
15	28.29	10.76	42.35	64.56	28.35	10.69	41.66	64.18	10.73	2.13	2.25	0.85
									Average	3.326	3.198	0.778
									Variance	2.264	2.722	0.374

ΔX: measuring errors on the ground of X_{1E}, X_{2E}, Y_E, Z_E and 2_1, 2_2, Y, Z

ΔE^*: measuring errors on the ground of X_{1E}, x_{2E}, Y_E, Z_E and 2_1, x_2^*, Y, Z

$\Delta(\Delta E)$: colour-difference of the measured and the calculated datas X_1 X_2, Y, Z and 2_1, x_2^*, Y, Z)

Is the tristimulus value X_2 necessary?

On the preceeding pages we have proved that the tristimulus values X_2 do not significantly deviate from the values 1/5, 982 times as great as the values Z. Accordingly the tristimulus value X_2 does not give any significant further information about the colour under examination as compared to the tristimulus value Z; that is why it is enough to know the tristimulus values X_1, Y, Z and X_2 is unnecessary for a definite characterization of the colours.

The tristimulus value X_2 is not only unnecessary but harmful as well because of the following viewpoints:

1. If we compare the connections (4) and (8), then:

$$X \approx X_1 + \frac{Z}{5.982} \tag{9}$$

On the other hand this means we mix the information X_1 and Z — independent from each other — gained about the colour examined; X will not be independent from Z.

2. The tristimulus colour-measuring devices are provided with 4 filters instead of 3 ones; it is an unnecessary surplus cost.

3. In the course of the measurements instead of 3 data 4 ones should be measured, registered and evaluated; this causes the unnecessary increasing of the measuring process.

Summary

In the article the author — on the base of theoretical considerations and measuring data — proves that in tristimulus colorimetry the tristimulus value X_2 can be defined with practically adequate accuracy from the value of the tristimulus value Z, since there is 98.36% correlation between the spectral colour-matching functions $\bar{x}(\lambda)$ and $\bar{z}(\lambda)$ in the range of $360 \leq \lambda$ $\lambda \leq 504$ nm. Consequently the measuring of the tristimulus value X_2, i.e. the application of the fourth colour filter seems to be unnecessary.

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