# DC-DC CONVERTER 

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#### Abstract

The paper briefly refers to the feasibilities of the switched condenser (S. C.) concept in power electronics. Its main objective is the description of a thyristor chopper as it is one of the application of the S . C . concept. Its main attractions stem from the fact that no forced commutation is needed, it operates with high frequency at and near full load and its thyristor current is partly sinusoidal.


## 1. Introduction

The switched condenser (S. C.) concept developed in the field of communication engineering and the feasibilities of the idea in power electronics will briefly referred to with the help of two basic circuits.

The main objective of the paper is the description of a thyristor chopper. Its operation is based on the S. C. concept. No forced commutation circuitry is needed. The recommended operation frequency around full load is several thousand Hertz. The thyristor derating factor is less at partly sinusoidal current than at trapezoidal one. The partly sinusoidal current is favourable from the viewpoint of Electro Magnetic Interference (EMI) compared to the trapezoidal one.

First a basic configuration with symmetrical supply voltage is discussed. It is not a practical circuit since its output voltage cannot be changed. However, the understanding of its operation helps the description of the mode of action of the chopper circuit developed from the basic configuration supplementing it by either two diodes or two thyristors.

## 2. Switch condenser concept

The analog IC technology widely applies the operational amplifier, the condenser and the resistance as principal components. The operational amplifiers and the condensers of excellent quality can be produced by MOS technology. The properties of the resistance are far not so favourable.

The switched condenser offers an attractive, ingenious solution of the problem in IC technology and involves an inherent, additional advantageous feature. The resistance is substituted by the switched condenser consisting of an electronic switch and a condenser. Remarkably good electronic switch can be produced by MOS technology.

What is actually the switched condenser [1]? A parallel and a series switched condenser circuit are shown in Figs 1.a and I.b, respectively. The switch $S$ connects terminal 0 to terminal 1 in the first half period $T / 2$ and to


Fig. I. For the explanation of the switched condenser concept
terminal 2 in the second half period $T / 2$. The switching frequency $f=1 / T$ can be either constant or it can be changed. Voltage $u_{i}$ and $u_{0}$ are supposed to be constant within one period $T$.

In the circuit of Fig. 1.a the charge of the condenser is $q_{i}=C u_{i}$ in the first half period and $q_{0}=C u_{0}$ in the second one. The average current:

$$
\begin{equation*}
i=\frac{q_{i}-q_{0}}{T}=C \frac{u_{i}-u_{0}}{T}=\frac{u_{i}-u_{0}}{R} . \tag{1}
\end{equation*}
$$

In the circuit in Fig. 1.b the charge of the condenser is $q=C\left(u_{i}-u_{0}\right)$ in the first half period, and in the second one the condenser is short-circuited and the charge is zero. The average current:

$$
\begin{equation*}
i=\frac{q}{T}=C \frac{u_{i}-u_{0}}{T}=\frac{u_{i}-u_{0}}{R} . \tag{2}
\end{equation*}
$$

From the viewpoint of average current $i$ the switched condenser can be considered in both circuits as a resistance (Fig. 1.c).

$$
\begin{equation*}
R=\frac{T}{C}=\frac{1}{f C} \tag{3}
\end{equation*}
$$

The paper tries to explore some of the feasibilities of the idea of switched condensers in power electronics.

## 3. Energetical considerations

Obviously, in IC technique the efficiency plays no decisive role. On the other hand, in power electronics it is one of the most important factors.

One theoretically possible way to realize the parallel switched condenser by thyristor switches in power electronics is shown in Fig. 2. Resistance $R$ is needed to limit the current. Unfortunately, the configuration cannot be applied, due to the extremely high power loss.

In order to avoid the high power losses and the high $\mathrm{d} i / \mathrm{d} t$ value, chokes rather than resistors are inserted in the circuits (Fig. 3). The free-wheeling thyristors drawn by dotted line clamp down the maximum and minimum


Fig. 2. Parallel switched condenser by thyristor switches and by series resistance


Fig. 3. Parallel switched condenser by thyristor switches and by series inductance
value of condenser voltage $u_{C}$ and provide a path for the energy trapped in choke $L$ towards the output or input condenser. Condenser $C_{i}$ and $C_{0}$ are needed because of the high frequency operation. Energy can be transported from the input to the output and vice versa. The configuration is the so-called parallel switched condenser circuit.

The energy taken by a current pulse from the input source and delivered by the next current pulse to the output is constant provided that the input voltage and the $C$ condenser voltage change are constant.

A simple version of the so-called series switched condenser circuit is shown in Fig. 4. It can deliver energy only from the input to the output.


Fig. 4. Series switched condenser circuit

Assuming $U_{i}=$ const. and $U_{0}=$ const. as well as $U_{i}>U_{0}$ and turning on thyristor $T_{i}$, a sinusoidal current pulse flows in the series ringing circuit $L-C$. The condenser voltage $u_{C}$ is increasing from its initial value $u_{C}=-U_{i}$. Reaching the value $u_{C}=+U_{i} T_{i}$ turns off and diode $D$ starts conducting the choke current $i$ which will be diminishing as a negative ramp function with speed $\mathrm{d} i / \mathrm{d} t=-U_{0} / L$. Later turning on thyristor $T_{0}$, the voltage $u_{C}$ is reversed and it takes its initial value $u_{C}=-U_{i}$ again.

A number of rectifier, chopper and inverter circuits can be built on the basis of S. C. concept. One of them is the chopper discussed here.

## 4. Chopper circuit

The chopper configuration shown in Fig. 5 is based on the series S. C. circuit (Fig. 4). Constant and smooth $u_{i p}$ and $u_{i n}$ input voltages ( $u_{i p}=-u_{i n}=U_{i}$ ) and $C_{i} \gg C, C_{0} \gg C$ are supposed. The positive upper side sinusoidal current pulse $i_{p}$ generates a positive voltage swing across the switched condenser $C$ while the negative, lower side current $i_{n}$ reduces the voltage $u_{C}$ (Fig. 6). The current pulses start after turning on either the main thyristor $T_{p}$ or $T_{n}$. Now there is no need for condenser voltage reversing circuit since the d. c. component of current $i_{C}$ is zero.


Fig. 5. DC-DC converter


Fig. 6. Symmetrical supply $\left(u_{\text {ip }}=-u_{\text {in }}=U_{i}\right)$.
$C_{i} \gg C$ and $C_{0} \gg C$
$C_{i} \gg C$ and $C_{0} \gg C$

Clamping thyristor $T_{c p}$ and $T_{c n}$ are inserted in the place of the freewheeling diode D . They can be fired at any time after $\left|u_{C}\right|>U_{i}$. The peak values of the condenser voltage $u_{c}$ are clamped by them at a lower value (Fig. 6). After turning on, for instance, thyristor $T_{c p}$ at angle $\alpha$, thyristor $T_{p}$ is turned off and the choke current $i_{p}$ commutates from $T_{p}$ to $T_{c p}$.

The input power is:

$$
\begin{equation*}
P_{i}=2 U_{i} \frac{1}{\omega T} \int_{0}^{x} i_{C} \mathrm{~d} \omega t=\frac{2 U_{i} I_{C_{p}}}{\omega T}(1-\cos \alpha), \tag{4}
\end{equation*}
$$

where $I_{C_{p}}$ is the peak value of the condenser current $i_{C}, T$ is the period, $\omega=1 / \sqrt{L C}$.

