THE EFFECT OF HYDRAULIC FLUID MODULUS VARIATION DUE TO AIR BUBBLES ON HYDRAULIC POSITIONAL SERVOMECHANISM

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Abstract

The presence of air bubbles in hydraulic fluid changes very strongly the fluid compressibility which gives rise to a negative fact in the system behavior. This paper discusses the problem of interest and gives some instructions to determine the critical volume ratio of air-oil mixture from the point of view of both transient response and steady-state of the system. The suitable operation pressure is also proposed for a required system behavior.

Introduction

Both in the hydraulic follower-systems and in the hydraulic positional servomechanisms the compressibility of the hydraulic fluid plays an important role in the system behavior. It can namely make a system unsteady which otherwise operates without fault in normal conditions. In general, the system behavior depends on several parameters and these parameters also depend more or less on each other. So for instance in the copying shaper with two-control edge valve, if the compressibility is negligible then the transfer function is of second-order (with the acceptable application of linearization). But, if this assumption is not satisfied, the transfer function becomes a third-order one consisting of a second-order proportional element and an integral one in cascade. The geometrical parameters of the system practically do not change significantly, whereas the hydraulic fluid, namely hydraulic oil temperature, viscosity, compressibility and mass density change continually, if we consider seriously the problem of interest. In addition to the parameter variations, they are not independent from each other. But if we take these into consideration, the problem becomes very difficult. In the engineering domain though viscosity and so fluid friction depends very strongly upon the temperature, nevertheless we shall not deal with them. In a system which is dynamically balanced in temperature the hydraulic fluid viscosity change is very small, even if air bubbles of small size arise from the oil in a certain way. We shall therefore not deal with them. This neglection may be proven as follows.

When the bubbles are compressed as a consequence they run hot and naturally give a certain heat quantity to oil. That is all very well but the specific heat of air is much lower than that of oil, similarly the ratio of specific weight of air to oil is very small. Therefore the temperature increment may be neglected [1]. One may determine this increment by the following expression

$$\Delta T=\frac{\frac{24}{427}\,\frac{n}{n-1}\,p_a v_a\left[\left(\frac{p_0}{p_a}+1\right)^{\frac{n-1}{n}}-1\right]\frac{c_l}{c_0}\,\frac{\gamma_l}{\gamma_0}\,\frac{v}{1-v}}{1+\frac{c_l}{c_0}\,\frac{\gamma_l}{\gamma_0}\,\frac{v}{1-v}} \tag{1}$$

where p_a = atmospherical pressure, [N/m^2] or [bar]
v_a = specific volume of environmental air [m^3/N]
p_0 = working pressure of oil in the system [N/m^2] or [bar]
n = polytropic ratio of air
c_0 and c_1 = specific heat of oil and air, respectively [J/NC°]
γ_l = specific weight of air [N/m^3]
γ_o = specific weight of pure oil [N/m^3]
v = volume ratio of air in the air-oil mixture.

From this we may ascertain that in practice ΔT is very small when the magnitude of v is not extremely valuable. Remember, v is always much smaller than unit, i.e., $v \ll 1$.

Some papers stated that in hydraulic systems, which operate with a high pressure, the so-called Diesel effect may occur [2], [3]. As a consequence the oil temperature increases, but according to experience such a phenomenon is rare enough and localized. It may be important only from the viewpoint of system reliability, because it effects adversely some fragile elements of the system (e.g. filler rings and piston-rings, and so on). To be sure, the Diesel effect accelerates the ageing process of hydraulic fluid due to the influence of local high temperature, but as a high speed process, which changes the oil viscosity and thus influences the system, we may neglect this fact in the following investigation. The temperature rise due to the Diesel effect and the change of oil viscosity may be evaluated by the Diesel combustion [4]. For example, let us suppose that this combustion is realized with an air-surplus of a magnitude of 1.25 and $v=0.1$. In a pressure condition of normal environment the temperature rise of the air-oil mixture does not exceed a magnitude of 0.11 °C. After this we shall not deal with either the oil temperature rise or the viscosity change, but only with the compressibility of oil and suppose that its state change is isothermal. For the sake of simplicity we shall investigate a simpler system, which in principle is generally valid for a more complicated system, too.

2. Investigation of system stability and dynamic parameters in the presence of air bubbles in oil

System stability

It is known that in the copying shaper with a two-control edge valve the transfer function of the open loop is as follows [5]

$$G(s)=\frac{Y(s)}{E(s)}=\frac{K}{s[C_2 s^2+C_1 s+1]} \tag{2}$$

where

$$\left.\begin{aligned} K&=\frac{2A_0K_pK_Q}{BK_Q+4A_0^2K_p} \quad \text{the loop gain}\\ C_2&=\frac{\beta VmK_p}{BK_Q+4A_0^2K_p}\\ C_1&=\frac{mK_Q+\beta-VK_pB}{BK_Q+4A_0^2K_p} \end{aligned}\right\} \tag{3}$$

and

in which

A_0 = the smaller area of actuator piston,
K_p = pressure-difference amplifying factor of valve,
K_Q = volumetric flow amplifying factor of valve,
B = friction factor of fluid,
β = compressibility factor of fluid,
m = mass which moves with the actuator cylinder,
V = the largest capacity of actuator cylinder.

