CUTTING TEMPERATURE
IN INTERMITTENT CUTTING

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Abstract

With increasing cutting speed, the cutting temperature increases only to a certain limit, and in case of intermittent cutting, it even reduces after a certain maximum, as stated by Salomon. A new empirical function is suggested, which describes the relationship between cutting speed and cutting temperature even at high cutting speeds and in case of intermittent cutting appropriately. The new function has been applied to non-metallic coat formation in case of intermittent cutting.

The intensification of cutting is limited by the thermal strength of the tools. It is therefore necessary that most accurate knowledge be obtained about the change of thermal stress acting upon the tool as the rate of metal removal, first of all the cutting speed, increases. In spite of the fact that this problem ranges in cutting theory among those which have been intensively investigated since Gottwein [1], it cannot be considered as settled. This work is intended to be a contribution to the answer.

Cutting temperature and the factors affecting it

The characteristics of temperature distribution on the surface of tools getting into contact with the workpiece and chips are shown in Fig. 1. It can be seen that the temperature is not uniformly distributed but changes even along the edge of the tool. Still, it is usual to speak of a definite cutting temperature that can be considered an average value. This temperature is usually measured by using the workpiece and the tool as the thermocouple [2], in which case internal currents compensate for the differences in thermopotential occurring the different points of the contact surface, that is the potential difference observable in the measuring circuit indicates the average temperature of the contacting tool surface. [3].

From the beginning of the process of metal removal, the cutting temperature changes every moment until a steady state takes place in the chip
Fig. 1. Temperature distribution on tool surface in contact with workpiece and chip.

Fig. 2. Setting in of thermal equilibrium of the chip root ($\theta_c$—temperature of tool body, $\theta_{ob}$—temperature of surface layer, $\theta_z$ total temperature rise in surface layer).

Fig. 3. Effect of cutting speed on cutting temperature a) Customary presentation; b) Theoretical characteristic applying to a wide speed range.
root. This process can practically be divided in two partial processes and thus in two ranges. A quasi-stationary state is soon taking place in the surface layer of the tool according to Fig. 2, which is then superimposed on the slower warming up of the whole tool body that is on the other range. Leaving the latter out of consideration, in the following let us point out that in some abrasion processes, e.g. in diffusion wear, the effect of a change of 30 to 40 K in temperature may be considerable. This fact shall be considered whenever the cutting temperature is used in planning technology.

The cutting temperature is affected also by the technological parameters. Fig. 3/a shows the generally accepted characteristic representing the effect of cutting speed on cutting temperature. This relationship is usually described using empirical formula

\[ T = C \nu^x, \]

where \( C \) and \( x \) are constants to be determined by measurements, the effect of other parameters being described by similar formulae.

Formula (1) has proven good in practice. It yields a result of proper accuracy in every case when the change in cutting speed remains within close limits. Even so, difficulties result from the fact that the value of exponent \( x \) may differ considerably in different measurements.

Figure 4 is a summing-up of the results of measurements by 13 research workers [4]. Distribution of the data set is illustrated by the curve in the

![Diagram](image)
middle. As seen, the set falls into two subsets depending on whether the tool used in the measurement was made of high-speed steel or hard metal. Separated for each tool material, the data show normal distribution and also the variation is considerably smaller. Remarkably, the mean value of \( x \) differs considerably, for the two types of material amounting to \( \bar{x} \approx 0.25 \) for hard metal and to \( \bar{x} \approx 0.40 \) for high-speed steel.

Before measurement, the equipment and thus also the tool are calibrated. In this way, the possibility of attributing the considerable difference in the value of \( \bar{x} \) to the difference in physical properties between both tool materials is excluded. As a sole explanation, the difference in average cutting speed achieved with the two materials offers itself. This means that \( x \) itself is also a function of cutting speed.

Actually, the cutting temperature changes according to different relationships in a wide range of cutting speed. According to Fig. 3/b, the deformation is considerable even at quite low cutting speeds, and thus also heating-up of the material is significant. Here let us remember the standard tensile test where the part of material experiencing heavy deformation gets heated. At low cutting speeds, the effect of the deformation rate is probably little, and therefore the cutting temperature changes here only slightly. The same results were published also by Zorev [5]. The speed range in question designated I in Fig. 3/b is investigated seldom; attention is attracted rather by continuously increasing cutting speeds.

So far we have dealt with range II mainly, to which formula (1), except for the remark concerning exponent \( x \), had been applicable. The central section of the characteristic shown in Fig. 3/b is in good agreement with the measurements of Makarov [6] who investigated the cutting temperature over a wide range of cutting speed. As shown in Fig. 5/a, the curve can be divided in two sections, and the values so determined for exponent \( x \) of formula (1) comply with the data of Fig. 4.