Being $C_{0} \gg C$, the output voltages are smooth, $u_{0 p}=u_{0 n}=U_{0}$. Neglecting the losses in the chopper, the output power $P_{0}=P_{i}$ and the output voltage:

$$
\begin{equation*}
U_{0}=\frac{P_{i}}{2 I_{L}}=\frac{U_{i}}{\omega T} \frac{I_{C_{p}}}{I_{L}}(1-\cos \alpha) \tag{5}
\end{equation*}
$$

where $I_{L}$ is the load current. Voltage $U_{0}$ can be changed by $U_{i}, T$ and $\alpha$ at constant load current. On the other hand, $I_{C p}$ depends both on the output voltage $U_{0}$ and on the peak value of the condenser voltage $\left|u_{c}(0)\right|$. Consequently, Equ. (5) is not the best form to judge the ways for changing the output voltage $U_{0}$. Somewhat deeper study of the circuit is needed.

## 5. Analysis

### 5.1 Basic configuration. No clamping

In the following study the clamping thyristor $T_{c p}$ and $T_{c n}$ are omitted in the circuit in Fig. 5. The resulting so called basic configuration is not a practical one. Its output voltage cannot be changed at constant load current in steady state. However, the understanding of its operation clarifies the behaviour of the chopper circuit shown in Fig. 5.

Again constant and smooth input voltages are supposed ( $u_{i p}=-u_{i n}=U_{i}$ ). On the other hand, now only the input condenser is chosen at much higher value than the other two: $C_{i} \gg C$ and $C_{i} \gg C_{0}$. It means that the output voltages $u_{0 p}$ and $u_{0 n}$ vary with the current pulse. Constant and smooth load current $i_{L}=I_{L}$ is assumed. As a consequence of perfect symmetry only the upper, positive side of the circuit is analysed.

At time $t=0$ thyristor $T_{p}$ is turned on. The value of the output and condenser voltages are $u_{0}(0)=u_{0 p}(0)$ and $u_{c}(0)<0$. The resultant voltage $U_{i}-u_{0}(0)-u_{C}(0)$ generates a sinusoidal current pulse in switched condenser $C$ :

$$
\begin{equation*}
i_{C u}=\frac{U_{i}-u_{0}(0)-u_{C}(0)}{\sqrt{L / C^{*}}} \sin \omega t, \tag{6}
\end{equation*}
$$

where $C^{*}=\frac{C C_{0}}{C+C_{0}}$ and $\omega=\frac{1}{\sqrt{L C^{*}}}$.
On the other hand, load current $I_{L}$ as an input step function generates a condenser current:

$$
\begin{equation*}
i_{C i}=\frac{C}{C+C_{0}} I_{L}(1-\cos \omega t) . \tag{7}
\end{equation*}
$$

The resultant condenser current:

$$
\begin{gather*}
i_{C}=i_{C u}+i_{C i},  \tag{8}\\
i_{C}=I_{C P} \sin \omega t+\frac{C}{C+C_{0}} I_{L}(1-\cos \omega t), \tag{9}
\end{gather*}
$$

where

$$
\begin{equation*}
I_{C_{p}}=\frac{U_{i}-u_{0}(0)-u_{\mathrm{C}}(0)}{\sqrt{L / C^{*}}} . \tag{9a}
\end{equation*}
$$

Equation (7) is obtained from the following node and loop equations ( $i_{0_{p}}=i_{0}$ ):

$$
\begin{equation*}
i_{C}=I_{L}+i_{0} . \tag{10}
\end{equation*}
$$

$$
\begin{equation*}
u_{0}=U_{i}-u_{C}-u_{L}=U_{i}-\frac{1}{C} \int i_{C} \mathrm{~d} t-L \frac{\mathrm{~d} i_{C}}{\mathrm{~d} t} . \tag{11}
\end{equation*}
$$

Differentiating the last relation:

$$
\begin{equation*}
i_{0}=C_{0} \frac{\mathrm{~d} u_{0}}{\mathrm{~d} t}=-\frac{C_{0}}{C} i_{C}-C_{0} L \frac{\mathrm{~d}^{2} i_{C}}{\mathrm{~d} t^{2}} . \tag{12}
\end{equation*}
$$

From Equs (10) and (12):

$$
\begin{equation*}
C^{*} L \frac{\mathrm{~d} i_{C}}{\mathrm{~d} t^{2}}+i_{C}=\frac{C}{C+C_{0}} I_{L} . \tag{13}
\end{equation*}
$$

Equation (7) is the solution of differential equation (13).
The condenser voltage:

$$
u_{C}(\omega t)=u_{C}(0)+\frac{1}{\omega C} \int_{0}^{\omega t} i_{C} \mathrm{~d} \omega t .
$$

Substituting here relation (9):

$$
\begin{gather*}
u_{C}(\omega t)=u_{C}(0)+I_{C_{p}} X_{C}(1-\cos \omega t)+ \\
+\frac{C}{C+C_{0}} I_{L} X_{C}(\omega t-\sin \omega t), \tag{14}
\end{gather*}
$$

where

$$
\begin{equation*}
X_{C}=1 / \omega C . \tag{14a}
\end{equation*}
$$

The output voltage variation in interval $0 \leq \omega t \leq \alpha_{e}$ is:

$$
u_{0}(\omega t)=u_{0}(0)+\frac{1}{\omega C_{0}} \int_{0}^{\omega t}\left(i_{C}-I_{L}\right) \mathrm{d} \omega t .
$$

$\alpha_{e}$ is the extinction angle of current $i_{c}$.
Substituting again relation (9):

$$
\begin{equation*}
u_{0}(\omega t)=u_{0}(0)+X_{C 0}\left[I_{C_{p}}(1-\cos \omega t)-\frac{C^{*}}{C} I_{L} \omega t-\frac{C^{*}}{C_{0}} I_{L} \sin \omega t\right], \tag{15}
\end{equation*}
$$

where

$$
\begin{equation*}
X_{C 0}=1 / \omega C_{0} . \tag{15a}
\end{equation*}
$$

The output voltage change in interval $\alpha_{e} \leq \omega t \leq \omega T$ is:

$$
u_{0}(\omega t)=u_{0}\left(\alpha_{e}\right)-\frac{1}{\omega C_{0}} \int_{z_{e}}^{\omega t} I_{L} \mathrm{~d} \omega t .
$$

$$
\begin{equation*}
u_{0}(\omega t)=u_{0}\left(\alpha_{e}\right)-I_{L} X_{C 0}\left(\omega t-\alpha_{e}\right) \tag{16}
\end{equation*}
$$

The average value of the input power:

$$
\begin{align*}
& P_{i}=\frac{2 U_{i}}{\omega T} \int_{0}^{\alpha_{e}} i_{C} \mathrm{~d} \omega t \\
& P_{i}=\frac{2 U_{i}}{\omega T}\left[I_{C_{p}}\left(1-\cos \alpha_{e}\right)+\frac{C^{*}}{C_{0}} I_{L}\left(\alpha_{e}-\sin \alpha_{e}\right)\right] \tag{17}
\end{align*}
$$

Assuming lossless circuit, the average output power $P_{0}=P_{i}$ and the average output voltage:

$$
\begin{equation*}
U_{0}=\frac{P_{i}}{2 I_{L}} \tag{18}
\end{equation*}
$$

Equation

$$
\begin{equation*}
U_{0}=U_{i} \tag{19}
\end{equation*}
$$

must be satisfied. The reason is as follows (Fig. 7):


Fig. 7. Time function of output voltage

Interval $0 \leq \omega t \leq \alpha_{e}$ :

1. The initial and final values of the choke current are zero. The average value of the choke voltage:

$$
\begin{equation*}
U_{L}=U_{L p}=\frac{1}{\omega T} \int_{0}^{x_{e}} u_{L p} \mathrm{~d} \omega t=\frac{L}{\omega T} \int_{i_{C}(0)}^{i_{C}\left(z_{e}\right)} \mathrm{d} i_{C}=0 \tag{20}
\end{equation*}
$$

2. As a consequence of the perfect symmetry the relation between the initial and final value of the condenser voltage $u_{C}$ is:

$$
\begin{equation*}
u_{c}(0)=-u_{c}\left(\alpha_{e}\right) \tag{21}
\end{equation*}
$$

3. Taking into consideration Equ. (21) and $i_{C}\left(\alpha_{e}\right)=0$ from Equ. (9), it can be shown that the average value of the condenser voltage:

$$
\begin{equation*}
U_{C}=\frac{1}{\omega T} \int_{0}^{x_{e}} u_{C} \mathrm{~d} \omega t=0 \tag{22}
\end{equation*}
$$

Conclusion: $U_{0}=U_{i}$ must be satisfied in this interval as a consequence of Equs (20) and (22).