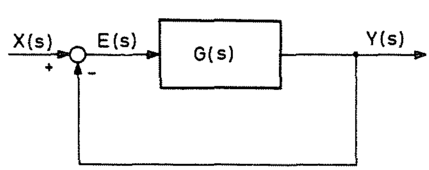

Fig. 1

After Fig. 1 the transfer function may be written

$$W(s)=\frac{Y(s)}{X(s)}=\frac{G(s)}{1+G(s)}=\frac{C_2s^3+C_1s^2+s}{C_2s^3+C_1s^2+s+K} \tag{4}$$

from which the characteristic equation for this system is

$$K(s)=C_2s^3+C_1s^2+s+K=0. \tag{5}$$

On the basis of [5] the necessary conditions of system stability are: $C_2>0$ and $C_1>0$. It is well known that after Routh-Hurwitz the sufficient conditions may be written in the inequalities as follows

$$C_1>0 \qquad \text{and} \qquad C_1-KC_2>0 \tag{6}$$

It is remarkable that in a given system the C_1 and C_2 quantities change due to air presence, namely they depend upon the magnitude of v. Therefore the stability conditions are not independent of v. The investigation of this problem is very interesting and important, because we may receive many instructive conclusions for the permissible limit of v (i.e., the magnitude of the critical v) and for the acceptable pressure for the normal system operation. Before dealing with these problems it is better to use the form of Bode for the sake of expressing the damping ratio and the time constant (or the natural frequency) of the second-order element, namely:

$$\left.\begin{aligned} \zeta_0 &= \frac{C_1}{2}\sqrt{\frac{1}{C_2}} \\ \text{and} \qquad T_0 &= \frac{1}{\omega_0} = \sqrt{C_2} \end{aligned}\right\} \tag{7}$$

where ζ_0 = damping ratio
T_0 = time constant
ω_0 = natural frequency

Returning to the stability criterion of the system we have:

$$\begin{aligned} C_1 &= \frac{mK_Q+\beta VK_pB}{BK_Q+4A_0^2K_p}>0 \\ C_1-KC_2 &= \frac{mK_Q+\beta VK_pB}{BK_Q+4A_0^2K_p}-\frac{2A_0K_pK_Q}{BK_Q+4A_0^2K_p}\cdot\frac{\beta VmK_p}{BK_Q+4A_0^2K_p}>0. \end{aligned} \tag{8}$$

In the physical sense the denominator of both C_1, C_2 and K is positive, because B, $K_p=p_0/\varepsilon_0$, $K_Q=\sqrt{2}\,k\sqrt{p_0}$ quantities are always positive (where ε_0 is a negative overlap and k is a quantity related to the flow factor of the valve). Moreover the numerator of C_1 is the same, i.e.

$$mK_Q+\beta VK_pB>0. \tag{9a}$$

Therefore the investigation of the stability problem is equivalent to the consideration of the inequality:

$$(mK_Q+a_1\beta)(BK_Q+d_1)-b_1c_1K_QB>0 \tag{9b}$$

where
$$\left.\begin{aligned} a_1 &= VK_pB \\ b_1 &= 2A_0K_p \\ c_1 &= VMK_p \\ d_1 &= 4A_0^2K_p. \end{aligned}\right\} \tag{9c}$$
and

Let us now consider the quantities of (9b), which depend upon the fluid compressibility. It is clear that in practice K_p does not change, but K_Q does. As a matter of fact

$$K_Q = \sqrt{2p_0}\,k = \sqrt{2p_0}\,\mu D\pi\sqrt{\frac{2g}{\gamma}} = C\sqrt{\frac{1}{\gamma}}$$

where

μ = the flow factor of the valve
D = the valve diameter
g = the acceleration of gravity
γ = specific weight of air-oil mixture
$C = \sqrt{2p_0}\,\mu D\pi\sqrt{2g}$ = the constant of the given system.

As a consequence k is a function of variable v, namely

$$k = k(\gamma(v)).$$

With the negligible error this expression may be written in the form:

$$k = k_0\sqrt{\frac{1}{1-v}} \tag{10}$$

where $k_0 = 2\pi D\sqrt{\frac{2g}{\gamma_0}}$, and γ_0 = specific weight of pure oil.

It is important to remember that

$$0 \leq v < 1$$

therefore

$$K_Q = K_{Q_0}\sqrt{\frac{1}{1-v}} \tag{11}$$

where

$$K_{Q_0} = \sqrt{2p_0}\,k_0. \tag{12}$$

As mentioned above we suppose that the air compression is an isothermal process and so

$$\beta = \frac{1 + E_0 \dfrac{p_a}{p_0^2} \dfrac{v}{1-v}}{1 + \dfrac{p_a}{p_0} \dfrac{v}{1-v}} \frac{1}{E_0} \tag{13}$$

when air bubbles are in the hydraulic fluid [1] and E_0 is the compressibility modulus of the pure oil (i.e. the compressibility of oil, which is free from air). After this the relation (9b) may be written in the form

$$av^3 + bv^2 + cv + d > 0 \tag{14}$$

where

$$\begin{aligned} a &= -c_2 g_2 \\ b &= c_2 g_2 + e_2 f_2 - b_2 g_2 - c_2 \\ c &= a_2 g_2 + b_2 g_2 + d_2 f_2 - b_2 + c_2 + e_2 \\ d &= a_2 + b_2 + d_2 \end{aligned} \tag{15}$$

and

$$\begin{aligned} a_2 &= mBK_{Q_0}^2 \\ b_2 &= md_1 K_{Q_0} \\ c_2 &= 0.5 md_1 K_{Q_0} \\ d_2 &= [(a_1 B - b_1 c_1) K_{Q_0} + a_1 d_1] e_1 \\ e_2 &= 0.5(a_1 B - b_1 c_1) K_{Q_0} e_1 \\ f_2 &= f_1 - 1 = E_0/p_0^2 - 1 \\ g_2 &= g_1 - 1 = 1/p_0 - 1 \\ e_1 &= 1/E_0 \end{aligned} \tag{16}$$

after using the following approximations

$$(1-v)^{-1/2} \approx 1 + 0{,}5v \tag{17}$$

and

$$p_a = 1 \tag{18}$$

because in general v is much smaller than unit and p_a may be taken to be 1 bar.

If we want to receive a more exact result, we do not use the (17) approximation, i.e. the linearization. With the convenient conversions we have the following inequality:

$$A^* v^3 + B^* v^2 + C^* v + D^* > 0 \tag{19}$$

where

$$A^*=b_2^2f_2^2h_2^2-2a_2g_2f_2$$
$$B^*=a_2^2g_2^2-2a_2g_2+2a_2f_2g_2-1+b_2^2d_2^2f_2^2-b_2^2h_2^2f_2^2-2f_2 \qquad (20a)$$
$$C^*=2a_2g_2+2a_2g_2-1+2a_2f_2+2b_2^2d_2f_2-b_2^2h_2^2 2f_2-1$$
$$D^*=a_2^2+2a_2+b_2^2d_2^2-b_2^2h_2^2$$

in addition

$$h_2=a_1d_1e_1. \qquad (20b)$$

With suitable engineering experience and the aid of a digital computer the physically right result may be obtained without difficulty. But it is easily seen that even if one keeps the first terms, namely the linear terms, the result is more exact than one from the term of (14).

After these, in a concrete case we may state when the system becomes labile due to the disturbing presence of air bubbles in the hydraulic fluid.

The change of system dynamic parameters due to the presence of air bubbles

We consider as system dynamic parameters the open loop gain, the time constant, and the damping ratio of the second-order element of the system transfer function.

a) *The open loop gain K*. From (3) it is known that

$$K=\frac{2A_0K_pK_Q}{BK_Q+4A_0^2K_p}=\frac{2A_0K_pK_{Qo}(1-v)^{-1/2}}{BK_{Qo}(1-v)^{-1/2}+4A_0^2K_p}. \qquad (21)$$

Using the following designations

$$f^*=2A_0K_pK_{Qo}$$
$$g^*=BK_{Qo}$$
$$h^*=4A_0^2K_p$$

we have the expression:

$$K=\frac{f^*}{g^*+h^*\sqrt{1-v}}. \qquad (22)$$

The examination of function K in the usual way is complicated enough, so for the sake of simplicity we shall use a simple graphical method. Due to their physical sense f^*, g^* and h^* are positive. Using the new variable $v=u^2-1$ the expression (22) can be written in the form:

$$\frac{1}{K}=\frac{g^*}{f^*}+\frac{h^*}{f^*}u.$$

With a convenient co-ordinate system we may estimate the variation of function K as shown in Fig. 2. It is visible that as v increases, so does K, and inversely. This fact is in compliance with reality, because the rise of K decreases the gain margin or the phase margin and therefore the system stability is getting worse, despite the fact that the "spring rate" of the system decreases (Remember, when the spring rate decreases, the "spring mass" decreases, too). Without compensation it is natural that the system swings more easily than in its original condition. A concrete case (as the result of the illustrative example in this paper) is shown in Fig. 4a.

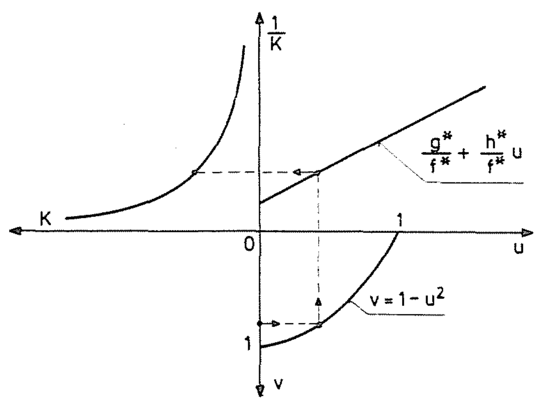

Fig. 2

b) *Time constant and natural frequency.* From expressions (3), (7) and (13) we have

$$\frac{1}{E_0}\,\frac{1+E_0\dfrac{1}{p_0^2}\,\dfrac{v}{1-v}}{1+(1/p_0)v/(1-v)}\,VmK_p \quad {}^{1/2}$$

$$T_0=\sqrt{C_2}=\left\{\frac{\beta VmK_p}{BK_Q+4A_0^2K_p}\right\}^{1/2}=\left\{\frac{\beta VmK_p}{\dfrac{BK_{Q0}}{\sqrt{1-v}}+4A_0^2K_p}\right\}^{1/2} \qquad (24a)$$

with regarding $p_a \approx 1$ [bar].