However, high-speed or even superhigh-speed cutting has attracted increasing interest recently. Under these conditions, the fact that the cutting temperature will increase with increasing deformation rates without, however, melting of the workpiece material, shall be taken into account. Partial melting of the material in the zone of flow on the face of the tool can be assumed, this being at the same time the upper limit of cutting temperature. This section of the theoretical function is illustrated by the range designated III in Fig. 3/b.
Fig. 5. Compliance of the empirical temperature function with the results of measurements a) for function (1) and b) for function (2)
Suggested new temperature function

Taking empirical formula (1) as a starting point, the generalization of the function will be carried out in two steps. Still dealing with section II of Fig. 3/b, let us first take only the transient thermal processes of intermittent cutting into consideration.

At the beginning of cutting, the cutting temperature increases as shown by curve a in Fig. 6, then, in case of intermittent cutting, cooling of the tool surface follows according to curve b. In milling, the rotating tool is efficiently cooled also by air while in case of a steady tool, the use of some coolant is assumed.

![Fig. 6. Cutting temperature in case of intermittent metal removal](image)

After cutting for time $t_1$, then cooling for time $t_2$, the tool will usually not assume its original thermal condition again, and therefore the subsequent temperature peaks designated $\vartheta_1, \vartheta_2$ will be continuously increasing. This is, however, limited according to experience and after a certain time the temperature peaks will become practically steady.

Let us approximate the temperature change in time by empirical functions

\[ \vartheta = \vartheta_{stac} \exp \left[ -\frac{\tau}{t} \right] \]  \hspace{1cm} (2)

upon warm-up and

\[ \vartheta = \pm \vartheta_{stac} \exp \left[ -\frac{t}{\tau'} \right] \]  \hspace{1cm} (3)

upon cooling,

$\vartheta_{st}, \tau$ and $\tau'$ being constants to be determined by measurement.

Temperature peak at the end of the first cutting period:

\[ \vartheta_1 = \vartheta_{stac} \exp \left[ -\frac{\tau}{t_1} \right] \]  \hspace{1cm} (4)
while after cooling, at the end of the entire cycle:

\[ g'_1 = g_1 \exp \left[ -\frac{t_2}{\tau'} \right]. \]  

(5)

At the end of the second cutting period, the temperature will be

\[ g_2 \approx g_1 + g_1 \left[ 1 + \exp \left( -\frac{t_2}{\tau'} \right) \right]. \]  

(6)

while after the n-th warm-up

\[ g_n \approx g_{n-1} \left[ 1 + \exp \left( -\frac{t_2}{\tau'} \right) \right] = \frac{g_1 \exp \left( -\frac{t_2 n}{\tau'} \right) - 1}{\exp \left( -\frac{t_2}{\tau'} \right) - 1}. \]  

(7)

Limit value of \( g_n \)

\[ g_\infty = \lim_{n \to \infty} g_n = g_{\text{stac}} \frac{\exp \left( -\frac{\tau}{\tau_1} \right)}{1 - \exp \left( -\frac{t_2}{\tau'} \right)}. \]  

(8)

In face milling

\[ t_1 = \frac{i}{v}, \quad t_2 = \frac{D\pi - i}{v}, \]  

(9a, b)

where \( D \) — diameter of milling cutter

\( i \) — machine arc length per revolution.

After all, the temperature after a sufficient number of revolutions of the milling cutter on the basis of (4), (8), and (9):

\[ g_\infty = C g \nu^x \frac{\exp \left( -\frac{\tau}{i} \right)}{1 - \exp \left( -\frac{D\pi - i}{v\tau'} \right)}. \]  

(10)

A remarkable feature of this function is its maximum at a cutting speed of \( v = v_{\text{max}} \), that can be determined from equation

\[ \frac{d g_\infty}{dv} = \left[ \frac{x}{v} - \frac{\tau}{i} \right] \left[ 1 - \exp \left( -\frac{D\pi - i}{v\tau'} \right) \right] + \]  

\[ + \frac{D\pi - i}{v^2\tau'} \exp \left( -\frac{D\pi - i}{v\tau'} \right) = 0 \]  

(11)
(10) and (11) can be reduced if we assume that
\[
\exp \left(-\frac{D\pi - i}{v}\right) \approx 0,
\]
(12)
as has been confirmed by experience in milling since, after all, temperature function
\[
\theta_x \approx \theta = C_g v^x \exp \left(-\frac{\tau}{i} \nu\right) = \theta_{\text{stac}} \exp \left(-\frac{\tau}{i} \nu\right)
\]
(13)
proved applicable. As has been demonstrated elsewhere, \( \tau = 0.00195 \text{s} \) was obtained for the milling of grade Ck 55 steel [7].