At the same time relation (Fig. 7):

$$
\begin{equation*}
u_{0}\left(\alpha_{e}\right)-U_{i}=U_{i}-u_{0}(0)=\Delta U \tag{23}
\end{equation*}
$$

can be proved on the basis of Equs $i_{C}\left(\alpha_{e}\right)=0$ [see Equ. (9)], $u_{C}\left(\alpha_{e}\right)=-u_{C}(0)$ [see Equ. (14)] and $u_{0}\left(\alpha_{e}\right)$ [see Equ. (15)].

Interval $\alpha_{e} \leq \omega t \leq \omega T$
The output voltage $u_{0}$ is decreasing as a ramp function (Fig. 7) and as a consequence of relation (23) the voltage-time area:

$$
\int_{x_{e}}^{\omega t}\left(u_{0}-U_{i}\right) \mathrm{d} \omega t=0 .
$$

$U_{0}=U_{i}$ must be satisfied in this interval again. Naturally $U_{0}=U_{i}$ holds in the whole period.

The calculation of the variables at given $C, C_{0}, T, U_{i}$ and $I_{L}$ goes as follows:

Being $i_{C}\left(\alpha_{e}\right)=0$, from Equ. (9):

$$
\begin{equation*}
U_{i}-u_{0}(0)-u_{C}(0)=I_{L} X_{C O} \frac{\cos \alpha_{e}-1}{\sin \alpha_{e}} . \tag{24}
\end{equation*}
$$

From Equ. (15):

$$
\begin{align*}
& u_{0}\left(\alpha_{e}\right)-u_{0}(0)=\frac{C^{*}}{C_{0}}\left[U_{i}-u_{0}(0)-u_{C}(0)\right]\left(1-\cos \alpha_{e}\right)- \\
&-I_{L} X_{C 0}\left(\frac{C^{*}}{C} \alpha_{e}+\frac{C^{*}}{C_{0}} \sin \alpha_{e}\right) . \tag{25}
\end{align*}
$$

From Equ. (16):

$$
\begin{equation*}
u_{0}\left(\alpha_{e}\right)-u_{0}(0)=I_{L} X_{C 0}\left(\omega T-\alpha_{e}\right), \tag{26}
\end{equation*}
$$

where the periodicy condition $u_{0}(\omega T)=u_{0}(0)$ has been applied.

Substituting Equs (24) and (26) into Equ. (25):

$$
\begin{equation*}
\omega T=\frac{C^{*}}{C_{0}}\left(\alpha_{e}-2 \frac{1-\cos \alpha_{e}}{\sin \alpha_{e}}\right) \tag{27}
\end{equation*}
$$

relation is deduced. It means that period $T$ as well as $C, C_{0}$ determine the extinction angle $\alpha_{e}$.

In the next two steps the switched condenser voltage amplitude $u_{c}\left(\alpha_{e}\right)$ and the maximum value of the output voltage $u_{0}\left(\alpha_{e}\right)$ are calculated from Equ. (14) and from Equ. (16), respectively.

$$
\begin{gather*}
u_{C}\left(\alpha_{e}\right)=\frac{I_{L}}{2 \omega\left(C+C_{0}\right)}\left(\alpha_{e}-2 \frac{1-\cos \alpha_{e}}{\sin \alpha_{e}}\right)=\frac{I_{L}}{2 C} T,  \tag{28}\\
u_{0}\left(\alpha_{e}\right)=U_{i}+\frac{1}{2} I_{L} X_{C 0}\left(\omega T-\alpha_{e}\right)=U_{i}+\Delta U . \tag{29}
\end{gather*}
$$

Relations (23) and (24) have been used in the derivation of Equs (28) and (29), respectively.

The result given in Equ. (28) can easily be explained. The current pulse $i_{C}$ delivers the charge $Q=C\left[2 u_{C}\left(\alpha_{e}\right)\right]$ from the input to the output condenser $C_{0}$ while the same amount of charge $Q=I_{L} T$ is taken from the condenser $C_{0}$ by the load current.

The interpretation of Equ. (29) is as follows (Fig. 7): The load current $I_{L}$ flowing across condenser $C_{0}$ in time $\Delta T=\left(T-\alpha_{e} / \omega\right) / 2$ delivers a charge $\Delta Q=I_{L} \Delta T$ and generates a voltage change $\Delta U=\Delta Q / C_{0}$.

Figure 8 summarizes the steps of calculation.
Introducing per unit system, Equs (28) and (29) take the form:

$$
\begin{gather*}
u_{C}^{\prime}\left(\alpha_{e}\right)=\frac{1}{\pi} I_{L}^{\prime} \omega T  \tag{28a}\\
u_{0}^{\prime}\left(\alpha_{e}\right)=1+\frac{1}{\pi} \frac{C}{C_{0}} I_{L}^{\prime}\left(\omega T-\alpha_{e}\right), \tag{29a}
\end{gather*}
$$

where

$$
\begin{equation*}
I_{L}^{\prime}=\frac{I_{L}}{\frac{2}{\pi} \omega C U_{i}} \tag{30}
\end{equation*}
$$

and

$$
\begin{aligned}
& u_{C}^{\prime}\left(\alpha_{e}\right)=u_{C}\left(\alpha_{e}\right) / U_{i} \\
& u_{0}^{\prime}\left(\alpha_{e}\right)=u_{0}\left(\alpha_{e}\right) / U_{i}
\end{aligned}
$$



Fig. 8. Steps of calculation

Figure 9 shows the variation $\alpha_{e}$ as a function of $\omega T C_{0} / C^{*}$ on the basis of Equ. (27). The order of magnitude of $\omega T C_{0} / C^{*}$ is at least $10^{3}$ degrees, that is, the extinction angle is somewhere in the range $200^{\circ} \geq \alpha_{e} \geq 180^{\circ}$.

It was proved that relation $U_{0}=U_{i}$ must be satisfied in steady state [Equ. (19)]. For the sake of simplicity let us suppose for a moment that a d.c. voltage source is connected in the place of condenser $C_{0}$ and its voltage


Fig. 9. Extinction angle $\alpha_{e}$ versus $\omega T C_{0} / C^{*}=\omega T\left(1+C_{0} / C\right)$
$U_{0}$ is smaller than $U_{i}$, that is, $\Delta=U_{i}-U_{0}$. After each current pulse the condenser voltage amplitude will increase by 24 value. Steady-state can be reached either in case $\Delta=0$ or by firing the clamping thyristor $T_{c p}$ and $T_{c n}$ in Fig. 5 at time when the condenser voltage $u_{C}$ takes a preset constant value.

### 5.2 Chopper with clamping

The clamping thyristor $T_{c p}$ and $T_{c n}$ in Fig. 5 are fired at angle $\alpha$ measured from the gating of the main thyristor $T_{p}$ and $T_{n}$, respectively after the voltage across them becomes positive. Otherwise all of the assumptions mentioned at the beginning of section 5.1 hold. Now steady-state can be ensured although the average output voltage $U_{0}<U_{i}$. The output voltage $U_{0}$ can be changed at constant input voltage as well.