Now we investigate and estimate how the time constant T_0 changes with v. In this case if we are interested in the change of T_0 then, instead of T_0, we may deal with T_0^2. After the convenient conversions (24a) becomes

$$T_0 = \left\{ \frac{\dfrac{1/v + E_0/p_0^2 - 1}{1/v + 1/p_0 - 1} \cdot \dfrac{VmK_p}{E_0}}{\dfrac{BK_{Q_0}}{\sqrt{1-v}} + 4A_0^2 K_p} \right\}^{1/2} . \tag{24b}$$

If we examine in detail the $(f_1 - 1)$ and $(g_1 - 1)$ expressions we can discover an interesting property of the numerator $S(v)$ of (24b). It is known that in the customary cases

$$f_1 - 1 = \frac{E_0}{p_0^2} - 1 = \text{const.} > 0$$

and also relation (25)

$$g_1 - 1 = \frac{1}{p_0} - 1 = \text{const.} < 0$$

is always valid. In addition, the $\frac{1}{v} + f_1 - 1$ and $\frac{1}{v} + g_1 - 1$ quantities give the two coinciding hyperbolas as shown in Fig. 3 because the only difference between

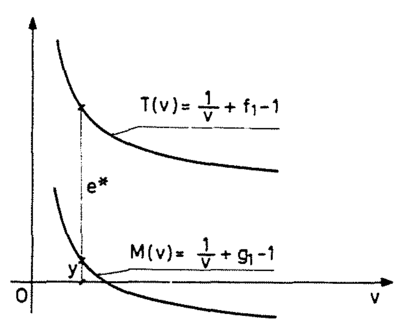

Fig. 3

them is again $f_1 - g_1 = e^* > 0$ namely the constant displacement along the vertical axis from each other. With the designations of Fig. 3 we have

$$f(v) = \frac{\dfrac{1}{v} + f_1 - 1}{\dfrac{1}{v} + g_1 - 1} = \frac{T(v)}{M(v)} \tag{26}$$

and therefore it may be written, that

$$f(v)=1+\frac{e^*}{y} \tag{27}$$

It is easy to see that e^* is a constant. The function $f(v)$ very rapidly tends to infinity at a certain finite value of v, therefore the numerator $S=T(v)/M(v)$ is also similar (i.e. $S\to\infty$). Now let us consider the denominator of (24b). This is a function of variable v where the rapid variation is evidently smaller than that of the numerator in the interval $0\le v\le v_h$. Consequently T_0 increases monotonously with the increasing of v in the same interval. For the purpose of a more exact result, let us solve the extreme problem of T_0^2 function in the interval of interest. We have

$$\frac{d}{dv}T_0^2=\frac{-\alpha v^2-\lambda v+\delta}{(g_2v+1)^2(Cv+M)^2} \tag{28}$$

where

$$\begin{aligned}\alpha&=ACf_2g_2\\ \lambda&=ACg_2\\ \delta&=Af_2M-Ag_2M-AC\end{aligned} \tag{29a}$$

Here

$$A=\frac{VmK_p}{A_0};\quad C=0.5BK_{Q_0}\quad \text{and}\quad M=BK_{Q_0}+4A_0^2K_p \tag{29b}$$

Because after the physical sense the cases:

$$v=-\frac{1}{g_2}=-\frac{1}{\frac{1}{p_0}-1}=\frac{p_0}{p_0-1}\ge 1$$

and (30)

$$v=-\frac{M}{C}=-\frac{BK_{Q_0}+4A_0^2K_p}{0.5BK_{Q_0}}\le 0$$

do not exist, therefore $\frac{d}{dv}T_0^2$ has not vanished only if equation

$$\alpha v^2+\lambda v-\delta=0 \tag{31}$$

is satisfied. In this case the function T_0^2 has its extreme value if we can prove that this extreme point exists in the semi open interval $0\le v<v_{kr}$. Then we may be sure of a monotonous increasing of T_0^2 in the same interval. For this purpose let us estimate the roots of equation (31), namely the values:

$$v_{1,2}=\frac{-\lambda\pm\sqrt{\lambda^2+4\alpha\delta}}{2\alpha} \tag{32}$$

We found that

$$\alpha = ACf_2 g_2 = \frac{VmK_p}{E_0} f_2 0.5BK_{Q_0} g_2 = \frac{VmK_p}{E_0}\left(\frac{E_0}{p_0^2}-1\right)0.5BK_{Q_0}\left(\frac{1}{p_0}-1\right)<0 \tag{33a}$$

