A SYSTEMATIC SURVEY OF THE MAINTENANCE MODELS

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Abstract

A systematic survey of the maintenance models has been carried out, which may help the maintenance engineer to have a total picture on maintenance policies. Maintenance policies discussed in the literature have been studied, inter-related and classified into three groups viz. (I) Replacement/repair at failure, (II) Planned maintenance (repair/replacement) at pre-determined intervals based on number of failures or otherwise and (III) Condition based maintenance. The above number of groups are necessary and sufficient for the purpose of methodical surveying of the publications on the topic.

1. Introduction

Poor maintenance may obviate the benefits of superior design and production technology. Demands for higher reliability, longer life, and shorter periods of downtime of the equipment are factors which work to increase the significance of the maintenance problem.

In many situations, failure of a unit during actual operation is costly and/or dangerous. If the unit is characterized by an increasing failure rate, it is advisable to replace it before it has aged too greatly. On the other hand too frequent replacement means excessive/unnecessary cost. Thus, one of the most important questions of maintenance policy is the need to balance the cost of failures against the cost of planned replacements. During the last few decades there has been a growing interest in maintenance policies for systems that are subject to stochastic failure. Several maintenance policies are discussed in the literature. A systematic study of the maintenance policies is necessary, which may help the maintenance engineer to have a total picture on maintenance policies. This in fact will enable the maintenance engineer to choose or develop an optimum maintenance policy for a given system.

A good survey on this subject is given in Barlow—Proschan [6], McCall John [30], Pierskalla and Voelker [47] and Sherif [50]. Barlow—Proschan and McCall John in their works have surveyed the area of maintainability up to 1965. Pierskalla and Voelker in their paper have tried to survey the results that were published mainly between 1965 and 1974 on maintenance models. Sherif in his valuable work has given a review of the literature related to optimal inspection and maintenance schedules of failing systems along with a bibliography of 818 publications. A systematic study of maintenance policies is necessary, which may help the maintenance engineer to have a comprehensive picture on maintenance policies. Recently Petrik, Chowdhury and Farkas [45, 46] have also studied the question. Classification given here is similar to the one discussed in Chowdhury and Farkas [12]. In this paper objective functions of the important maintenance policies are also given.

2. Classification

Maintenance policies discussed in the literature have been studied, inter-related and classified into three groups as follows:

(I) Replacement/repair at failure;

(II) Planned maintenance (repair/replacment) at predetermined intervals based on number of failures or otherwise;

(III) Condition based maintenance.

This classification gives the best way to analyse the very wide and rich literature on the subject. The above number of groups are necessary and sufficient for the purpose of systematic surveying of the publications on the topic.

2.1. Replacement/repair at failure policy

The model underlying replacement/repair at failure policy can be described by a simple renewal process. It is to be noted that if the failure leads to catastrophe then this policy should not be used in any case. It is found that the optimal maintenance policy for systems that are not having increasing failure rate (IFR) is to maintain at failure. Under this policy action is taken only after failure has actually occured. We may express the cost for a replacement/repair at failure (RRAF) during the intervals between two successive regeneration points as follows:

$$C_{RRAF} = C_R + C_d$$

where C_{RRAF} is the cost of repair or replacement at failure, C_R is the active repair cost and C_d is the downtime cost. The repair cost has two main factors. One is the cost of the materials, spare parts, payment for the maintenance crew etc. The other factor is the cost of diagnosis of the failure. In case of increasing failure rate, replacement/repair at failure policy is optimum if the following condition is satisfied:

$$C_{RRAF} \leqq C_{PM}$$

i.e.,

$$C_R + C_d \leqq C_{PM}$$

where C_{PM} is the cost incurred due to planned maintenance policy.

2.2. Planned maintenance (repair/replacement) at predetermined intervals based on number of failures or otherwise

2.2.1. *Block replacement policy.* The basic block replacement policy is presented in Barlow and Proschan [64]. Under this policy all components of a given type are replaced simultaneously at times KT ($K=1, 2, 3, \ldots$) independent of the failure history of the system. It is also assumed that failed components are replaced at failure. The main advantage of this policy lies in its simplicity because no recording of times of failure and ages of items is required. Block replacement policies have also been investigated by Savage [48], Welker [55], Drenick [18], Flehinger [20] and Barlow, Hunger, Proschan, Rosenblatt, Weiss and Wolman [5]. In Barlow and Proschan [6] optimum block replacement interval that minimizes the expected cost per unit of time is obtained by using the following equation:

$$T_0 m(T_0) - M(T_0) = \frac{C_2}{C_1} \tag{1}$$

where T_0 = optimum block time
$M(T)$ = renewal function
$m(T)$ = renewal density
C_1 = expected cost of failure
C_2 = expected cost for exchanging non-failed item

The main drawback of the block replacement policy is that at planned replacement times we might replace practically new items. To overcome this undesirable feature, various modifications have been advocatad. In the first modified block replacement model (Barlow and Hunter [8]), a failed unit is no longer replaced but is instead given a minimal repair. By minimal repair, we mean that the repair, needed to put the failed item back into operation, has no effect on its remaining life time. This repair action is mathematically equivalent to replacing the failed item by another working item of the same age. This modified block replacement policy is also known as periodic replacement with minimal repair at failure. Such a policy might apply to a complex system such as a computer, airplane, etc. Replacement or overhaul occurs at times T, $2T$, $3T$, ... etc. In case of periodic replacement with minimal repair at failure policy, the task is to select optimum interval T so as to minimize

$$C(T) = \lim_{t\to\infty} \frac{c_1 EN_1(t) + c_2 EN_2(t)}{t} \tag{2}$$

where c_1 = cost of minimal repair,
c_2 = cost of replacement, $[0, t]$,
$N_1(t)$ = number of replacement in $[0, t]$