Function (13) reaches its maximum at cutting speed
\[
v_{\text{max}} = \frac{x}{\tau} i
\]
(14)
and this maximum can be calculated by means of relationship
\[
\theta_{\text{max}} \approx C_g v^x \exp (-x) = \theta_{\text{stac}} e^{-x}.
\]
(15)
The family of curves shown in Fig. 7/a illustrates function \( \theta(v, i = \text{const}) \) associated with different machining arc lengths \( i \). As compared with continuous cutting \( i = \infty \), the maximum values of temperature peaks developing in face milling are 2.71 times lower according to (15).

As to practical application, the shape of curve \( i - v \) of function (13), associated with constant temperature \( \theta = \text{const} \), is also of interest. After transposition
\[
i = \frac{v}{\ln \frac{\theta}{C_g}} \cdot \frac{\tau}{x},
\]
(16)
Remarkably, this function has a discontinuity in case of
\[
\ln v - \ln \left(\frac{\theta}{C_g}\right)^\frac{1}{x} = 0
\]
(17)
that means that the lower limit for cutting speed
\[
v = \left(\frac{\theta}{C_g}\right)^\frac{1}{x} = v_{\text{min}}
\]
(18)
is the value where temperature \( \theta \) can still develop, that is the asymptote of function \( v - i \). This is the case of continuous cutting as illustrated in Fig. 7/b.
Fig. 7. Change of temperature peaks as a function of a) cutting speed and b) machining arc length in face milling according to formula (15)
It can be confirmed that in the forward peak of the curve the cutting speed is exactly \(v_m\), and that

\[v_{\text{max}} = e v_{\text{min}} = \frac{\pi i}{\tau},\]  

(19)

where \(e\) — basic quantity of the natural logarithm. Arc length \(i_{gr}\) associated with the forward peak is also a limit value, no temperature \(\theta\) developing over an arc length shorter than \(i_{gr}\).

It follows simply that

\[i_{\text{min}} = \frac{\tau}{\pi} v_{\text{max}}.\]  

(20)

Returning to Fig. 3/b, a formula suited to describe range III will be increasingly needed. Function

\[\theta = \theta_s \exp \left[ -\frac{\alpha}{v^\beta} - \frac{\tau}{i} \right] \]  

(21)

or in a more general sense, function

\[\theta = \theta_s \exp \left[ -\frac{\alpha}{v^\beta} - \frac{\tau}{i} \right] = \theta_{\text{stac}} \exp \left[ -\frac{\tau}{t} \right] \]  

(21/a)

seems to be suited for this purpose,

where \(\theta_s\) — melting point of chip material in the zone of flow
\(\alpha\) and \(\beta\) — constants to be determined by the measurement of cutting temperature.

In respect of transient thermal processes, the behaviour of this function is similar to that of function (13). There exists a maximum in temperature, where, neglecting now calculations, at cutting speed

\[v_{\text{max}} = \left[ \frac{i \alpha \beta}{\tau} \right]^{\frac{1}{1+\beta}},\]  

(22)

\[\theta_{\text{max}} = \theta_s \exp \left[ -\frac{\alpha}{v_{\text{max}}^\beta} - \frac{\tau}{i} v_{\text{max}} \right].\]  

(23)

The temperature peak developing at different arc lengths \(i\) as a function of cutting speed is shown in Fig. 8/a.

Here function \(i - v\) at temperature \(\theta = \text{const}\)

\[i = \frac{v^{1+\beta} \tau}{v^\beta - v_{\text{min}}^\beta} A,\]  

(24)
Fig. 8. Change of temperature peaks as a function of a) cutting speed and b) machining arc length in face milling according to formula (21)
where

\[ A = \ln \frac{\theta_s}{\theta} \quad (25) \]

\[ v_{\min} = \left[ \frac{\alpha}{A} \right]^{\frac{1}{\beta}}. \quad (26) \]

Here also, the cutting speed associated with the forward peak of the curve is equal to \( v_m \), and

\[ v_{\max} = (1 + \beta)^{\frac{1}{\beta}} v_{\min}. \quad (27) \]

Neglecting calculations again, limit value \( i_{gr} \) of the machining arc length where a temperature \( \theta \) can still just develop

\[ i_{\min} = \frac{(1 + \beta)^{\frac{1}{\beta}}}{\beta} v_{\min} \tau. \quad (28) \]

Shown in Fig. 8/b are the \( i-v \) curves associated with temperature \( \theta = \text{const} \). It can be seen that also these curves are similar to function (13).