This inequality is also valid in case of the most modern copying shaper. The situation of δ is similar

$$\delta = AMf_2 - AMg_2 - AC = \frac{VmK_p}{E_0} f_2(BK_{Q_0}+4A_0^2K_p) -$$

$$-\frac{VmK_p}{E_0} g_2(BK_{Q_0}+4A_0^2K_p) - \frac{VmK_p}{E_0} 0.5BK_{Q_0} =$$

$$= \frac{VmK_p}{E_0}\left[BK_{Q_0}+4A_0^2K_p - 2\frac{BK_{Q_0}p_0^2}{E_0} - \frac{8A_0^2K_pp_0^2}{E_0} - \frac{BK_{Q_0}p_0}{E_0}\right.$$

$$\left. - \frac{4A_0^2K_pp_0}{E_0} - \frac{0.5K_{Q_0}p_0^2}{E_0}\right]<0 \tag{33b}$$

It is now easily seen that at $p<100$ bar relation

$$BK_{Q_0} \approx \frac{8A_0^2K_pp_0^2}{E_0}$$

is also valid and besides

$$4A_0^2K_p < \frac{2BK_{Q_0}p_0^2}{E_0} + \frac{BK_{Q_0}p_0}{E_0} + \frac{4A_0^2K_pp_0}{E_0} + \frac{0{,}5K_{Q_0}p_0^2}{E_0} =$$

$$= 4A_0^2K_p\left(\frac{2BK_{Q_0}p_0^2}{4A_0^2K_pE_0} + \frac{BK_{Q_0}p_0}{4A_0^2K_pE_0} + \frac{p_0}{E_0} + \frac{0{,}5K_{Q_0}p_0^2}{4A_0^2K_pE_0}\right)$$

because K_{Q_0} differs by several orders of magnitude from K_p.

Consequently the discriminant of Eq. (31) is positive and greater than λ^2. Thus there are one negative root of smaller magnitude (which is not of interest) and one positive root. This fact indicates the monotonous increase of T_0, as well as that of T_0^2 in the interval of interest. This result confirms again what was observed from the variation of T_0. It is well known that in this case the natural frequency necessarily decreases, because it is reciprocal to T_0. Our illustrative example also proves this nature of T_0 (Fig. 4b) iii *The damping ratio* ζ_0. According to expression (7)

$$\zeta_0 = \frac{C_1}{2}\sqrt{\frac{1}{C_2}} =$$

$$= \frac{1}{2}\cdot\frac{mK_{Q_0}\dfrac{1}{\sqrt{1-v}} + \dfrac{1}{E_0}\,\dfrac{1+E_0\dfrac{1}{p_0^2}\dfrac{v}{1-v}}{1+\dfrac{1}{p_0}\dfrac{v}{1-v}}VK_pB}{\left\{\left(BK_{Q_0}\dfrac{1}{\sqrt{1-v}}+4A_0^2K_p\right)\dfrac{1}{E_0}\,\dfrac{1+E_0\dfrac{1}{p_0^2}\dfrac{1}{1-v}}{1+\dfrac{1}{p_0}\dfrac{1}{1-v}}VK_pB\right\}^{1/2}} \tag{34}$$

In detail

$$\zeta_0 = \frac{1}{2}\,\frac{\dfrac{mK_{Q_0}}{\sqrt{1-v}} + \dfrac{VK_pB}{E_0}\,\dfrac{1+(E_0/p_0^2-1)v}{1+(1/p_0-1)v}}{\left\{\left(\dfrac{BK_{Q_0}}{\sqrt{1-v}}+4A_0^2K_p\right)\dfrac{VmK_p}{E_0}\,\dfrac{1+(E_0/p_0^2-1)v}{1+(1/p_0-1)v}\right\}^{1/2}}$$

For the sake of simplicity we shall investigate the behavior of ζ_0^2 instead of ζ_0. Using the following designations:

$$a_5 = mK_{Q_0}; \qquad b_5 = \frac{VK_pB}{E_0}; \qquad e_5 = BK_{Q_0}$$
$$f_5 = 4A_0^2K_p; \qquad g_5 = \frac{VmK_p}{E_0} \tag{35}$$

we have

$\zeta_0^2 = f^*(v) =$

$$= \frac{a_5^2 + 2a_5^2v + 2a_5v^2 + \dfrac{(2a_5+v)b_5(1+f_2v)}{1+d_5v} + b_5^2\dfrac{1+2f_2v+f_2^2v^2}{1+2d_5v+d_5^2v^2}}{(e_5g_5+0.5e_5g_5v)\dfrac{1+f_2v}{1+d_5v} + f_5g_5\dfrac{1+f_2v}{1+d_5v}} \tag{36}$$