In Tilquin and Cléroux [54] a periodic replacement with minimal repair at failure is considered, in which the standard cost structure (Barlow and Hunter [4]) is modified by the introduction of a term which takes adjustment costs into account. Here also a unit is replaced at times T, $2T$, $3T$, ... Between these periodic replacements it may happen to fail and it is immediately repaired in such a way that its failure rate is not disturbed. It costs c_1 to repair a failed unit and c_2 to replace it at age T. A cost $c_3(ik)$ is suffered at age ik, $i=1, 2, 3, \ldots$; $k>0$. All the costs are assumed to be greater than or equal to zero. The cost structure used here was first introduced in Cléroux and Hanscom [13]. Nakagawa [36] summarizes four models of modified periodic replacement with minimal repair at failures when the scheduled replacement time is specified. If a failure occurs just before the replacement time, then the three models are: (1) a unit remains as it is until the replacement time comes, (2) a unit is replaced by a spare (as often as necessary) until the replacement time comes. Here spares are not identical to the unit but they have the same functions as the failed unit and failed spares are scrapped without repairing, (3) a unit is replaced by a new unit. Here a new unit is statistically identical to the failed unit. In the fourth model a unit is replaced at failure or at time T_1, whichever occurs first, after it has reached the age $T(T_1 \geqq T)$.

In a block replacement model considered by Cox [16], an item which fails close to the time of the scheduled block replacement is not replaced and remains idle until block replacement occurs. A penalty, assumed to be a linear function of idle time, is taken into account. Crookes [17] in one of his models follows similar lines. In his model a unit which fails at any time within the interval is not replaced until the next block replacement. Both of these articles contain a mathematical error which has been corrected by Blanning [11]. Woodman [57] suggested the use of dynamic programming to find the optimal policy for the preceding two models. In a modified block replacement model with two variables considered by Nakagawa [39], failed units are replaced by a new unit during (O, T_0), and after T_0, if a failure occurs in an interval (T_0, T), then the replacement is not made in this interval and the unit remains failed until the scheduled time T. Using the results of renewal theory (see Barlow and Proschan [6]), the mean cost rate of the model is derived and the optimum T_0^* and T^* to minimize the cost rate are obtained. This model is similar to Cox [16] and Crookes [17]. In the model it is assumed that the replacement times are negligible. The mean cost rate is represented with the help of the following equation:

$$C(T_0, T) = \left[c_1 N(T_0) + c_2 + c_3 \int_0^{T-T_0} L(T_0, t)\, dt\right] / T \tag{3}$$

where,

c_1, c_2 — cost of replacement at failure and of scheduled replacement $(c_1 > c_2)$

c_3 — cost rate for time elapsed between failure and its replacement

$$N(t) = \sum_{k=1}^{\infty} F^{(k)}(t)$$

$F^{(k)}(t)$ — k-fold convolution of F with itself; $F^{(k)}(t) = \int_0^t F^{(k-1)}(t-u)\, dF(u)$, $(k=2, 3, \ldots)$, and $F^{(1)} = F$

$L(x, t)$ — distribution of $\gamma(x)$

$\gamma(x)$ — random variable denoting the remaining life of the unit at time x in a renewal process

T_0, T_0^* — replacement-for-failure interval and its optimum solution; T_0 is a time point until the unit is replaced at failure

T, T^* — scheduled replacement-time instant of the unit after T_0 and its optimum solution; $T \geqq T_0$.

Under block replacement policy, items are replaced at regular intervals of time and on failure. The replacement is made by new items. This policy is wasteful since items which are almost new are also replaced at the scheduled time of replacement. To overcome this Bhat [10] has suggested a model of used item replacement policy. In his model failed item is replaced by used item, which has been removed earlier after attaining the age T. Here T is the interval between two consecutive planned replacements. In the model it is assumed that (*a*) used item will cost much less than the new item, even if we include expenses of minimal repair, (*b*) there is an unlimited supply of used items and no item is used more than twice, (*c*) life-time distribution is continuous, and (*d*) probability density function is differentiable a number of times. The total cost of replacement during $(0, t)$ is

$$C_1 N_u^*(t) + c_2 N_u(t)$$

where c_1 = cost of replacement at failure,

$c_2 (< c_1)$ = cost of removal,

$N_u(t)$ = number of removals during $(0, t) \cdot N_u(t)$ also represents the number of replacements by new items. Evidently $N_u(t) = [t/T]$,

$N_u^*(t)$ = number of failures during $(0, t)$.

The asymptotic cost per unit time of used item replacement policy is equal to

$$UI(T) = (c_1 H_u(T) + c_2)/T \tag{4}$$

where $H_u(T)$=expected number of renewals for a modified renewal process (as defined in Cox [16]).

If $H_e(t)$ is the renewal function of a modified renewal process with initial distribution $F(x)$ and the remaining distribution is negative exponential with mean λ^{-1}, then the upper bound for unit cost $UI(T)$ is given by

$$U(T) = [c_1 H_e(T)+c_2]/T \tag{5}$$

$$\left(H_u(t) = F(t)+\int_0^t H_T(t-x)\,dF(x) \leqq F(t)+\int_0^t (t-x)\lambda\,dF(x) = H_e(t)\right).$$

The minimum value of $U(T)$ is:

$$U(T_0) = c_1 h_e(T_0) \tag{6}$$

where $h_e(T)$ = derivative of $H_e(T)$

T_0 = for given c_1 and c_2 the value T that minimizes $U(T)$.

The block replacement policy with used items introduced by Bhat [10] is further extended by Tango [52, 53] and Murthy and Nguyen [35]. Bhat's model is still costly since under this policy we have to replace the failed items by used items even when there remains much time until the next planned replacement. That is why Tango [52] considers a more flexible policy where either new or used items can be utilized:

(i) Items are replaced by new ones at times KT, $K=1, 2, 3, \ldots$

(ii) It items fail in $T(K-1)T, (KT-v)$, they are replaced by new items and, if they fail in $[KT-v, KT)$, they are replaced by used items of age T, where $0 \leqq v \leqq T$. The former region is called as now item region and the latter used item region.

The expected cost during $[0, t)$ is expressed as

$$C(t) = c_1 EN_1(t)+c_1' EN_2(t)+c_2 EN_3(t) \tag{7}$$

where c_1 = replacement cost for replacing a failed item by a new item,
$c_1'(<c_1)$ = replacement cost for replacing a failed item by a used item,
$c_2(<c_1)$ = replacement cost for replacing a nonfailed item by a new item,
$N_1(t)$ = number of failures in the new item region during $[0, t)$
$N_2(t)$ = number of failures in the used item region during $[0, t)$,
$N_3(t)$ = number of planned replacements by new items during $[0, t)$

In the model optimum value of v and T is determined.

Tango's [52] policy creates used items of age varying randomly from v to T. However, it uses only used items of age T and discards used items of age less than T. This policy is not rational because it discards superior used items of age smaller than T and uses only inferior used items of age T. Murthy and Nguyen [35] further extends the above block replacement policy of Tango [52] to the case where failed items in

$[KT-v, KT)$, $K=1, 2, 3, \ldots$ are replaced by used items with age varying from v to T, as opposed to replacement by used items of age T only. In their model similar assumptions are used as in Tango's model, i.e. (a) there is an unlimited supply of used items (with failure distribution $\hat{F}(t/v, T)$ as opposed to $F_T(t)$ in Tango's model), (b) no used item is used more than once, (c) used items cost less than new items, and (d) the lifetime distribution of new items is continuous, and differentiable and has an increasing failure rate. Let $F(t)$ be the failure-time distribution of new items. The failure-time distribution of used items is a function of $F(t)$, v and T. Let $\hat{F}(t/v, T)$ be the failure-time distribution of used items, and let $T_1=T-v$. A used item of age $(v+x)$ is created in a cycle if it starts operating at time T_1-x $(0<x<T_1)$ and survives until time T. $\hat{F}(t/v, T)$ can be expressed as follows

$$\hat{F}(t/v,T)=\frac{F(T+t)-F(T)+\int_0^{T_1}\{F(v+x+t)-F(v+x)\}\,dM(T_1-x)}{1-F(t)+\int_0^{T_1}\{1-F(v+x)\}\,dM(T_1-x)} \tag{8}$$

where $M(t)$=renewal function for the renewal process with failure distribution $F(t)$.

The expected cost per unit time for an infinite time span is given by

$$C(T, v) = \{c_1 M(T-v)+c_1' \hat{M}_D(v/v, T)+c_2\}/T \tag{9}$$

where $\hat{M}_D(v/v, T)$=renewal function for the modified renewal process, and c_1, c_1', c_2 carry the same meaning as in Tango [52].

The optimal policy is given by T^* and v^* which minimizes $C(T, v)$. T^* and v^* can be obtained by solving

$$\frac{\partial C(T, v)}{\partial T}=0 \quad \text{and} \quad \frac{\partial C(T, v)}{\partial v}=0$$

It is very difficult to obtain T^* and v^* analytically.

The difficulty with the block replacement policy with used items is that the renewal function of the used item generally becomes a very complicated function of age T (planned replacement interval), which often makes it difficult to obtain the optimum solution of block replacement policy with used items even by computer. To overcome this undesirable situation, Tango [53] has modified his earlier model Tango [52], as follows:

(i) Operating items are replaced by items A at time KT $(K=1, 2, 3, \ldots)$.

(ii) If operating items fail in $[(K-1)T, KT-v)$, they are replaced by items A, and if in $[KT-v, KT)$, they are replaced by items B, $0\leqq v\leqq T$.

In this modified model, items B should be cheaper and thus less durable than items A. The s-expected cost rate under this policy is given by

$$C(T, v) = B(T)+(1/T) \int_0^v L(T, x)\,dx, \quad \text{for} \quad 0 \leqq v \leqq T \tag{10}$$

where $B(T)$ = s-expected unit cost under block replacement policy
$=[C_A M_A(T)+C_p]/T$,
T = planned replacement interval,
v = replacement-by-item-B interval,
$L(T, v) = m_A(T-v)(C_B Q(v)-C_A)$,
m_A = renewal density of item A,
M_A = renewal function of item A,
C_A = cost of a failure replacement by item A,
C_B = cost of a failure replacement by item B,
C_p = cost of a planned replacement by item A,
$Q(t)=1+M_B(t)-M_{AB}(t)$,
M_{AB} = renewal function for a delayed renewal process with cumulative density function $F_A(t)$ and remaining $F_B(t)$,
M_B = renewal function of the item B.

In a modified block replacement model considered by Berg and Epstein [7], failed items are still instantaneously replaced after failure, but items possessing age b or less at scheduled block replacement points T, $2T$, $3T$, ..., are not replaced by new items but are instead permitted to be retained in service; b is a number between 0 and T. Thus at the points T, $2T$, $3T$, ..., some of the items will have age zero (following the age replacement) and some of the items will have age x, $0<x\leqq b$. Here the time points T, $2T$, $3T$, ..., are no longer regeneration points as in the ordinary block replacement policy. This makes, the mathematical treatment much more complicated and Berg and Epstein [7] develops new techniques to get the optimum solution. If the probability density function is an Erlang distribution with m stages, i.e.,

$$F(t) = \lambda e^{-\lambda t} \frac{(\lambda t)^{m-1}}{(m-1)!} \quad t \geqq 0$$

then the expected cost of following a modified block replacement policy (introduced by Berg and Epstein [7]) can be expressed as follows:

$$C(b, T) = \frac{c_1}{4T}\left[2T - \frac{(1-e^{-2T})e^{-b}(1+b)}{1-e^{-2T}(1-e^{b}(1-b))}\right] + \\ + \frac{c_2}{T} e^{-b}\left[1+\frac{b}{2}+\frac{b(1+b)e^{-2T+b}}{2-2e^{-2T}(1-e^{b}(1-b))}\right] \tag{11}$$

where c_1 = cost of making an unscheduled replacement of a failed item,

c_2 = cost per item of scheduled (block) replacement,

T = length of the interval between replacements.

In principle it is possible to find the values of b^*, T^* which minimize $C(b, T)$ by computing $\partial C(b, T)/\partial b$ and $\partial C(b, T)/\partial T$, setting them equal to zero and solving the two equations for b^*, T^*. Of course this is not a practical way of finding the optimal values of b^*, T^* and it is simpler to use a computer routine.

Models described in Woodman [57], Bhat [10], Berg and Epstein [7], Tango [52], Murthy and Nguyen [35] etc., have often been limited to specified lifetime distributions. Alley and Lin [3] have worked out a general block replacement policy in which general lifetime distributions are accomodated. Here also previously used components which have been replaced at block replacement time are retained for possible future replacement of failing items. The model is as follows:

(i) Items are replaced instantaneously upon failure.

(ii) At scheduled block replacement times KT, $(K=1, 2, \ldots)$ only those items whose ages exceed a specified limit L are replaced.

(iii) Items failing in the interval $[(K-1)T, KT-v]$ are replaced by new items; those failing in $[KT-v, KT]$ are replaced by used items. The former interval is labelled the "new item region" and the latter interval is the "used item region".

T, L and v are decision variables for obtaining minimumcost maintenance under the generalized block replacement policy. It is assumed $0 \leqq v \leqq L \leqq T$. The generalized block replacement policy assures that at time points $T, 2T, 3T, \ldots$, some items will have zero age (following scheduled replacement) and the remaining items will have age x $[v+1, v+2, \ldots, L]$. The generalized block replacement policy introduced by Alley and Lin [3] assumes that (*a*) failures are immediately detected and replaced, and (*b*) no item is installed more than twice. The total expected system maintenance cost of following a generalized block replacement policy is:

$$C(t) = N \sum_{i=1}^{3} c_i E[N_i(t)]$$

where c_1 = unit cost for new item replacement,

c_2 = unit cost for used item failure replacement,

c_3 = unit cost for scheduled block replacement,

$N_1(t)$ = number of failures per component in the new item region (system is composed of N items),

$N_2(t)$ = number of failures per component in the used item region,

$N_3(t)$ = number of items whose age exceeds L at scheduled block replacement times KT.

The objective of the model is to minimize the total expected unit cost over an infinite time span, i.e., to minimize the following equation:

Generalized block replacement policy

$$(T, L, v) = \lim_{t\to\infty} \frac{C(t)}{t}. \quad (12)$$

Beichelt [9] considers a system having two failure modes: (1) mode-1: failures are removed by minimal repair, and (2) mode-2: failures are removed by replacement. Beichelt's model assumes that (a) all failure events are s-independent and completely self-announcing (including mode), (b) in case of preventive maintenance and mode-2 failures the system is renewed, (c) all maintenance actions take only negligible times, and (d) $0 \leqq c_1 < c_3 < c_2$ (where c_1, c_2, c_3 are costs of a minimal repair, renewal after mode-2 failure, renewal by preventive maintenance respectively). In this model maintenance is carried out according to the mode of failure. At the moments T, $2T$ $3T$, ... preventive maintenance is performed. The long run loss rate is:

$$C(T) = (c_1 M(T) + c_2 N(T) + c_3)/T \quad (13)$$

where $M(T)$ = s-expected number of mode-1 failures in $(0, t)$,
$N(T)$ = s-expected number of mode-2 failures in $(0, t)$,
T = fixed preventive maintenance interval.

If $m(t)$ and $n(t)$ exist (where $m(t) = dM(t)/dt$ and $n(t) = dN(t)/dt$), then a maintenance interval $T = T_0$ which minimizes $C(T)$ satisfies the following equation:

$$\int_0^t [c_1(m(T) - m(t)) + c_2(n(T) - n(t))]\, dt = c_3. \quad (14)$$

This policy with $p(x) = p$, $0 < p \leqq 1$ is formally equivalent to basic block replacement policy (Barlow and Proschan [6]), if a modified cost of renewal on mode-2 failures $c_2(p) = (\bar{p}c_1 + pc_2)/p$ is introduced (where $p(x)$ and $\bar{p}(x)$ are the fraction of mode-2 or mode-1 failures at age x, $\bar{p}(x) = 1 - p(x)$).

2.2.2. *Age Replacement Policy.* Under an age replacement policy, we replace at failure or at the end of a specified time interval, whichever occurs first (cf. Barlow and Hunter [4] or Barlow and Proschan [6]). Age replacement makes sense when a failure replacement costs more than a planned replacement and the failure rate is strictly increasing. In the above mentioned model it is shown that if F, the distribution of time-to-failure, has a strictly increasing failure rate then there exists a unique t_0^* such that expected cost per unit time is minimized if the unit is replaced at age t_0^* or at failure, whichever occurs first. In case of age replacement policy the expected cost per unit of time for an infinite time span (Barlow and Proschan [6]) is as follows:

$$C(t_0) = [c_1 F(t_0) + c_2 \bar{F}(t_0)] \Big/ \int_0^{t_0} \bar{F}(t)\, dt \quad (15)$$

where $\bar{F}=1-F$. If $t_0=\infty$ then the policy corresponds to replacement at failure only and the expected cost is $C(\infty)=\lambda c_1$. If $t_0\to 0$ then $\lim_{t_0\to\infty} C(t_0)=\infty$. Let us assume that there exists a density $f(t)$ of the failure time distribution $F(t)$ and let $\omega(t)=$ $=f(t)/\bar{F}(t)$ be the failure rate. Then, differenciating (15) with respect to t_0 and setting it equal to zero, we get:

$$\omega(t_0)\int_0^{t_0}\bar{F}(t)\,dt-F(t_0)=c_2/(c_1-c_2). \tag{16}$$

It is shown in Osaki and Nakagawa [43] that if the failure rate $\omega(t)$ is continuous, monotonically increasing and $\omega(\infty)>K$, then there exists a finite and unique t_0^* which satisfies (16), and it minimizes the expected cost $C(t_0)$, where $K=\lambda c_1/(c_1-c_2)$. Here it is assumed that the failure time of each unit is independent and has an identical distribution $F(t)$ with finite mean $1/\lambda$.

Nakagawa [40] derives five formulae which give upper or lower bounds of an optimum time t_0^* (t_0^* satisfies eq. 16) when the failure rate is increasing: (1) If t_1 satisfies the following equation:

$$\omega(t_0)=\lambda c_1/(c_1-c_2), \tag{17}$$

then t_1 exists uniquely and $t_0^*<t_1$. (2) If t_2 satisfies the following equation:

$$\bar{F}(t_0)/\int_{t_0}^{\infty}\bar{F}(t)\,dt=\lambda c_1/(c_1-c_2), \tag{18}$$

then t_2 exists uniquely and $t_0^*>t_2$. (3) If t_3 satisfies the following equation:

$$t_0 f(t_0)-F(t_0)=c_2/(c_1-c_2), \tag{19}$$

then $t_0^*\cdot<t_3$. It is supposed that, if no solution exists then $t_3=\infty$, and if many solutions exist then t_3 is the smallest one. (4) If t_4 satisfies the following equation:

$$t_0\omega(t_0)-\int_0^{t_0}\omega(t)\,dt=c_2/(c_1-c_2), \tag{20}$$

then t_4 exists uniquely and $t_0^*>t_4$. (5) If t_5 satisfies the following equation:

$$F(t_0)[(1/\lambda)\,\omega(t_0)-1]=c_2/(c_1-c_2), \tag{21}$$

then t_5 exists uniquely and $t_0^*<t_5$.

Fox [21] considers an age replacement with discounting. Here it is assumed that the cost of a planned (failure) replacement is $c_1(c_2)$, where $0<c_1<c_2$. Continuous discounting is used, with the loss incurred at the time of replacement and the total loss equal to the sum of the discounted losses incurred on the individual stages. (A stage is the period starting just after a replacement and ending just after the next replacement.) Let us suppose that a stage starts at time t and we set a replacement

interval a where throughout the sequel we assume $a \in [0, \infty]$. If replacement actually occurs at $t+x$, then the loss incurred on that stage is

$$L(a, x, t) = \begin{cases} c_1 e^{-\alpha(t+a)}, & \text{if} \quad x = a, \\ c_2 e^{-\alpha(t+x)}, & \text{if} \quad x < a, \end{cases} \tag{22}$$

where α is a positive discount rate.

Since it is assumed that the planning horizon is infinite, the (total) risk at each stage is the same, except for a discount factor. Assuming that the failure distribution is continuous, there exists a replacement interval that minimizes the risk.

Fox [22] works out an adaptive age replacement policy. Loss structure considered here is the same as the one introduced in Fox [21]. In the model it is assumed that the failure distribution belongs to the family

$$F_\lambda(y) = \begin{cases} 1 - e^{-\lambda y^k}, & y \geqq 0 \\ 0, & \text{elsewhere} \end{cases} \tag{23}$$

$$k > 1 \quad \text{and known.}$$

For fixed λ, we have a Weibull distribution with known shape parameter and strictly increasing failure rate $\lambda k y^{k-1}$. It is assumed that λ has a fixed (but unknown) value λ_0. It is further assumed that there is at hand a prior distribution G with specified parameters which is modified after each stage according to Bayes's rule. If G has density

$$g(\lambda; b, c) = \begin{cases} b^c \lambda^{c-1} e^{-b\lambda}; \ \Gamma(c), & \lambda \geqq 0 \\ 0, & \text{elsewhere} \end{cases} \tag{24}$$

the posterior density in case of planned replacement at a (failure replacement at x) is again a gamma density $g(\lambda; b+a^k, c)[g(\lambda; b+x^k, c+1)]$. Thus there is a natural conjugate prior distribution. Barlow and Proschan [6] along with several others have treated the case where the failure distribution is known and the criterion is expected cost per unit time. In that case, the optimal replacement interval to set is found as an elementary application of renewal theory. With unknown failure distribution, if the loss were (literally) undiscounted cost per unit time, the problem of finding a suitable adaptive policy effectively reduces to the preceding case, since one could ignore the loss in any finite transient period while one learned about the failure distribution. With discounting, there is a tradeoff between minimizing expected loss with respect to one's current prior distribution for λ as if future information obtained about the failure distribution were to be ignored, and acquiring maximal information about the failure distribution so as to minimize future losses.

Glasser [24] adapts the general solution of age replacement policy to three special cases: when the probability distribution of times-to-failure can be assumed to have the form of (1) a truncated normal distribution, (2) a Gamma distribution, and

(3) a Weibull distribution. For each model a graph is presented in Glasser [1967] that enables one to read off, aproximately, the optimal solution to the problem.