The time functions of $i_{c}$ and that of $u_{C}$ are shown in Fig. 6. The main thyristor $T_{p}$ is turned on at $\omega t=0$ and the clamping thyristor $T_{c p}$ is fired at $\omega t=\alpha$ when $u_{C}(\alpha)>U_{i}$. Equ. (9) for the condenser current $i_{C}$, Equ. (15) for the condenser voltage $u_{C}$ and Equ. (14) for the output voltage $u_{0}$ are still valid in the interval $0 \leq \omega t \leq \alpha$. Now $\left|u_{C}(0)\right|=u_{C}(\alpha)$ is the clamped condenser voltage amplitude.

Right after firing the clamping thyristor the condenser voltage stops changing, its current $i_{C}$ is zero. The choke current $i_{p}$ reaches its zero value at the extinction angle $\alpha_{e}$. It is assumed that the next main thyristor is gated only after the choke current has declined to zero. In the interval $\alpha \leq \omega t \leq \alpha_{e}$ the choke current $i_{p}$ in the ringing circuit $T_{C_{p}}-L-C$ is:

$$
\begin{equation*}
i_{p}(\omega t)=-\frac{u_{0}(\alpha)}{X_{L}^{*}} \sin x_{t}+\left[i_{C}(\alpha)-I_{L}\right] \cos x_{t}+I_{L} \tag{31}
\end{equation*}
$$

where

$$
\begin{gather*}
X_{L}^{*}=\omega_{0} L,  \tag{32}\\
\omega_{0}=1 / \sqrt{C_{0} L},  \tag{33}\\
x_{t}=\frac{\omega_{0}}{\omega}(\omega t-\alpha) . \tag{34}
\end{gather*}
$$

The output voltage:

$$
\begin{equation*}
u_{0}(\omega t)=u_{0}(\alpha) \cos x_{t}+\left[i_{C}(\alpha)-I_{L}\right] X_{L}^{*} \sin x_{t} . \tag{35}
\end{equation*}
$$

In fact, beside $U_{i}$ and $I_{L}$ the firing angle $\alpha$ and the period $T$ can be selected at a given circuit. All other variables, namely the extinction angle $\alpha_{e}$, the condenser voltage $u_{C}$ and current $i_{c}$, the output voltage $u_{0}$ and current $i_{0}$ as well as the choke current can be calculated from $U_{i}, I_{L}, \propto$ and $T$. However,
from mathematical point of view the selection of $\alpha_{e}$ rather than $T$ is rewarding. It means that besides $U_{i}, I_{L}$ and $\alpha$ the extinction angle $\alpha_{e}$ will be preselected. The rest of the variables will be calculated from them.

### 5.2.1 Calculation of instantaneous values

It has been shown [Equ. (27)] that the period $T$ can be calculated from $\alpha_{e}$ in the case when the clamping thyristor are not fired. The picture is more complex with clamping thyristor gated. The value $T$ depends not only on $\alpha_{e}$ but on $\alpha, U_{i}$ and $I_{L}$ too as we will see.

In order to calculate $T$ six equations are needed at the beginning. They are as follows: $i_{C}(\alpha), i_{p}\left(\alpha_{e}\right)=0, u_{C}(\alpha)=-u_{C}(0), u_{0}(\alpha), u_{0}\left(\alpha_{e}\right)$ and $u_{0}(\omega T)=u_{0}(0)$.

$$
\begin{gather*}
i_{C}(\alpha)=I_{C p} \sin \alpha+\frac{C}{C+C_{0}} I_{L}(1-\cos \alpha),  \tag{9b}\\
i_{p}\left(\alpha_{e}\right)=-\frac{u_{0}(\alpha)}{X_{L}^{*}} \sin x+\left[i_{C}(\alpha)-I_{L}\right] \cos x+I_{L}=0,  \tag{30a}\\
u_{C}(\alpha)=-u_{C}(0)=u_{C}(0)+I_{C p} X_{C}(1-\cos \alpha)+ \\
+\frac{C}{C+C_{0}} I_{L} X_{C}(\alpha-\sin \alpha)  \tag{14b}\\
u_{0}(\alpha)=u_{0}(0)+\dot{I}_{C p} X_{C 0}(1-\cos \alpha)-I_{L} X_{C 0}\left(\frac{C^{*}}{C} \alpha+\frac{C^{*}}{C_{0}} \sin \alpha\right),  \tag{15b}\\
u_{0}\left(\alpha_{e}\right)=u_{0}(\alpha) \cos x+\left[i_{C}(\alpha)-I_{L}\right] X_{L}^{*} \sin x,  \tag{35a}\\
u_{0}(\omega T)=u_{0}(0)=u_{0}\left(\alpha_{e}\right)-I_{L} X_{C 0}\left(\alpha T-\alpha_{e}\right), \tag{26a}
\end{gather*}
$$

where

$$
\begin{equation*}
x=\frac{\omega_{0}}{\omega}\left(\alpha_{e}-\alpha\right) . \tag{36}
\end{equation*}
$$

Substituting Equ. (9b) into Equs (30a) and (35a) as well as taking into consideration Equ. (9a), the following matrix equation is obtained from Equs (30a), (14b), (15b) and (35a) in per unit:

$$
\begin{equation*}
\mathbf{A u}=\mathbf{B} \mathbf{v} \tag{37}
\end{equation*}
$$

where

$$
\mathbf{A}=\left[\begin{array}{llll}
1 & -1 & a 13 & 0  \tag{38}\\
a 21 & 1 & 0 & 0 \\
a 31 & a 32 & 1 & 0 \\
a 41 & a 42 & a 43 & 1
\end{array}\right]
$$

$$
\begin{align*}
& \mathbf{u}^{T}=\left[\begin{array}{llll}
u_{C}^{\prime}(\alpha) & u_{0}^{\prime}(0) & u_{0}^{\prime}(\alpha) & u_{0}^{\prime}\left(\alpha_{e}\right)
\end{array}\right]  \tag{39}\\
& \mathbf{B}=\left[\begin{array}{cc}
-1 & b 12 \\
1 & b 22 \\
b 31 & b 32 \\
b 41 & b 42
\end{array}\right]  \tag{40}\\
& \mathbf{v}^{T}=\left[\begin{array}{ll}
1 & I_{L}^{\prime}
\end{array}\right] .  \tag{41}\\
& a 13=-\sqrt{\frac{C_{0}}{C^{*}}} \frac{1}{\sin \alpha} \operatorname{tg} x,  \tag{42}\\
& a 21=\frac{2 \frac{C}{C^{*}}-(1-\cos \alpha)}{1-\cos \alpha} \text {, }  \tag{43}\\
& a 31=-\frac{C^{*}}{C_{0}}(1-\cos \alpha),  \tag{44}\\
& a 32=\frac{C^{*}}{C_{0}}(1-\cos \alpha)-1,  \tag{45}\\
& a 41=-\sqrt{\frac{C^{*}}{C_{0}}} \sin \alpha \sin x \text {. }  \tag{46}\\
& a 42=-a 41 \text {, }  \tag{47}\\
& a 43=-\cos x \text {, }  \tag{48}\\
& b 12=-\frac{2}{\pi} \frac{C}{C^{*}}\left[\frac{C}{C+C_{0}} \frac{1-\cos \alpha}{\sin \alpha}+\frac{1-\cos x}{\sin \alpha \cos x}\right],  \tag{49}\\
& b 22=\frac{2}{\pi} \frac{C}{C_{0}} \frac{\alpha-\sin \alpha}{1-\cos \alpha},  \tag{50}\\
& b 31=-a 31 \text {, }  \tag{51}\\
& b 32=-\frac{2}{\pi} \frac{C}{C_{0}}\left(\frac{C^{*}}{C} \alpha+\frac{C^{*}}{C_{0}} \sin \alpha\right),  \tag{52}\\
& b 41=-a 41 \text {, }  \tag{53}\\
& b 42=\frac{2}{\pi} \sqrt{\frac{C^{*}}{C_{0}}}\left[\frac{C}{C_{0}}(1-\cos \alpha) \sin x-\frac{C}{C^{*}} \sin x\right] . \tag{54}
\end{align*}
$$