After conversions and reduction, and using the linear terms of Taylor's series expansion in the neighbourhood of zero the function $f^*(v)$ may be written in the following form

$$f^*(v) = \frac{a_5(1+2b_5)}{g_5(e_5+f_5)} + \frac{b_5^2}{e_5 g_5} +$$

$$+ \frac{a_5 b_5 g_5 (2c_5+1)(e_5+f_5) - a_5 g_5 (1+2b_5)(e_5 c_5 + f_5)}{g_5^2 (e_5+f_5)^2} +$$

$$+ \frac{b_5 c_5 e_5 g_5 - (0.5 e_5 + d_5 g_5) b_5^2}{(e_5 g_5)^2} v \tag{37}$$

Since the value of v is usually small we may estimate the variation of $f^*(v)$ by means of sign examination of the coefficient of v in formula (37). After substitution of (35) into (37) the coefficient of v is

$$\frac{mB^2 V^2 K_p^2 K_{Q_0}}{E_0^2} \left(\frac{E_0}{p_0^2} - 1 \right) - \frac{0.5 BmVK_p K_{Q_0}}{E_0} - \frac{V^2 BmK_p}{E_0^2} \left(\frac{1}{p_0} - 1 \right)$$

from which it is easily seen that the following inequalities are always satisfied:

$$-\frac{0.5BmVK_pK_{Q_0}}{E_0} < 0$$

and (38)

$$BK_{Q_0}\left(\frac{E_0}{p_0^2} - 1 \right) - \left(\frac{1}{p_0} - 1 \right) < 0$$

Consequently the damping ratio ζ_0 decreases when v increases. Physically this would be expected. This character of ζ_0 may be seen in Fig. 4c (as the result of our illustrative example).

Conclusions and discussion

Summarizing the received result it may be stated that the presence of air bubbles in oil and the increase of the volume ratio of an oil-air mixture gives rise to a negative fact as to transient response and system steady-state, because the increase of v enlarges the open loop gain K (which makes the system more easily unstable) and time constant T_0, and in addition decreases damping ratio ζ_0. All these are detrimental to the system operation quality.

It is to be remarked that from the received results we may deduce the permissible value of v in a given system, and if the presence of air bubbles is inevitable then we can choose a system which operates with a higher pressure.

Table 1

v	K
0	987.654320988
0.0024	988.826968072
0.0049	990.052946586
0.0074	991.283515621
0.0099	992.518703945
0.0124	993.75854058
0.0149	995.003054801
0.0174	996.252276147
0.0199	997.506234414

Table 2

v	T_0
0	0.000384900179
0.0025	0.000408276393
0.005	0.000430486929
0.0075	0.000451704465
0.01	0.000472063588
0.0125	0.000491671594
0.015	0.000510615655
0.0175	0.000528967726
0.02	0.000546788018

Table 3

v	ζ
0	0.330372654036
0.002	0.31620763097
0.004	0.303861793387
0.006	0.292982361301
0.008	0.283305293417
0.01	0.274628524921
0.012	0.26679444918
0.014	0.259678092796
0.016	0.253178919464
0.018	0.247215016171
0.02	0.241718884519

Illustrative example

Let us consider a concrete system with the following technical data:

$A_0 = 20[\text{cm}^2] = 2 \cdot 10^{-3}[\text{m}^2]$; $B = 1000[\text{Ns/m}]$

$\gamma_0 = 8100[\text{N/m}^3]$; $p_0 = 20[\text{bar}] \approx 2 \cdot 10^6[\text{N/m}^2]$

$m = 2[\text{kg}]$; $p_a = 1[\text{bar}] \approx 10^5[\text{N/m}^2]$

$\beta_0 = E_0^{-1} = 5 \cdot 10^{-8}[\text{m}^2/\text{N}]$

$V = 2.4 \cdot 10^{-4}[\text{m}^3]$

$K_p = 2 \cdot 10^{10}[\text{N/m}^3]$ (according to the amplification factor of force $B_0 = 8 \cdot 10^3[\text{N/m}]$)

$K_{Q_0} = 4[\text{m}^2/\text{s}]$ (in accordance with the amplification factor of speed $D_0 = 10^3[1/\text{s}]$)