Scheaffer [49] extends the standard age replacement model by including an age-dependent cost. Age-dependent cost functions frequently occur in practice. For example, the cost of keeping a unit in service may increase with age because adjustments must be made on the unit from time to time, because the action of the unit may slow down as the unit ages, or simply because replacement costs may increase due to depreciation or wear. In the model unit life is assumed to be a random variable, denoted by X, with distribution function F, which is continuous and such that $F(0)=0$. Let the length of time unit i operates in the system be denoted by Z_i and $Z_i = \min(X_i, T)$, where T is the fixed age at which a replacement is to be made. A unit is replaced by another having the same life distribution so that $\{Z_i;\ i=1, 2, \ldots\}$ forms a renewal process. Let

$$S_k = \sum_{i=1}^{k} Z_i,$$

$N(t)$ = the maximum integer k such that $S_k \leqq t$,
$N_1(t)$ = the number of units that have failed up to time t,
$N_2(t)$ = the number of units replaced before failure up to time t,
c_1 = the cost of replacing a failed unit,
c_2 = the cost of replacing a unit which has not failed, $\infty > c_1 > c_2 > 0$.

A cost factor which increases with the age of the unit is added by introducing a factor proportional to Z^α; $\alpha > 0$. Then the total cost up to time t is given by

$$C(t) = c_1 N_1(t) + c_2 N_2(t) + c_3 \Big[\sum_{i=1}^{N(t)} Z_i^\alpha + (t - S_{N(t)})^\alpha\Big] \tag{25}$$

where c_3 is a constant of proportionality ($c_3 > 0$). The expected cost per unit time over an infinite time interval will be used as the criterion for evaluating replacement policies. That is, the optimum value of T will be the one which minimizes

$$\lim_{t\to\infty} \frac{E[C(t)]}{t}$$

Scheaffer gives examples of the optimal solution when the unit life distribution is exponential.

Cleroux and Hanscom [14] considered a very similar model. In the model the age replacement policy which minimizes the average expected cost per unit time over an infinite time span is obtained in the case where the cost structure involves a term which takes into account adjustment costs, depreciation costs or interest charges which are suffered at fixed intervals of time of equal length. The optimal policy is shown to be nonrandom and sufficient conditions are given for it to be finite. Let unit life X be a random variable with continuous and increasing failure rate distribution function F with finite mean μ and such that $F(0)=0$. The unit is replaced at failure or at age T, whichever occurs first. A cost c_1 is suffered for each failed unit which is replaced, a cost $c_2<c_1$ is suffered for each non-failed unit which is exchanged, and a cost $c_3(ik)\geqq 0$ is suffered at age ik, $i=1, 2, 3, \ldots$ where $c_3(0)=0$. The costs c_1, c_2, $c_3(ik)$, $i=1, 2, 3, \ldots$ are known costs and the sequence $\{c_3(ik)\}$ which can be called the sequence of costs of keeping the unit operating need not be an increasing sequence. In the model it is assumed that the replacements and the adjustments are made instantly and that the adjustments do not change the life characteristics of the unit. Let

$N_1(t)$ = the number of failures in $[0, t]$,
$N_2(t)$ = the number of changes of nonfailed units during $[0, t]$,
$N(t)$ = $N_1(t)+N_2(t)$,
Z_i = $\min(X_i, T)$ = the length of time the ith unit operates in the system;
T = the fixed age at which a replacement is to be made.

The total cost suffered during $[0, t]$ is

$$C(t) = c_1 N_1(t)+c_2 N_2(t)+\sum_{i=1}^{N(t)} W_i$$

where W_i is the associated adjustment cost (here adjustment costs mean all the costs that are suffered at time $k, 2k, 3k, \ldots$) during renewal process Z_i (since a unit is replaced by an identical unit, $\{Z_i;\ i=1, 2, \ldots\}$ forms a renewal process). The problem is to find T that minimizes the following:

$$A(T) = \lim_{t\to\infty} \{E[C(t)]/t\}.$$

The concept of an age-dependent cost structure is further generalized by Wolff and Subrahmanian [56].

Ingram and Scheaffer [25] consider the problem of estimating optimum age replacement interval T_0 by a statistic T_n obtained from minimizing a consistent estimator, $C_n(T)$, of $C(T)$, where $C(T)$ is the limiting expected cost per unit time. The method worked out by Ingram and Scheaffer depends on the availability of a uniformly and strongly consistent estimator of the underlying distribution function. Such estimators considered are the maximum likelihood estimators and the minimum variance unbiased estimators of the Weibull and Gamma distribution functions with unknown scale parameters, the empirical distribution function and the maximum likelihood under the restriction of increasing failure rate.

Mine and Nakagawa [31] consider an age replacement problem of a unit with two failure modes (a unit fails by catastrophic or degradation failure mode. The catastrophic failure stops the operation of a unit suddenly and completely; the degradation failure deteriorates the performance of a unit with time.), in which it is replaced at any failure or age t_0. The unit fails according to a mixed distribution $a_1F_1(t)+a_2F_2(t)$, where $a_1\geqq 0$, $a_2\geqq 0$, and $a_1+a_2=1$. Mine and Nakagawa [31] develop an optimum age replacement policy using the results of Osaki and Nakagawa [43] since a mixed distribution is a special case of a general failure distribution.

In Beichelt and Fischer [8] a generalized age replacement policy is developed in which if on a fixed interval $(0, T)$ no type 2 failures (type two failures are failures that are to be removed by replacement) occur, then the system is renewed at time T by preventive maintenance and time is returned to zero. If at system age x, $0<x<T$, a type 2 failure occurs, then the unit is renewed by repair and time is returned to zero. Minimal repairs are carried out after each type 1 failure (type one failures are failures that are/can be removed by minimal repair).