The four components of column matrix $\mathbf{u}$ can be calculated from Equ. (37) at given $I_{L}^{\prime}, \alpha$ and $\alpha_{e}$. Finally $\omega T$ is determined from Equ. (26a):

$$
\begin{equation*}
\omega T=\frac{\pi}{2} \frac{C}{C_{0}} \frac{1}{I_{L}^{\prime}}\left[u_{0}^{\prime}\left(\alpha_{e}\right)-u_{0}^{\prime}(0)\right]+\alpha_{e}=f\left(I_{L}^{\prime}, \alpha, \alpha_{e}\right)+\alpha_{e} . \tag{55}
\end{equation*}
$$

In contrast to Equ. (27) now $\omega T$ depends not only $\alpha_{e}$ but on $\alpha$ and $I_{L}^{\prime}$ as well.

### 5.2.2 Power, output voltage

The average value of the input power:

$$
\begin{equation*}
P_{i}=\frac{2 U_{i}}{\omega T} \int_{0}^{\pi} i_{C} \mathrm{~d} \omega t . \tag{56}
\end{equation*}
$$

The voltage change of the switched condenser during current conduction (Fig. 6):

$$
\begin{equation*}
u_{C}(\alpha)-u_{C}(0)=2 u_{C}(\alpha)=\frac{1}{\omega C} \int_{0}^{\alpha} i_{C} \mathrm{~d} \omega t \tag{57}
\end{equation*}
$$

From the last two equations:

$$
\begin{equation*}
P_{i}=\frac{2 W}{T}=\frac{4 C U_{i} u_{C}(\alpha)}{T}, \tag{58}
\end{equation*}
$$

where $W$ is the energy transported by the circuit from the input to the output by one $i_{C}$ current pulse. Keeping $u_{C}(\alpha)$ at constant value by the clamping thyristors, $W$ is constant as well.

In case of lossless chopper the average output power:

$$
\begin{equation*}
P_{0}=P_{i}=\frac{4 C}{T} U_{i} u_{C}(\alpha), \tag{59}
\end{equation*}
$$

or

$$
\begin{equation*}
P_{0}=2 U_{0} I_{L} . \tag{59a}
\end{equation*}
$$

Substituting Equ. (14):

$$
\begin{equation*}
P_{0}=\frac{2 U_{i}}{\omega T}\left[I_{C_{p}}(1-\cos \alpha)+\frac{C^{*}}{C_{0}} I_{L}(\alpha-\sin \alpha)\right] . \tag{60}
\end{equation*}
$$

The average value of the output voltage from $P_{0}=2 U_{0} I_{L}$ :

$$
\begin{equation*}
U_{0}=\frac{U_{i}}{\omega T}\left[\frac{I_{C_{p}}}{I_{L}}(1-\cos \alpha)+\frac{C^{*}}{C_{0}}(\alpha-\sin \alpha)\right] \tag{61}
\end{equation*}
$$

or

$$
\begin{equation*}
U_{0}=\frac{W}{\Pi_{L}}=\frac{2 C}{T} \frac{U_{i} u_{\mathrm{C}}(\alpha)}{I_{L}} . \tag{62}
\end{equation*}
$$

In per unit

$$
\begin{gather*}
U_{0}^{\prime}=\frac{\pi}{\omega T} \frac{u_{C}^{\prime}(x)}{I_{L}^{\prime}} .  \tag{63}\\
P_{0}^{\prime}=2 U_{0}^{\prime} I_{L}^{\prime} . \tag{59b}
\end{gather*}
$$

The following conclusions can be drawn from Equs (59) . . (63) for a given circuit:

According to Equ. (59) the output voltage can be changed by $U_{i}, u_{\mathrm{C}}(\alpha)$ and $T$. In practice $U_{i}$ is usually constant. In order to keep the voltage stress of the components on low value, $u_{C}(\alpha)$ is chosen near at value $U_{i}$ by selecting the right firing angle $\alpha$. It means that the basic way for changing $P_{0}$ is the variation of period $T$.

The same statements hold for changing the output voltage $U_{0}$ except that $U_{0}$ depends on the value of the load current $I_{L}$ as well [Equ. (62)].

The most important conclusion one has to keep in mind is that the configuration is a constant energy delivery device if $U_{i}=$ const. and $u_{C}(\alpha)=$ const. The energy $W$ transported by each current pulse is constant.

Equation (61) takes the same form as Equ. (5) when $\mathrm{C}_{0} \gg C^{*}$, that is, when the output voltage is smooth, ripple free. From Equs (61) and (63):

$$
\begin{equation*}
I_{C_{p}}^{\prime}=\frac{1}{1-\cos \alpha}\left[\pi u_{C}^{\prime}(\alpha)-I_{L}^{\prime} \frac{C^{*}}{C_{0}}(\alpha-\sin \alpha)\right] . \tag{64}
\end{equation*}
$$

$I_{C p}^{\prime}$ is the amplitude of the current pulse flowing as a response to the voltages $U_{i}^{\prime}, u_{0}^{\prime}(0)$ and $u_{c}^{\prime}(0)$ [Equ. (9a)]. From Equ. (9):

$$
\begin{equation*}
I_{L p}^{\prime}=\frac{C^{*}}{C_{0}} I_{L}(1-\cos \alpha) . \tag{65}
\end{equation*}
$$

Assuming $x \leq \pi, I_{L p}^{\prime}$ is the maximum value of the condenser current component flowing as a response to the load current $I_{L}^{\prime}$.

### 5.2.3 Smooth output voltage. Discontinuous current condition

It is assumed that the choke current is discontinuous, that is, it reaches its zero value before the same main thyristor is turned on again.

Increasing the capacitance $C_{0}$ of the output condenser, output voltage $u_{0}$ becomes more and more smooth. On the other hand in the case of
$C_{0} \gg C^{*}, I_{C p}^{\prime} \gg I_{L_{p}}^{\prime}$ and with good approximation:

$$
\begin{equation*}
I_{C_{p}}^{\prime}=\frac{1}{1-\cos \alpha} \pi u_{C}^{\prime}(\alpha) \tag{64a}
\end{equation*}
$$

and

$$
\begin{equation*}
u_{0}^{\prime}(\omega t)=U_{0}^{\prime} \tag{66}
\end{equation*}
$$

The instantaneous value of the output voltage does not change. The condenser current is pure sinusoidal in interval $0 \leq \omega t \leq \alpha$ (Fig. 6).

Now the relation among $u_{c}^{\prime}(\alpha), \alpha$ and $U_{0}^{\prime}$ from Equ. (14) is:

$$
\begin{equation*}
u_{C}^{\prime}(\alpha)=\left(1-U_{0}^{\prime}\right) / \beta \tag{67}
\end{equation*}
$$

where $\beta=\frac{1+\cos \alpha}{1-\cos \alpha}$.
Substituting $u_{i}^{\prime}(\alpha)$ from Equ. (64a) into Equ. (63):

$$
\begin{equation*}
U_{0}^{\prime}=\frac{1}{\omega T} \frac{I_{C p}^{\prime}}{I_{L}^{\prime}}(1-\cos \alpha) \tag{68}
\end{equation*}
$$

and into Equ. (67):

$$
\begin{equation*}
I_{C_{p}}^{\prime}=\frac{\pi}{1+\cos \alpha}\left(1-U_{0}^{\prime}\right)=\frac{\pi}{1-\cos \alpha} u_{c}^{\prime}(\alpha) . \tag{69}
\end{equation*}
$$

From the last two equations:

$$
\begin{equation*}
U_{0}^{\prime}=\frac{\pi}{\pi+\beta \omega T I_{L}^{\prime}} \tag{70}
\end{equation*}
$$

Selecting the values $\alpha, \omega T$ and $I_{L}^{\prime}$, the unknows $U_{0}^{\prime}, I_{C p}^{\prime}$ and $u_{C}^{\prime}(\alpha)$ can be determined from Equs (70), (69) and (67).

In the interval $\alpha \leq \omega t \leq \alpha_{e}$ the choke current:

$$
i_{p}^{\prime}=i_{C}^{\prime}(\alpha)-U_{0}^{\prime}(\omega t-\alpha) \pi / 2
$$

At the extinction angle $\omega t=\alpha_{e}$ :

$$
i_{p}^{\prime}\left(\alpha_{e}\right)=0=i_{C}^{\prime}(\alpha)-U_{0}^{\prime}\left(\alpha_{e}-\alpha\right) \pi / 2
$$

that is

$$
\begin{equation*}
\alpha_{e}=\frac{2}{\pi} \frac{I_{C p}^{\prime}}{U_{0}^{\prime}} \sin \alpha+\alpha . \tag{71}
\end{equation*}
$$

The maximum value of the output voltage:

$$
U_{0 \max }=U_{i}
$$

and that of the energy when $U_{0}=U_{i}$ and $-u_{C}(0)=U_{i}$ :

$$
\begin{equation*}
W_{\max }=2 C U_{i}^{2} \tag{72}
\end{equation*}
$$

The maximum value of the output power $\left(\alpha_{e}=\alpha=\pi\right)$

$$
\begin{equation*}
P_{0 \max }=\frac{2 W_{\max }}{T_{\min }}=\frac{4 C U_{i}^{2}}{I_{\min }}=\frac{2 \omega C U_{i}^{2}}{\pi} \tag{73}
\end{equation*}
$$

and that of the load current:

$$
\begin{equation*}
I_{L \max }=\frac{P_{0 \max }}{U_{i}}=\frac{1}{\pi} \omega C U_{i}=\frac{1}{\pi} \frac{U_{i}}{X_{C}} \tag{74}
\end{equation*}
$$

$U_{i}$ and $P_{0 \text { max }}$ have been selected as base quantities. The base quantity for the current is $2 I_{L \max }=\frac{2}{\pi} \frac{U_{i}}{X_{C}}$ [see Equ. (30)]. The rated value of $I_{L}^{\prime}$ is 0.5 . The output power in per unit [Equ. (63)]:

$$
\begin{equation*}
P_{0}^{\prime}=\frac{2 \pi}{\omega T} u_{C}^{\prime}(\alpha) \tag{75}
\end{equation*}
$$

When the output quantities are at their rated values and $\omega T=\omega T_{\min }=2 \pi$, $\alpha=\pi$ then $u_{c}^{\prime}(\alpha)=1, \alpha_{e}=\pi, I_{C_{p}}^{\prime}=\pi / 2$. It is a theoretical limit case since the turn-off time for both thyristors are zero as it will be seen soon.

The time function of $i_{C}^{\prime}, i_{p}^{\prime}$ and $u_{C}^{\prime}$ is shown in Fig. 10a and Figures b and c present the time functions of thyristor voltages $u_{T p}$ and $u_{T n}$, respectively. The turn-off time for the main thyristors (Fig. b):

$$
\begin{equation*}
t_{\mathrm{off}}=\frac{T}{2}-\frac{1}{\omega}(\alpha-\Delta \alpha) \tag{76}
\end{equation*}
$$

and that for the clamping thyristors:

$$
\begin{equation*}
t_{\mathrm{offc}}=T-\alpha_{\mathrm{e}} / \omega \tag{77}
\end{equation*}
$$

Figure 11 shows the variation of $I_{C_{p}}^{\prime}$ and $U_{0}^{\prime}$ as a function of $\alpha$ drawn on the basis of Equs (64a) and (67). Parameter is $u_{c}^{\prime}(x) . U_{0}^{\prime}$ is approaching zero at $\alpha=\alpha_{0}$. In the case of $u_{c}^{\prime}(\alpha)=1$ the value $\alpha_{0}=90^{\circ}$. The extinction angle $\alpha_{e} \rightarrow \alpha$ at $\alpha_{0}$.

From Equs (70) and (75):

$$
\begin{equation*}
u_{C}^{\prime}(\alpha)=\frac{\omega T I_{L}^{\prime}}{\pi+\omega T I_{L}^{\prime} \beta(\alpha)} \tag{78}
\end{equation*}
$$

Figure 12 presents $\alpha_{e}$ and $\omega T r_{L}$ in the function of $\alpha$. The parameter is $u_{C}^{\prime}(\alpha)$. $\omega T>x_{e}$ at any firing angle $x$ and $u_{C}^{\prime}(x)$ even in some cases of load current


Fig. 10. Time functions for discontinuous current conduction $C_{0} \gg C$. Scale is changed in b ) and c) compared to a)


Fig. 11. The average output voltage $U_{0}^{\prime}$ and the peak value of the condenser current $I_{c_{p}}^{\prime}$ versus $\alpha$ for smooth output voltage


Fig. 12. $\alpha_{e}(x)$ and $\omega T I_{L}^{\prime}(\alpha)$ for smooth output voltage
$I_{L}^{\prime}>1 . I_{L}^{\prime}=1$ is twice as high as the rated load current. It means that the configuration works only in discontinuous current conduction mode.

The most important quantities belonging to preselected $\alpha$ and $\omega T I_{L}^{\prime}$ can be determined from Figs 11 and 12.

On the basis of the previous relations three more expressions are given here, namely from Equs (68) and (71):

$$
\begin{equation*}
\omega T=\frac{\pi}{2}\left(\alpha_{e}-\alpha\right) \frac{1-\cos \alpha}{\sin \alpha} \frac{1}{I_{L}^{\prime}} . \tag{79}
\end{equation*}
$$

From Equs (67) and (75):

$$
\begin{equation*}
U_{0}^{\prime}=\frac{2 \sin \alpha}{2 \sin \alpha+\left(\alpha_{e}-\alpha\right)(1+\cos \alpha)} \tag{80}
\end{equation*}
$$

and with the last equation:

$$
\begin{equation*}
P_{0}^{\prime}=\frac{4 I_{L}^{\prime} \sin \alpha}{2 \sin \alpha+\left(\alpha_{e}-\alpha\right)(1+\cos \alpha)} . \tag{81}
\end{equation*}
$$

The next chapter uses the three equations for comparison between the various functions obtained by supposing smooth d. c. output voltage on the one hand and on the other determined them in the more general case when the output voltage varies during the period.

## 6. Computer results

Figures 13-21 present the numerical results in per unit for the condenser ratios $C_{0} / C=10,20$ and 40 as well as for firing angles $\alpha=90^{\circ}, 120^{\circ}$ and $160^{\circ}$. The parameter in each case is the extinction angle $\alpha_{e}$. The variables are plotted as a function of the load current $I_{L}^{\prime}$. The "a" figures show the variation of the output voltages $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha)$ and $u_{0}^{\prime}\left(\alpha_{e}\right)$ as well as that of the condenser voltage amplitude $u_{c}^{\prime}(\alpha)$ [Equ. (37)]. The "b" figures show $\omega T$ [Equ. (55)], the variation of the average value of the output voltage $U_{0}^{\prime}$ [Equ. (63)] and that of the output power $P_{0}^{\prime}[E q u$. (59a) $]$ as a function of $I_{L}^{\prime}$. The " $b$ " figures include the results calculated on the basis of Equs (79), (80) and (81) for smooth output voltage drawn by heavy lines for the sake of comparison.