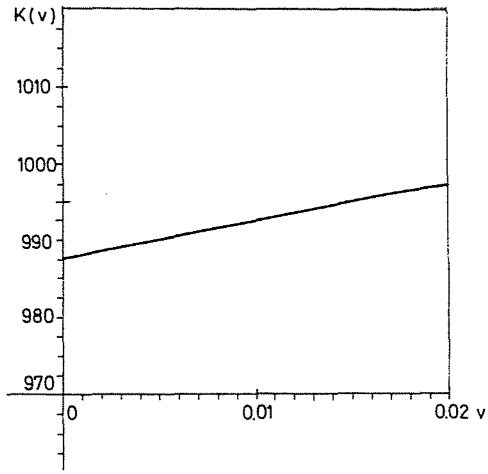

Fig. 4a. Open loop gain

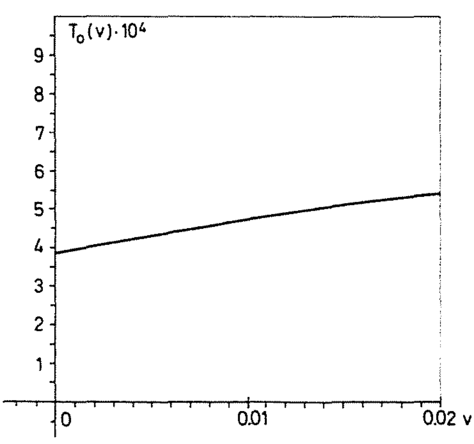

Fig. 4b. Time constant

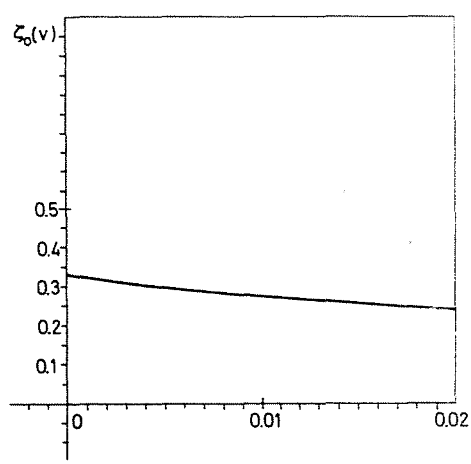

Fig. 4c. Damping ratio

After these the quantities of (9c) and (15) are

$a_1 = 4.8 \cdot 10^5$; $e_1 = 5 \cdot 10^5$; $a_2 = 3.2 \cdot 10^7$; $f_2 = 49$

$b_1 = 8 \cdot 10^4$; $f_1 = 50$; $b_2 = 2.56 \cdot 10^9$; $g_2 = -0.95$

$c_1 = 9.6 \cdot 10^3$; $g_1 = 5 \cdot 10^{-2}$; $c_2 = 1.28 \cdot 10^9$

$d_1 = 3.2 \cdot 10^6$; $d_2 = -1.4582 \cdot 10^9$

from where the coefficients of the first approximation equation receive the following magnitudes

$$a=1.216 \cdot 10^9; \qquad b=-3.767248 \cdot 10^{10}$$

$$c=-7.596368 \cdot 10^{10}; \quad d=1.133376 \cdot 10^9$$

and the physically possible root of this equation is

$$v=0.0148$$

(see Fig. 5a and b, and Tabs. 4a and b)
namely in percentage:

$$v=1.48\%$$

Table 4/a

v	P
−2	−9.6074
−1.75001	−4.7364
−1.50001	−1.13794
−1.25001	1.30197
−1.00001	2.69733
−0.75001	3.16215
−0.50001	2.81042
−0.25001	1.75615
−1.0E−5	0.11333
0.24999	−2.00403
0.49999	−4.48194
0.74999	−7.2064
0.99999	−10.06341
1.24999	−12.93895
1.49999	−15.71905
1.74999	−18.28969
1.99999	−20.53688
2.24999	−22.34661
2.49999	−23.60489
2.74999	−24.19772
3	−24.01109
3.25	−22.931
3.5	−20.84347
3.74	−17.63448
4	−13.19003
4.25	−7.39613
4.5	−0.13878
4.75	8.69603

Table 4/b

v	P
−0.002	0.12851
−0.001	0.12092
0	0.11333
0.001	0.10573
0.002	0.09812
0.003	0.09051
0.00399	0.08289
0.005	0.07526
0.006	0.06762
0.007	0.05997
0.008	0.05232
0.009	0.04466
0.01	0.03699
0.011	0.02931
0.012	0.02163
0.013	0.01394
0.014	0.00624
0.015	−0.00146
0.016	−0.00918
0.017	−0.0169
0.018	−0.02463
0.01899	−0.03236
0.01999	−0.0401

At values of v higher than 1.48% the system operation is unstable. It is evident that we do not necessarily use the equation (18) because, as we have seen, the magnitude of v is very small and therefore the approximative error due to Taylor's series expansion cannot be shown.

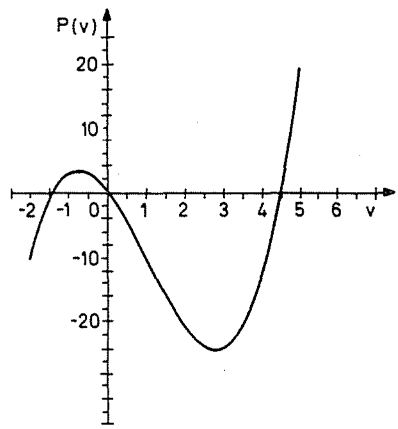

Fig. 5a. Plot of polynomial $P(v)$

The example proves numerically the previously mentioned statements from the open loop gain K, the time constant and the damping ratio, in spite of that as the functions of v. The tables and plots are produced by calculation on the EMG 777 computer, operating in BASIC language.

It is important to remark that by the other investigation method*, namely the phase-space method, the result received is totally identical (see Fig. 6a and b). There, as $v=1.48\%$ the system steady-state response is a permanent oscillation!