Function (21) applies to steady thermal state in the following shape:

\[ \theta = \theta_s \exp \left( -\frac{\alpha}{v^\beta} \right). \quad (29) \]

This is rather sophisticated as compared with (1) but it can still be simply handled by the use of up-to-date instruments of computer engineering. This function describes range III according to Fig. 3/b appropriately, and it can be adapted to cutting temperatures involved in higher cutting speeds used increasingly also in practice today.

The relation between the constants of functions (13) and (21), and/or (29) can be deduced from condition

\[ \frac{d\theta_{(13)}}{dv} = \frac{d\theta_{(21)}}{dv}. \quad (30) \]

Again neglecting details, we obtain that

\[ x = \frac{\alpha \beta}{v^\beta}, \quad (31) \]

which clearly expresses the feature of exponent \( x \) that it changes inversely as compared with \( v \). (Fig. 4).

Function (29) adapts itself sufficiently to the measurements summed up in Fig. 5 according to Fig. 5/b, and the exponent calculated from data \( \alpha, \beta \) valid in
Table 1

<table>
<thead>
<tr>
<th>Description</th>
<th>Function for range II</th>
<th>Function for range III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting temperature in transient state</td>
<td>( \theta = C_\theta v^x \exp \left( -\frac{\tau}{v} \right) )</td>
<td>( \theta = \beta \exp \left( -\frac{\tau}{v^\beta} \right) )</td>
</tr>
<tr>
<td>Cutting temperature in steady state</td>
<td>( \theta_{\text{max}} = C_\theta v^x )</td>
<td>( \theta_{\text{max}} = \beta \exp \left( -\frac{\tau}{v^\beta} \right) )</td>
</tr>
<tr>
<td>Maximum temperature</td>
<td>( \theta_{\text{max}} = \theta_{\text{max}} e^{-x} )</td>
<td>( \theta_{\text{max}} = \theta_{\text{max}} \exp \left( \frac{\beta}{(1 + \beta)\tau} \right) )</td>
</tr>
<tr>
<td>Cutting speed for max. temperature</td>
<td>( v_{\text{max}} = \frac{\tau}{x} )</td>
<td>( v_{\text{max}} = \left( \frac{\beta}{(1 + \beta)\tau} \right) )</td>
</tr>
<tr>
<td>Machining arc length as a function of cutting speed for ( \beta = \text{const} )</td>
<td>( i = \frac{v}{\ln v - \ln v_{\text{min}}} \cdot \frac{\tau}{x} )</td>
<td>( i = \frac{v^{1 + \beta}}{v^{\beta} - v_{\text{min}}^{\beta}} \cdot \frac{\tau}{\beta} )</td>
</tr>
<tr>
<td>Minimum cutting speed for ( \beta = \text{const} )</td>
<td>( v_{\text{min}} = \left[ \frac{\beta}{C_v} \right]^{-\frac{1}{\beta}} = \frac{v_m}{e} )</td>
<td>( v_{\text{min}} = \left( \frac{\beta}{A} \right) = \frac{v_m}{\left(1 + \beta\right)^{\frac{1}{\beta}}} )</td>
</tr>
<tr>
<td>Minimum machining arc</td>
<td>( i_{\text{min}} = \frac{v_{\text{max}}}{x} )</td>
<td>( i_{\text{min}} = \frac{(1 + \beta)^{1 + \beta}}{\beta} \cdot v_{\text{min}} \tau )</td>
</tr>
<tr>
<td>Relation between the two functions</td>
<td>( x = \frac{\beta}{v^\beta} )</td>
<td></td>
</tr>
</tbody>
</table>

the present case as a function of cutting speed is in good agreement with the values of \( x \) of the series of measurement divided in two sections in Fig. 5/a.

Table 1 facilitates a survey of the characteristics of both functions suggested for the cutting temperature.

The Salomon myth

Salomon carried out some high-speed milling tests in the 1920s. He found that function \( \theta - v \) had a maximum. Illustrated diagrammatically in Fig. 9 are what Salomon enclosed to the patent specification [8] summing up his
experiences and conclusions. According to the diagram, the thermal strength of the tool prevents the cutting speed to be increased only in range $v_a - v_b$ while in case of $v > v_b$, the tool can be used again.