Nakagawa and Yasui [38] consider an age replacement policy with Weibull failure times. They give upper and lower bounds of an optimum age replacement interval in terms of replacement costs and parameters of a Weibull distribution.

Murthy and Maxwell [33] consider the age replacement for an item when the replacements are drawn from a mixture of two types (1 and 2) with life probability functions known. The decision maker also knows the fraction (P_i) of each type in the mixture. However, he is unaware of whether a specific replacement item is of type 1 or 2. Such a situation would arise when a dealer sells items under his own brand label after buying them from two different manufacturers. Thus, the items loose their individual identity with regards to the manufacturer. The decision maker has to select an optimum age replacement policy when the items are drawn from the mixture, so as to minimize the s-expected cost rate for an infinite operation. The policy can be based on either (I) knowledge of life probability density functions, P_i (where P_i is the fraction of type i in mixture; $P_1+P_2=1$; $i=1, 2$), and no knowledge of the replacement item being of type 1 or 2. Or (II) knowledge of life probability density functions, P_i, and a further knowledge of the replacement items being type 1 or 2. This extra knowledge is obtained by testing, with an associated cost. Murthy and Maxwell examine conditions under which policy II is better than policy I — i.e. obtaining the extra information at a cost is more economical.

In Murthy and Nguyen [34] age replacement policy with imperfect preventive maintenance is considered. The imperfect nature of preventive maintenance is modeled as follows: whenever the unfailed system is subjected to preventive maintenance, the outcome — (a) the system is like new, at a cost c_2 with probability $(1-p)$, (b) the system fails instantaneously with probability p, so that a cost c_1+c_2 is incurred in renewing the system (where c_1 is the cost for failure renewal and c_2 is the cost for preventive maintenance renewal), (c) $(c_2/c_1)<1$, otherwise preventive maintenance is unreasonable, a priori. The other assumptions are (i) the planning horizon is infinite, (ii) the preventive maintenance and failure-repair times are negligible. They obtain the optimum age replacement policy that minimizes the s-expected cost rate.

Yoo and Sung [58] discuss under the following assumptions an optimal age-replacement policy for equipment monitored by a stocastically failing indicator: (a) failure of the equipment and of the indicator are mutually statistically independent, (b) the equipment has a finite mean life, (c) replacements occur instantaneously, (d) both the equipment and the indicator are simultaneously replaced by s-identical new ones, (e) it is more costly to replace the equipment after failure than at age replacement interval T, (f) the planning horizon is infinite, (g) equipment failures can be identified by the indicator during operation (if the indicator fails, the equipment state can be checked after the operation is completed), (h) a detected failure can occur when the indicator does not fail until after the equipment fails, (i) an undetected failure can occur if the indicator fails before the equipment, (j) the equipment is replaced either when failure is detected or at age T, (k) when undetected failures occur, the equipment is not replaced until age T so that system-down cost is incurred during the undetected period. The decision variable is the optimal replacement time for which the mean cost is minimized over an infinite time span. This model infact generalizes one of Barlow and Hunter [4], in that it duplicates their model when the indicator is perfect.

2.2.3. *Planned Maintenance Using Optimal Number of Failures.* Makabe and Morimura [27], [28], [29] proposed a maintenance policy where a unit is replaced at k-th failure. Morimura [32] made several extensions of the model. In his model the system is replaced at time T or at n-th failure after its installation, whichever occurs first, where T is a positive constant and previously specified. The system undergoes only minimal repair at failures. This policy is in fact a modification of policy II (periodic replacement with minimal repair at failure) introduced by Barlow and Hunter [4]. This model is also considered by Nakagawa and Kowada [41]. Park [44] determines the optimal number of minimal repairs before replacement. This question is also discussed by Nakagawa [42].

Adachi and Kodama [2] discuss the following policy: corrective maintenance is performed for (1) and (2), and preventive maintenance for (3), where

(1) the $(n+1)$-th failure occurs before the operating time reaches T_1 hour $(0<T_1\leqq\infty)$ regardless of the downtime at intervening failures,
(2) the first failure occurs after the total operating time reaches T_1 hour,
(3) the total operating time reaches T_2 hour $(T_1\leqq T_2\leqq\infty)$ without failure after T_1 hour,

whichever occurs first, but for intervening failures minimal repairs are performed. It is assumed that (a) the failure rate of the system is not disturbed by minimal repair, (b) the system is as good as new immediately after the corrective and preventive maintenance and (c) T_1 and T_2 are constant.

Nakagawa [37] discusses the following policy:

(A) A unit has two types of failures when it fails: Type 1 failure occurs with probability α and is corrected with minimal repair, and whereas Type 2 failure occurs with probability $1-\alpha$ and a unit has to be replaced. If the k-th type 1 failure occurs before type 2 failure, then a unit is replaced preventively.

(B) A system has two types of units: when unit 1 fails, it undergoes minimal repair, and when unit two fails, a system has to be replaced. If unit 1 fails at k times before unit 2 failure, then the system is replaced preventively.

Nakagawa derives the expected cost rates for each model and obtains the optimal numbers of k^* to minimize the cost rates when the hazard rate is monotonically increasing.

2.2.4. Planned Corrective Maintenance Using Cycle Time. In Fischer [19] cycle time c_0 is determined. If failure occurs at time $c_f > c_0$, then restoration is performed and the system is brought back to its original state. If failure occurs at time $c_f < c_0$; then minimal repair is performed. This minimal repair simply brings back the system to a workable condition and the system failure rate is not changed.

Some of the maintenance policies belonging to the group planned maintenance at predetermined intervals based on number of failures, or otherwise are shown in Figure 1.

2.3. Condition Based Maintenance Policy

In order to increase company profitability and decrease production losses due to system failure, mainly in the large, complex systems, it is common to perform preventive maintenance activities at predetermined intervals. These types of policies are for example block replacement and age replacement policy.