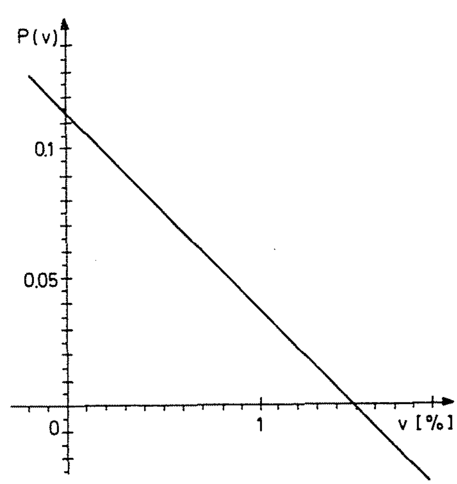

Fig. 5b. Expanded view of plot $P(v)$

* In the following work of the same author: "Digital simulation of hydraulic positional servomechanism considering the hysteresis backlash and the hydraulic fluid compressibility" (manuscript)

```
5  PRINT "HE3COH"
6  INPUT H
10 T0=0
20 Y0=0
30 Z0=0
40 D=5.0E-6
50 T1=0.01
51 T2=0.0005091
52 B=0.514042
53 K=994.55
54 R=0
55 X0=0.1
60 R0=1
70 H1=0.25
80 N1=25
81 M5=50
82 N5=0.05
83 L5=5.0E-5
84 M=0
85 N=0
86 L=0
```

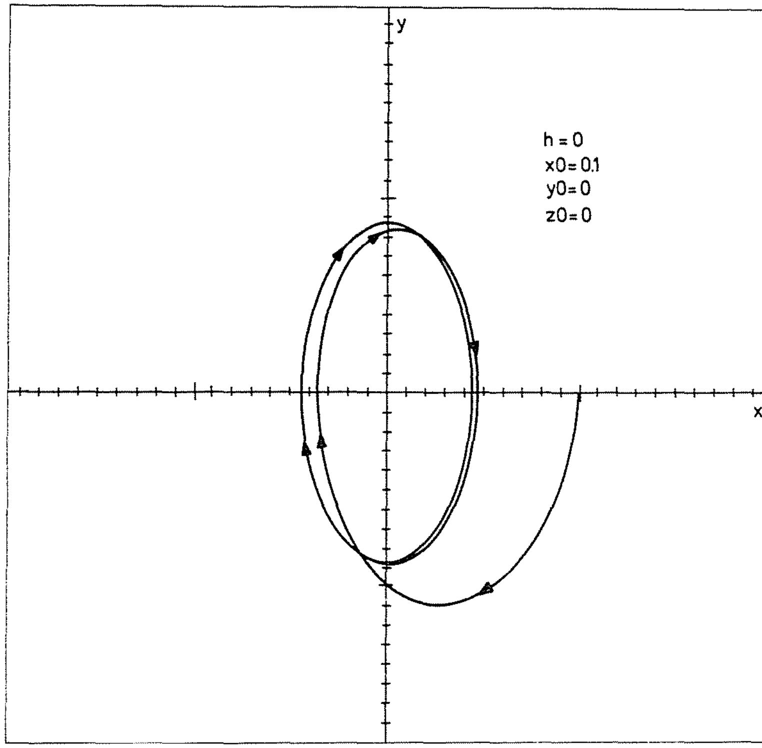

Fig. 6a

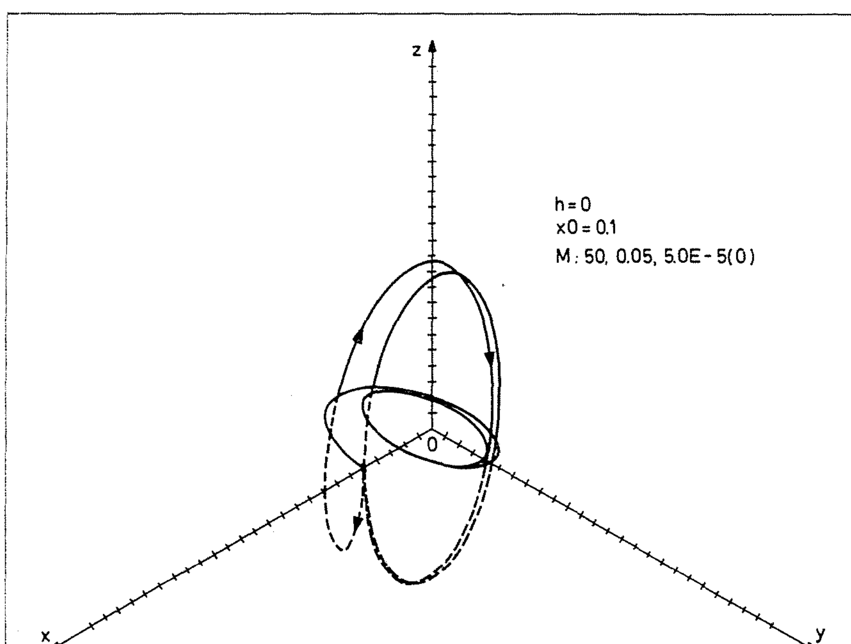

Fig. 6b

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