This statement has been quoted by many research workers in the last 60 years [9—12] and in technical literature one can often find the diagram of Fig. 10. Then the majority of those referring to Salomon left out of consideration that it was milling that had been investigated originally, and the myth of the Salomon curve became prevailing in literature. This is especially true for the recent 10 to 20 years when high-speed cutting has become a matter of increasing interest. The Salomon curve suggests that, independently of the method of machining, similarly to the cutting force, also the cutting temperature will be reduced as the cutting speed increases. Although the validity of this theorem has been queried since [13] and neither theoretical nor experimental proof has been produced by anybody, even recent literature refers regularly to the theorem.

![Fig. 9. Cutting temperature as a function of cutting speed according to Salomon [8]](image)

![Fig. 10. Cutting temperature as a function of cutting speed in case of machining of different metals [11, 12]](image)
On the basis of the considerations outlined above as well as in the knowledge of the suggested formulae, Salomon's 60-year old experiences be easily explained for the temperature conditions of intermittent cutting. As suggested by the patent specification, Salomon's tests were undoubtedly based on milling, that means on intermittent cutting. In this case, the point in question is obviously the transient heat phenomenon taken as a basis for suggesting formula (13) and/or (21). Thus the measurements of Salomon are the experimental proof of these formulae.

If temperature $T_{gr}$ is the upper limit of the thermal stress to which the tool is exposed, then it is the zone indicated in Fig. 11 that should be avoided instead of the prohibited speed range $v_a - v_b$ given in Fig. 9.

Coat formation in face milling

As an application of the temperature functions suggested, the conditions contributing to non-metallic coat formation in face milling have been investigated.

In the last two decades, considerable interest has been excited by the observation that under favourable conditions, a coat may be produced on the tool surface by the oxide-silicate inclusions in steel, multiplying the service life of the tool (e.g. [14–16]). Such a coat-forming steel has been machined by means of face milling cutter in order to study coat formation under conditions of intermittent cutting [7].
The constants of temperature functions (13) and (21) are based on the measurement results of different authors. These constants are summed up in Table 2.

Arc length \( i = \infty \) was approximated by adjustment of \( i = 288 \) mm. It was found that under these conditions, \( v_i = 288 = 0.88 \) m/s is the limit cutting speed at which non-metallic coat formation takes place on the tool face. With this cutting speed and with the use of (18)

\[
v_{\text{min}} = \left[ \frac{\eta}{C} \right]^{\frac{1}{x}} = v_{i_0} \exp \left[ - \frac{\tau v}{i_0 - \chi} \right] = 0.88 \exp \left[ - \frac{0.00195 \cdot 0.88}{0.228 \cdot 0.19235} \right] = 0.856 \text{ m/s},
\]

or, with (21) and (26) reduced,

\[
v_{\text{min}} = \frac{\alpha}{A} = \left[ \frac{\alpha}{\frac{\alpha + \tau}{v_{i_0}} + \frac{\tau}{i_0}} \right]^{\frac{1}{\beta}} =
\]

\[
\left[ \frac{0.49186}{0.49186 + \frac{0.00195}{0.88}} \right]^{\frac{1}{0.51537}} = 0.861 \text{ m/s}.
\]

In the knowledge of \( v_{\text{min}} \), the limit speed for non-metallic coat formation as a function of \( i \) can be calculated by means of (16) and/or (24). The agreement of measured and calculated values was sufficient as shown in Fig. 12. Hence, in order to obtain an anti-wear coat formation on the tool surface, the range of operation shall be that to the right from the curves.
Cutting temperature is an important basic data of technological planning. In literature, empirical formulae are usually used for the cutting temperature changing as a function of cutting speed, applying a simple power function for this purpose. However, in a wide range of cutting speed, the exponent of the power function cannot be considered constant, moreover, the characteristic of this function is not satisfactory either. Therefore, a new function corresponding also to the recently used increasing values of cutting speed should expediently be introduced.

In case of intermittent metal removal, the surface layer of the tool is heating up, then cooling, periodically. As a result, cutting temperature increases first with increasing cutting speeds and then starts decreasing due to the short period of heating-up which are becoming shorter and shorter. This phenomenon recognized by Salomon experimentally must not be considered to be a general characteristic of high-speed cutting as it applies only to intermittent cutting. The temperature function constructed for intermittent cutting has been successfully used for the milling of non-metallic coat forming steels.

Fig. 12. Limit cutting speed for non-metallic coat formation in case of face milling ($s = 0.1$ mm/revolution, $a = 2.5$ mm, $\kappa = 70^\circ$, $\gamma = 6^\circ$, steel grade Ck 55, HB = 220, tool: P 35)
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