If too short a preventive maintenance interval is selected, the machine will be overhauled unnecessarily, with consequent loss of production and possible human errors during reassembly. Moreover, it should be remembered that a machine seldom deteriorates in a few minutes, followed by a sudden failure. Generally, failure is the culmination of slow deterioration over months or years. An acceptable compromise is to perform maintenance at irregular intervals, determined by the real condition of the equipment or its components. To establish these intervals, one must know the actual condition of the machine at a given time, and its deterioration trend over a period of time. A procedure based on this premise is known as condition based maintenance. Condition based maintenance requires a unit to be inspected at intervals or monitored continuously and replaced just before its failure, the moment of failure being predicted from measurements of prognostic characteristics which is the object of the monitoring process. The new unit, again, will be subject to the same rule. Effective application of condition based maintenance does not require detailed knowledge of the probability distribution of time to failure, though complete knowledge of it would make the determination of (near) optimal inspection

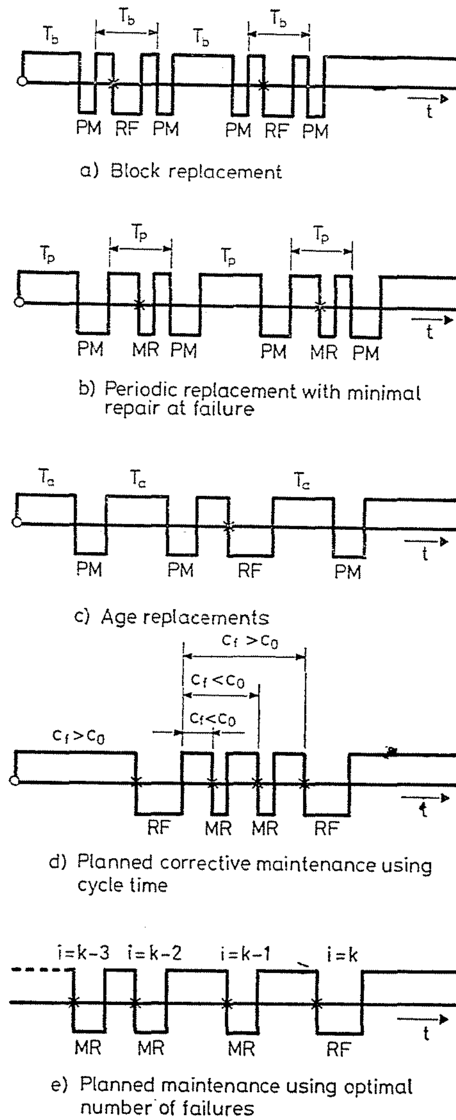

Fig. 1

intervals possible, as demonstrated by Sherwin [51]. The condition of the system is determined with the help of inspection. While studying the maintenance policies we come across three different types of condition based maintenance models. Namely,

(A) models for systems having monotonously changing parameters of deterioration,

(B) models for Markovian systems,

(C) models for semi-Markovian systems.

We can talk about two different types of models for systems having monotonously changing parameters of deterioration. One is the condition based maintenance policy using physical parameters and the other is the condition based maintenance policy using reliability parameters. We are giving below a brief description of these two types of condition based maintenance policies.

A.1. Condition based maintenance policy using physical parameters. The desirability of this policy, monitoring technique used, and its periodicity, will depend on the deterioration characteristics of the equipment studied and the costs involved. At one extreme, replaceable items such as brake pads can be checked simply at short intervals and at little expense. At the other extreme complex replaceable items, e.g. engines might require expensive strip down for visual examination (which might in itself cause subsequent failures). It is with items of this type that sophisticated condition monitoring, e.g. vibration monitoring, shock pulse monitoring, oil analysis, thermography etc. can be used to great advantage. The cost of instrumentation may be justified by high repair and unavailability costs [Collacott [15], Kelly and Harris [26]]. The applicability of condition based maintenance policy crucially depends on the existence of a prognostic characteristic, for example, tread depth, and its measurability [Geurts, 23].

A.2. Condition based maintenance policy using reliability parameters. It is not always possible to select parameters that influence the life time of the system in the best way. Sometimes, due to the lack of easily observable physical characteristics the question arises why not divide the complex system into suitable functional parts and reliability parameters be assigned to each critical part of the system. With the help of this procedure equipments that have adequante information regarding their failures, may be maintained. Such information may be collected by conducting experiments or from servicing data. By observing the changes in the values of the reliability parameters, in terms of time, we can know the actual condition of the given system. The value of the reliability parameters are checked at predetermined intervals, if necessary by varying the operating conditions or by thoroughly inspecting the system. The criteria for using the condition based maintenance policy based on reliability parameters are the following:

1. Consequences of the failure is not catastrophic.
2. Failures occur relatively seldom (the system reliability is high).
3. Failure is easily observable and the system can be repaired easily.
4. Condition based maintenance policy based on reliability parameters is economical.
5. Operation is well organized (computer, experts, reliability information).

3. Conclusion

Maintenance policies discussed in the literature have been studied, inter-related and classified into three broad groups vic. (I) Replacement/repair at failure, (II) Planned maintenance (repair/replacement) at pre-determined intervals based on number of failures or otherwise and (III) Condition based maintenance. This classification gives a good way to analyse the very wide and rich literature on the subject. The above number of groups are necessary and sufficient for the purpose of systematic surveying of the publication.

Many of the maintenance models discussed in the literature have been prepared with a view to the convenience of working with them. For the sake of mathematical convenience, in many cases the assumptions made in the models are over-simplification of the practical problems. Thus they (models) are usually applicable only to some situations. Practical application of most of these models is quite difficult.

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