MEASUREMENT OF SCULPTURED SURFACES ON A THREE-DIMENSIONAL COORDINATE MEASURING MACHINE

K. SULYOK and T. SZALAY

Department of Production Engineering,
Technical University, H-1521 Budapest

Received June 11, 1985
Presented by Prof. Dr. M. Horváth

Abstract

The checking of sculptured surfaces is one of the most important tasks in the field of three-dimensional coordinate measurement. The measuring method developed by us is applicable to coordinate measuring machines. There is no need for special software and for the approximate description of the checked surfaces. This is an all-purpose computational approximation method. The subroutines for description and drawing are dependent on the measured surfaces.

Introduction and examination of the possibilities of measurement

Nowadays the most important task in production engineering is to increase efficiency. As computer-aided production comes into general use, the costs and production periods decrease. The steps leading to the establishment of an automated plant are: machine-tools with NC/CNC/DNC-controlled path-generator; manufacturing cells; adaptable manufacturing systems, computer-aided integrated production. The steps of this process are at the same time economical achievements, too. The number of necessary machines and technological processes, the personnel, the area, the production time decrease. At the same time, within the production time, the ratio of cutting time increases.

The modernization of production technology changes the measurement technology [1], and the traditional tasks of checking the machine parts are also extended. For example, sculptured surfaces can be mentioned, where with developing the production method, more cheking is required as well. The accuracy to shape and dimension of the finished product is influenced by the accuracy of both design and production. The factors to be considered are the following:

— the accuracy of the part and of the geometrical model describing the surfaces of the allowance form the degree of waviness permissible for the surface elements;
— the faults in the interpolation and approximation of controlled tool paths and processing lines;

— the condition of the tools and their wear during processing;
— faults resulting from the static, dynamic, and thermodynamic characteristics of the tools;
— faults resulting from the static, dynamic, and thermodynamic characteristics of the workpieces and of the machine-tool [2].

For the adoption of the method developed by the authors it is not necessary to know the production technology of the measured part, but it is necessary to know it in order to utilize the measured data.

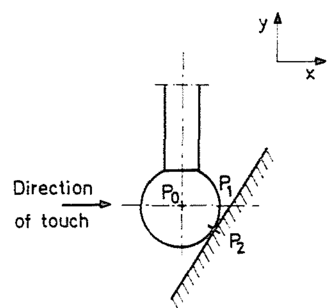

Fig. 1. Conditions of touching sculptured surfaces

To design and perform the measurements, the authors used a manually controlled DKM 1–300 DP three-dimensional coordinate measuring machine.

In a dynamic mode of operation, with the application of a three-dimensional probe head it is possible to solve the following tasks of measurement:
— distance measurement in the Cartesian coordinate system;
— indirect angular measurement;
— circle, cylinder, cone, and sphere;
— difference of shape (straightness, flatness, circularity, cylindricity);
— measuring the difference of position (parallelity, perpendicularity).

With the probe system integrated into the measuring machine, other forms, for example, sculptured surfaces can be measured as well, but in its basic set-up the measuring machine is not capable of processing the data. Unfortunately, with this probe system the surface normal of the measured point cannot be determined either. At the switching signal emitted at the moment of touch the spherical radius which lies in the direction of touching is to be added to the coordinate of the centre of the probe. In Figure 1 this corresponds to point P_1, though the surface of the workpiece was touched in point P_2. The actual tangent point, P_2, can naturally differ in all three of its coordinates both from the centre of the feeler (P_0) and from point P_1 which is in the direction of touch.

The problem arisen can be solved in two ways. On the one hand by developing a probe and measuring system capable of determining the normal of the tangent surface point, and then generating a correction from this, automatically, and on the other by developing a touching process with which the surface normal can be calculated.

Since the first solution is not feasible for the time being—at least for the authors—the second solution was attempted. Considering the characteristics of our measuring machines, the latter solution is reduced to the measurement of a point of (X, Y) coordinates of a given surface. The set of values Z belonging to the spread (X, Y) of the workpiece is known. The surface has to be touched with the probe sphere, so that the tangent point should be identical with the point to be measured, or should approximate it.

The real tangent point can be calculated from the radius of the probe sphere, the coordinates of its centre as well as the normal vector of the surface. The position and radius of the probe sphere are used by the measuring programs of the three-dimensional coordinate measuring machine, consequently they are known. Our method is suitable to determine the normal vector of the surface, outlined as follows.

Checking non-analytic surfaces

Planes, secondary surfaces are regarded as analytic surfaces. