PROBLEMS OF FATIGUE CRACK GROWTH MEASUREMENT UNDER RANDOM LOAD

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Abstract

Constant amplitude fatigue tests have been used in practical engineering for many years; they are well developed and satisfactory. However, under random loading a number of factors may have an unexpected and often significant effect on the crack growth response. Hence these factors require a detailed consideration. This paper deals with some basic aspects of the random loading process and points out certain changes occurring within a wide range of stress intensities, such as are observed during fatigue of large structures, for example, pressure vessels, ships, oil platforms and bridges.

Introduction

Examinations of structural failures have revealed that 90% of such failures are due to fatigue fracture. To understand and examine such catastrophic behaviour of fractures, science of linear elastic fracture mechanics (LEFM) has been adopted.

LEFM was originally developed for study of brittle fractures due to monotonic loads before occurrence of general yielding but it has been extended to fatigue fracture analysis. Fatigue fracture occurs due to initiation of cracks at defects or areas of high stress concentration. Such cracks propagate due to load variations experienced in service life.

The most common parameter in LEFM is stress intensity factor (SIF) K which is given by:

$$K=\sigma\sqrt{\pi a}\cdot Y \tag{1}$$

where σ is the applied stress, $2a$ is crack length, Y is geometrical factor.

Ahead of crack tips the material undergoes yielding and so plastic zones are developed. The size of the zone is dependent on more factors. For plain stress Irwin [1] has derived the equation for plastic zone radius, given by

$$r_{\mathrm{pl}}=\frac{1}{2\pi}\left(\frac{K}{R_e}\right)^2 \tag{2}$$

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r_{pl} for plain strain case is $\frac{1}{3}$ of plain stress. Rice has carried out an accurate calculation of r_{pl} and has found the difference to be approximately $\frac{2}{3}$. Further analysis was performed by Czoboly et al. [2].

Fatigue life is dependent on how a crack growth is controlled under different load conditions. The factor which describes the behaviour of material is crack growth rate, which depends on the stress field around the crack, the size and shape of the crack, environment and material itself [3].

Various propagation laws have been suggested. These laws are from growth models based on dimensional analysis or dislocation theories, and cyclic strain and damage accumulation. It is accepted that crack growth is dependent on ΔK.

The Paris equation is the most common law describing growth rate in terms of stress intensity variation, i.e.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\Delta K^n \tag{3}$$

where ΔK is the SIF range, N the number of load cycles, C and n are material parameters.

Much of the data obtained during fatigue tests have been fitted by this equation. The growth rate plot, however, has a sigmoidal shape approximated by three parts. Each part describing a particular growth behaviour. For low SIF (1st stage) growth rate is slow and there is a level of SIF (threshold) below which no growth is expected. The middle range is the most significant level of growth, described by eq. (3). As K is increased the crack propagation accelerates (stage three).

The requirements for the specimen sizes set by LEFM are that the plastic zone should be very small with respect to crack length and that the thickness and uncracked ligament be greater than $2.5\,(K_{Ic}/R_e)^2$ where K_{Ic} is the fracture toughness of the material.

Because of the plastic zone there are residual stresses built up in front of the crack tip which modify the stresses, so the SIF is changed.

Loading in fatigue tests

Fatigue test programmes which have been designed are performed by constant amplitude (CA) loads due to ease of operation and data analysis simplicity. The main factors involved are, maximum load, load range and frequency of signal.

Some of the test programmes which have been carried out have produced very useful conclusions:

(i) Tensile overload causes retardation in crack growth. The delay time is dependent on overload ratio (maximum load of overload/maximum load of *CA*).

(ii) Compressive overload causes acceleration of crack growth mainly due to crack tip sharpening.

(iii) A compressive overload following a tensile overload negates its retardation effect.

(iv) Multiple overload has a bigger effect than single overload but the delay time ensuing does not hold a linear relationship with the numbers of overload.

(v) The hold time at the maximum level of an overload increases the delay time.

(vi) Magnitude of overload increases the threshold stress intensity factor.

(vii) Increase of stress ratio reduces the threshold SIF range.

(viii) A corrosive environment enhances all the secondary effects such as the frequency and mean load (with respect to steel).

Although *CA* loads are vastly used they have no resemblance to service loads. Structures and components behave differently to service loads and growth rates are not the same. To reduce the gap between service loads and *CA* loads, block loading has been adopted. Some investigators have studied particular service loads and calculated the number of occurrences for varying load levels and so produced a load spectrum. These load spectra have been used to generate blocks of constant amplitude loads. This technique was first used by Gassner. In the Netherlands Schijve has carried out many tests on aircraft materials under such loads [4].

To test components for fatigue, the most severe load expected during the life of the component is analysed and an envelope spectrum is formed from which the programme load to test the components is generated.

Nevertheless the load sequence effect is lost under such loadings and to introduce some kind of load interaction effect under laboratory conditions some researchers have randomised the sequence of load occurrences. Comparison of results under such load conditions reveal that still some discrepancies exist between these and service load conditions.

Service load fatigue tests have been possible by recording of these loads and using them as input for testpieces. However, to get all the benefits of this test a very long record is required and to estimate fatigue life a record as long as that during the life of a component is necessary, which is impractical. One of the major problems involved with recording and play-back is the loss of frequencies within the original signal. These problems have led engineers to design equipment to simulate signals resembling service loads.

The most conspicuous characteristics of service loads is that they vary haphazardly with time. This means they are non-deterministic unlike *CA* loads. Although these loads are random in nature it is possible to identify them by statistical parameters if and only if they fall within a special category-stationary process.

It is believed that most phenomena in nature are stationary or weakly stationary. So the parameters necessary to define stationary random processes are, mean value, root mean square (*rms*) value, probability distributions and spectral density function.

Although various simulation techniques have been developed it is possible to categorise these techniques into two groups, namely digital and analogue methods. Their difference lies on choice of parameters involved in random data or processes. Amplitude variation (probability distribution) is the main parameter for digital method, while analogue technique uses the frequency spectrum and therefore it is frequency related. The digital technique examines the load through cycle by cycle method (counting range) with distribution of peaks in mind.

Load interactions

To study sudden changes of growth rate due to load variations, Jacobi et al., have carried out tests which reveal the behaviour of material within the plastic zones. These changes are strongly influenced by absorbed specific fracture energy as shown in [5], [6] for any material.

Plastic zone size is mainly due to maximum load and load range in each cycle. Maximum load causes the material to monotonically strain harden with high residual stresses set up at the crack tip. Within this zone a small zone is found due to stress variation per cycle. The material in this zone is cyclically strain hardened (or softened) depending on material properties and undergoes reversed plastic deformation.

An equilibrium exists between the two zones. The sizes of the monotonically and cyclically strained zones, as well as the level of residual stresses depend not only on the maximum and mean load, but also on the loading sequence. The variation of growth rate can be explained on this basis.

At the crack tip by increasing load the material experiences a large tensile strain and cracks. Therefore, for a very short distance the growth rate is very high. As the crack tip travels through the rest of the zone, it is surrounded by less effect on stress state. The growth rate is dropped until a stable condition is reached for propagation rate.

With reduction of mean load the stress intensity factor variation at the crack tip falls below the level that would have existed if the specimen was initially loaded with the present mean level. Therefore, the growth rate is reduced below this level.

The two plastic zones, due to the reduction of mean load are smaller than previous ones. It takes a longer time for the crack to grow through the first cyclically deformed zone; but growth rate through the plastic zone due to previous load level's maximum load is faster and increases as the crack tip reaches the end of this zone, where it becomes stable and follows the same level of growth rate expected under the same condition, without any changes of load level.

The case of a single overload is examined next.

With the application of a single overload a new plastic zone is created because of maximum load reached during the overload. The residual stresses within this zone are much higher than previously produced. The crack tip is suddenly under a high tensile strain — a temporary increase in growth rate is observed. But the stress intensity factor variation ahead of the crack tip is drastically reduced due to residual deformations. Therefore, the propagation rate is first dropped and then starts to increase as the crack tip reaches material that is less affected by the overload. Thence the growth rate increases until it reaches the old level before the overload.

By examination of these conditions, the importance of load interactions is clarified. It must be understood that in application of service loads the test-piece or component does not experience any load condition similar to constant amplitude loadings. After an overload in tension a totally different compressive overload can follow which generally reduces the effect of the tensile overload, and enhances the growth rate. Subsequently what can be expected during a random load test could be quite unpredictable.

Random processes

Any variable which is a function of one or more independent variables such that no definite relationship can be observed between is random. So, if load-time history is examined and no form of equation describes the variation, the load is random.

Random loads or variables can be defined by statistical parameters. If these parameters are time dependent, then the variable is non-stationary and if constant, the process is stationary. We are mainly concerned with the stationary process. Two inherent properties of random processes are probability distribution and spectral density function. To describe these properties completely three separate factors are necessary, position, scale and shape factor.

Most processes in nature have Gaussian normal distribution. For this distribution the mean and *rms* are the position and scale factors, while the shape factor has an exponential equation and a bell shape look. It is possible to define the spectral density function in the same way but the common approach is by Fourier transform integrals of autocorrelation function $R(\tau)$ where

$$R(\tau)=E(x(t)x(t+\tau)) \tag{4}$$

Spectral density function is a measure of distribution of power within the signal at different frequencies. In fact the total power can be measured by calculating the area underneath of the spectral density function.

In principle signals are generated to have uniform spectrum for all frequencies present. This approach is used to excite all the frequencies of the testpiece with equal power to reveal the dominant natural frequencies of the testpiece. Each system has one or two predominant natural frequencies which is depicted in power spectrum of testpiece.

Power spectrum of such systems are modelled by a narrow band filter. The output of the filter is a Rayleight random load which looks like a sinowave of varying amplitude, and its frequencies are clustered around the centre frequency.

The above parameters described are sufficient to define a stationary Gaussian process. But some forms of modification are necessary to analyse a random load. For a normal distribution the range of variation of signal is from $-\infty$ to $+\infty$. But this is impossible in the case of a simulated signal, as its maximum and minimum are limited. Therefore a new parameter called clipping ratio (crest factor), defined as maximum peak divided by *rms* value of signal, is adopted to show differences between two signals. The distribution of peak levels is as important as the maximum number of peaks occurrence and the extreme level of peak. For a constant spectrum signal it is very difficult to work out the number of peaks. For peak frequency calculations the level crossing frequency approach is used. For a sinusoidal signal the number of peaks equals the number of mean level crossings with positive gradient. For a narrow band signal the same result is observed, it very much resembles the sinusoidal signal. In the case of a broad band signal this assumption is not taken and so to show the effect of a variation of peaks, a new parameter is introduced, irregularity factor, defined as the ratio of frequency of peak divided by frequency of mean level crossing.

The peaks are distributed by Rayleigh distribution, and Rayleigh probability density function is defined as,

$$\rho(x)=\frac{x}{\sigma_x^2}\exp\left(\frac{-x^2}{2\sigma_x^2}\right) \tag{5}$$

where x is the variable and σ_x^2 is the variance of x.

Stress intensity factor and analysis of data

Specimens used for fatigue tests are generally plates with one or two notches. With these starter notches through the thickness, the growth parameter is one dimensional even though the crack grows in a plane [7].

For a part through crack the situation is different due to variation of growth along the crack boundary. As the crack grows perpendicular to its crack front, for a semielliptic crack a two-dimensional analysis is adopted, i.e. growth at surface being different at depth.

Measurement of surface crack growth is carried out by microscope, but depth measurement is much more complicated. The method used is the potential drop technique which is achieved by measurement of voltage drop across the crack opening. Although the method is gaining popularity very rapidly, it still has its own inherent problems. One of the unsolved problems is inaccuracy of measurements close to the surface crack tip. The other significant problem is the requirement of a chart to correct the calculated data.

To analyse the crack growth data (surface and depth) must be calculated. The SIF for a semielliptical crack is much more complicated than through cracks. Irwin in 1960 suggested a formula for SIF of a part through crack

$$K_{\varnothing}=\sigma\frac{\sqrt{\pi\cdot c}}{\sqrt{Q}}f_1 f_2(1-k^2\sin^2\varnothing)^{\frac{1}{4}} \tag{6}$$

σ = applied stress in tension.
Q = shape factor.
f_1 = front surface correction factor (effect of thickness).
f_2 = back surface correction factor (effect of thickness).
k^2 = $1-\left(\frac{a}{c}\right)^2$. Where a is the depth and c the half length of the semielliptic crack.
$\varnothing$ = the central angle of the crack front (see Fig. 1).

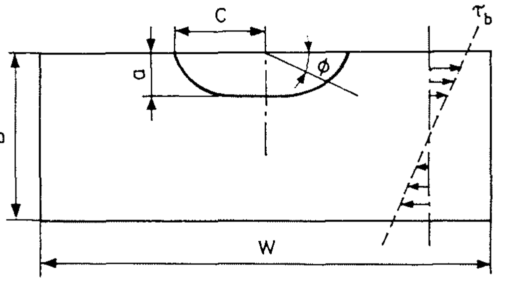

Fig. 1. Half-elliptical crack in a bended specimen. σ_B varies $\perp$ to the plane of the paper

$K_{\emptyset}$ was calculated for a crack in an infinite body, so there are no width corrections included. Therefore it is necessary to modify $K_{\emptyset}$ for a specimen of finite width.

Changes in growth can be observed at different stages of fatigue life. It is observed that for thin bend specimens any initial shape will change to a particular form after some growth. This effect was observed from a graph of shape versus thickness ratio a/t. Nevertheless, no evidence has been found for the thick specimen to follow this behaviour [8], [9].

Crack shape (a/c) change in bend and tension specimens vary. This can be due to stress variation across the crack plane. In bending the a/c ratio varies approximately linearly with thickness ratio a/t while in tension it increases with a/t until it reaches a particular configuration and starts decreasing. However, such effects are expected to change if thickness is increased further. It is necessary to examine the stress situation at the deepest point of the crack. The crack front experiences two different situations when propagating through thickness. This can easily be discerned by examining the Fig. 1.

Assuming that the neutral plane lies half way accross the thickness (this could not be true as the presence of the crack changes the position) and stress variation being linear across the thickness with the upper surface in tension and bottom one under compressive stress. This situation will affect the depth growth rate very severely as the crack front crosses the neutral plane, and the result in thick plates will be more significant.

As previously stated the magnification factors f_1 and f_2 vary in bend and tension testpieces. For bend specimens the f_1 and f_2 vary linearly with thickness ratio a/t, which for tension cases is not so. Magnification factor for depth varies linearly with depth ratio and its gradient is dependent on aspect ratio $\left(\frac{2c}{a}\right)$. However, for surface cracks this parameter is only linear up to thickness ratios of 0.8. Generally two different SIF values are necessary

$$K_c = \frac{\sigma_B\sqrt{\pi c}}{\sqrt{Q}} f_{1c} f_{2c} f_{WB}$$
$$K_a = \frac{\sigma_B\sqrt{\pi c}}{\sqrt{Q}} f_{1a} f_{2a} f_{WB} \tag{7}$$

f_{WB} is the width correction factor for bend specimens.

Fatigue crack growth data have been correlated by the Paris law of crack growth which is a function of stress intensity factor range. The Paris equation may be modified to include such effects as mean load, maximum load, etc.

$$\frac{da}{dN} = C\Delta K^m K^n_{max} \tag{8}$$

Forman's equation includes the mean effect by use of stress ratio R with K_{IC} the fracture toughness, and is given by

$$\frac{da}{dN} = \frac{C\Delta K^m}{(1-R)K_{IC} - \Delta K} \tag{9}$$

This equation is a good fit for intermediate growth rate but for SIF range close to the threshold does not prove to be useful. The effect of maximum stress can be included by use of

$$R = \frac{K_{min}}{K_{max}} \tag{10}$$

therefore

$$\Delta K = (1-R)K_{max} \tag{11}$$

and

$$\frac{da}{dN} = \frac{C\Delta K^{m-1} K_{max}}{K_{Ic} - K_{max}} \tag{12}$$

However, mean load effect cannot be important for steel alloys as mentioned by some investigators.

To adapt the above formulae to fit growth rate data obtained under random loading, they have to be modified. The stress intensity range can be replaced by *rms* value of SIF, K_{rms} and K_{max} can be defined in terms of K_{mean} and K_{rms}.

$$K_{max} = K_{mean} + K_{rms}$$

therefore

$$\frac{da}{dN_{random}} = CK^m_{rms} K^n_{max}$$

or using the above equation in the modified Forman operation

$$\frac{da}{dN} = CK^m_{rms}\left(\frac{K_{mean} + K_{rms}}{K_{IC} - (K_{mean} + K_{rms})}\right)$$

or

$$\frac{da}{dN} = CK^{m-1}_{rms} \quad \frac{\lambda}{\dfrac{K_{IC}^{-\lambda} - \lambda}{K_{rms}}}$$

$$\lambda = 1 + Q$$

$$Q = \frac{K_{\text{mean}}}{K_{rms}} = \frac{P_{\text{mean}}}{P_{rms}},$$

where P is the load.

If $\dfrac{\dfrac{\lambda}{K_{IC}^{-\lambda} - \lambda}}{K_{rms}}$ is kept constant, then the equation given above becomes a modified Paris formula applicable to random loading.

The parameters which will be monitored during random fatigue tests, are, the mean and *rms* values of load, the power spectra of input load and displacement of specimen, and the surface and depth crack lengths.

The effect of *rms* at a given mean and vice versa is going to be examined. The growth rate of the material at low *rms* will probably be part of the program and the variation of crack shape is to be examined.

Conclusions

1) Part-through cracks are of great importance in real engineering structures.

2) The crack shape is highly sensitive to load-alterations, such as occur during random loading.

3) Fatigue tests with constant stress or strain amplitudes are very seldom equivalent to real loading conditions.

4) The shape and size of the plastic zone ahead of the crack as well as the residual stresses within the zone are the main influencing factors of crack propagation. These quantities are dependent on the load sequence, too. A statistical approach has been recommended for a better approximation, however, further study is necessary on the subject.

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