As it was expected the output voltage $U_{0}^{\prime}$ can be reduced by increasing $\omega T$ and by decreasing $\alpha$ in order to keep $u_{C}^{\prime}(\alpha)$ near unity. Figure 22 drawn on the basis of the same equations used for plotting Figs 19, 20 and 21 confirms the previous statement. The unequality $1<u_{C}^{\prime}(\alpha)<1.3$ holds for the curves in


Fig. 13. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(x), u_{0}^{\prime}\left(x_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(x)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 14. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(x), u_{0}^{\prime}\left(x_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(x)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 15. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(\alpha_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(x)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. I6. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(x), u_{0}^{\prime}\left(x_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(x)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 17. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(x_{e}\right)$ and the peak value of the condenser voltage $u_{c}^{\prime}(x)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 18. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(\alpha_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(\alpha)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 19. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(\alpha_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(\alpha)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 20. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(\alpha_{e}\right)$ and the peak value of the condenser voltage $u_{C}^{\prime}(\alpha)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)


Fig. 21. Instantaneous values of the output voltage $u_{0}^{\prime}(0), u_{0}^{\prime}(\alpha), u_{0}^{\prime}\left(x_{e}\right)$ and the peak value of the condenser voltage $u_{c}^{\prime}(\alpha)$ versus load current $I_{L}^{\prime}$ (Fig. a). Average value of the output voltage $U_{0}^{\prime}$, output power $P_{0}^{\prime}$ and the period $\omega T$ versus $I_{L}^{\prime}$ (Fig. b)

Fig. 22. The smaller the firing angle is the bigger the change in the output voltage is during the period. The difference between the average output voltages $U_{0}^{\prime}$ calculated for smooth and for varying output voltage is quite substantial at low firing angle $\alpha$ and it approaches zero as the firing angle gets closer to $180^{\circ}$.


Fig. 22. Average value of output voltage $U_{0}^{\prime}$ versus firing angle $\alpha$
The peak condenser current $I_{C_{p}}^{\prime}$ as a function of $\alpha$ can be seen in Fig. 23 [Equ. (64)]. The curves $I_{C_{p}}^{\prime}(\alpha)$ at $C_{0} / C>10$ does not deviate substantially from those shown in Fig. 23. $I_{C_{p}}^{\prime}$ increases by the reduction of $\alpha . I_{C_{p}}^{\prime}=\pi / 2$ at $\alpha=180^{\circ}$ and doubles itself at $\alpha=90^{\circ}$ at no-load.


Fig. 23. Peak value of condenser current $I_{\mathcal{C}_{p}}^{\prime}$ versus firing angle $\alpha$
The time functions of $u_{0}^{\prime}, u_{C}^{\prime}, i_{C}^{\prime}$ and $i_{p}^{\prime}$ are plotted at four different operation points in Figs 24-27. The condenser voltage amplitude:

$$
u_{C}^{\prime}(\alpha)=\frac{1}{\pi} \int_{0}^{\alpha} i_{C}^{\prime} \mathrm{d} \omega t
$$



Fig. 24. Time functions of the output and condenser voltages $u_{0}^{\prime}, u_{C}^{\prime}$ as well as that of condenser and choke current ( $i_{C}^{\prime}$ and $i_{p}^{\prime}$ )


Fig. 25. Time functions of the output and condenser voltages $u_{0}^{\prime}, u_{C}^{\prime}$ as well as that of condenser and choke current ( $i_{C}^{\prime}$ and $i_{p}^{\prime}$ )
and it is near unity in each case. It means, that the condenser current integral is near constant, that is, $I_{C_{p}}^{\prime}$ has to be increased as $\alpha$ is reduced. On the other hand, $\omega T$ is several times as high at $\alpha=90^{\circ}$ as it is at $\alpha=180^{\circ}$. Being $I_{L}^{\prime}$ constant ( $I_{L}^{\prime}=0.5$ ), the average value of current $i_{p}^{\prime}$ must be constant as well. It is ensured by ballooning the tail of the current pulse $i_{p}$ from $\alpha$ till $\alpha_{e}$ at


Fig. 26. Time functions of the output and condenser voltages $u_{0}^{\prime}, u_{C}^{\prime}$ as well as that of condenser and choke current ( $i_{c}^{\prime}$ and $i_{p}^{\prime}$ )


Fig. 27. Time functions of the output and condenser voltages $u_{0}^{\prime}, u_{C}^{\prime}$ as well as that of condenser and choke current ( $i_{c}^{\prime}$ and $i_{p}^{\prime}$ )
small $\alpha$. It can be happened only if the output voltage is reduced since then the rate of the current change $\mathrm{d} i_{p} / \mathrm{d} t=-u_{0} / L$ is small as well. It is the mechanism how the reduction of $\alpha$ and the increase of $\omega T$ produce smaller output voltage.

## 7. Comparison

It is rewarding to compare the suggested circuit with the chopper shown in Fig. 28a [2]. The power transfer is carried out by the switched condenser $C$ here as well. Likewise the thyristors are turned-off, owing to series capacitor commutation.

On starting, $T 1$ and $T 4$ are first pulsed to charge $C$ from $u_{C}=-U_{i}$ to $u_{C}=+U_{i}$ via the load. Assuming constant load current, $u_{C}$ increases as a ramp function (Fig. b). Owing to series-capacitor commutation the load current commutates from the thyristors to diode $D$ at angle $\alpha_{e}$ (Fig. c). Thyristor $T 1$, $T 4$ are reversed biased until $C$ has discharged completely. Firing thyristors $T 2$, $T 3$ at $\pi$, the load current commutates from the diode to the thyristors and


Fig. 28. Chopper with trapezoidal current (Fig. a) and its time functions (Figs b. . .e)
the condenser is recharging in the opposite direction until $u_{C}=-U_{i}$. Seriescapacitor commutation turns-off the thyristors again. Diode $D$ conducts the load current from ( $\pi+\alpha_{e}$ ) until $2 \pi$ and the cycle repeats itself.

Assuming $\alpha_{e}=\pi$ at rated output power $P_{0 n}$, the average value of the [output voltage $U_{0}=U_{i}$ and] $P_{0 n}=I_{L n} U_{0}=I_{L n} U_{i} . I_{L n}$ is the rated load current.

The thyristor stresses of the circuits with sinusoidal current (Fig. 5) and with trapezoidal current (Fig. 28a) at rated load is given in Table 1. In both cases $\alpha_{e}=\pi$, and the operation frequency is the same. The clamping thyristors are replaced by diodes in Fig. 5, that is, $u_{c}^{\prime}(\alpha)=1$. The same volt-ampere products for the sum of thyristors results twice as high output power in the case of sinusoidal current than with trapezoidal current. In fact, the comparison is even more favourable for the circuit with sinusoidal current at rated load since the average current handling capability of the fast thyristors is $a>1$ times higher at sinusoidal current than with trapezoidal one at repetition frequency around 5 kHz . The ratio "a" can be as high as $1.5-2$. Further advantages are that the condenser current flows across only one thyristor instead of two, the output voltage is more-less smooth and the current is sinusoidal. It provides better conditions from EMI point of view.

Nevertheless the picture is not as nice as it seems to be from Table 1. The drawbacks of the circuit with sinusoidal current are included in Table 2. $u_{C}^{\prime}(x)=1$ is assumed again. One of the most important drawbacks is that the voltage stress of the thyristors is not constant, it is twice as high at zero output voltage than at rated one. The diode cost is twice and two chokes as well as two output condensers are needed in the circuit with sinusoidal current. The term "circuit with sinusoidal current" for the circuit in Fig. 5 is not accurate any more. The sinusoidal current stops flowing at angle $\alpha$ in the thyristor and the current commutates from the thyristor to the diode. The commutation

Table 1

|  | Sinusoidal current (Fig. 5) | Trapezoidal current <br> (Fig. 28a) |
| :---: | :---: | :---: |
| Turn-off time* | T/4 | T/4 |
| Volt-ampere product per thyristor* | $I_{\text {Ln }} U_{i}$ | $\left(1 / 2 I_{L n}\right) U_{i}$ |
| Volt-ampere product for the sum of thyristors* | $2 I_{L n} U_{i}$ | $4\left(1 / 2 I_{L n}\right) U_{i}$ |
| Rated output power | $I_{L \sim N}\left(2 U_{i}\right)$ | $I_{L n} U_{i}$ |
| Average current handling capability of thyristors | Higher | Lower |
| Number of thyristors simultaneously on |  | 2 |
| Electro Magnetic Interference | Lower | Higher |
| Output voltage ripple | Lower | Higher |

Table 2
Sinusoidal current Trapezoidal current
(Fig. 5)
(Fig. 28a)

| Maximum value of thyristor voltage | $2 U_{i}$ | $U_{i}$ |
| :--- | :---: | :---: |
| Maximum value of volt-ampere product for the sum of | $4 I_{L_{n}} U_{i}$ | $2 I_{L n} U_{i}$ |
| thyristors | $2 L_{L_{n}} U_{i}$ | $2 I_{L L} U_{i}$ |
| Volt-ampere product per diode | $4 I_{L_{n}} U_{i}$ | $2 L_{L-2} U_{i}$ |
| Volt-am?ere product for the sum of diodes | Yes | No |
| It has two chokes and two output condensers | Yes | No |
| Peak current of the thyristor increases with $U_{0}$ |  |  |

is the main source of EMI. The peak current of the thyristor increases with the reduction of the output voltage, that is, that of $\alpha$ (Fig. 23) and it can be twice as high as the peak current at rated load.

The current pulse delivers constant energy. The repetition frequency $f=1 / T$ must be reduced with the reduction of the output power. It increases substantionally the voltage ripple in the output voltage (Figs 13-21). The main source for the output voltage ripple is not the reduction of the load current but rather that of the average output voltage.

## 8. Configuration with subcircuits

The ripple of the output voltage can be better suppressed by dividing the circuit in Fig. 5 into $n$ subcircuits (Fig. 29) where $n=2,3,4 \ldots$ The set-up of each subcircuit is the same as that of the circuit in Fig. 5. $n$ separate d. c. supply voltages are needed. Each subcircuit operates in the same way as the one drawn in Fig. 5.


Fig. 29. Converter with subcircuits

Be assumed $n=4, C_{0} / C_{r}=10, u_{c}^{\prime}(\alpha)=1$ and for the sake of simplicity approximately smooth output voltage.

Table 3 presents the time functions of the condenser current at three output power: $P_{0}^{\prime}=1,1 / 2,1 / 4$. In each case the output voltage $U_{0}^{\prime}=1$ and the number of switched condenser is $n=1,2,4$. The current shapes shown for $n=1$ and $n=2$ can be realized with four switched condenser as well but the potential benefit of the four subcircuits would not be exploited. The ripple in the resultant condenser current:

$$
i_{C}^{\prime}=\sum_{k=1}^{4} i_{C k}^{\prime}
$$

is drasticly reduced with four subcircuits. At the same time $C_{0} / C_{r}$ is increased up to 40 where $C_{r}$ is the capacitance of the switched condenser in one subcircuit. The ripple of the output voltage is considerable reduced.

Similar condenser current curves can be drawn when the output power is changed by the reduction of the output voltage $U_{0}^{\prime}$ rather than that of the load current. The only difference is that the current pulses are truncated, they stop flowing from firing angle $\alpha$ on (Fig. 11). $\alpha$ drops at near $110^{\circ}$ at $U_{0}^{\prime}=0.5$ and at $98^{\circ}$ when the output voltage is $U_{0}^{\prime}=0.25$.

As a numerical example let us take the following rated values for one subcircuit: $P_{0}=25 \mathrm{~kW}, U_{0}=250 \mathrm{~V}, f=5 \mathrm{kHz}$ and $\left(C_{0} / n\right) / C_{r}=10$. The rated load current $I_{L n}=50 \mathrm{~A}, I_{C_{p}}=\pi I_{L n}=157 \mathrm{~A}, C_{r}=20 \mu \mathrm{~F}$ and $n L=50 \mu \mathrm{H}$. Ten subcircuits connected in parallel result 250 kW rated output power. Then the phase shift $\varphi$ among the condenser current pulses at rated output power is $\varphi=\frac{\omega T}{10}=36^{\circ}$. The minimum value of the resultant choke current is with $\left(1-\cos 18^{\circ}\right)=0.0489$, that is, $4.89 \%$ less then its peak value. The ripple in the resultant choke current and in the output voltage is rather low. To acquire some sense of the dynamic response of the system let us calculate the time $\Delta t$ needed to reach the rated output voltage starting from the state $U_{0}=0$ at noload when the output condensers are charged with rated current. The time $\Delta t=250 \mathrm{~V} \times 2 \times 10^{-3} \mathrm{~F} / 500 A=1 \mathrm{~ms}$. It is rather fast.

## Conclusion

The paper describes a high frequency thyristor chopper. Its main attractions are that no forced commutation is needed and its thyristor current is partly sinusoidal.

The mathematical investigation is carried out both in the case of smooth output voltage ( $C_{0}=\infty$ ) and in the case of fluctuating output voltage ( $C_{0} \neq \infty$ ). A number of computer results are presented.

Table 3

| $\begin{gathered} P_{0}^{\prime}=1 \\ U_{0}^{\prime}=1 ; I_{L}^{\prime}=\frac{1}{2} \\ \omega T=2 \pi \end{gathered}$ |  $i_{C}=\sum_{k=1}^{4} i_{C k} ; \quad \frac{C_{0}}{C_{r}}=10$ |  <br> $n=2$ $\begin{gathered} i_{c a}=i_{C 1}+i_{c 2}^{\prime} \\ \frac{c_{0}}{C_{r}}=20 \\ i_{c b}=i_{c 3}+i_{c 4} \end{gathered}$  <br> $n=4$ $\begin{aligned} & \frac{C_{0}}{C_{r}}=40 \\ & \varphi=\frac{\omega T}{C} \end{aligned}$ |
| :---: | :---: | :---: |
| $\begin{gathered} P_{0}=\frac{1}{2} \\ U_{0}=1 ; l_{i}=\frac{1}{4} \\ \omega T=2(2 \pi) \end{gathered}$ |  |   |
|  |  |  |
| $\begin{gathered} P_{0}=\frac{1}{4} \\ U_{0}=1 ; L_{i}=\frac{1}{8} \\ \omega T=4(2 \pi) \end{gathered}$ |  |  |
|  |  | $n=4$ $Q=\frac{\omega T}{L}$ |

The chopper suggested in the paper has been compared with an other chopper known from the literature. The latter one needs no forced commutation as well. The advantages and disadvantages are pointed out.

Finally, the chopper built up from a number of subcircuits is introduced. This configuration helps to suppress the output ripple.

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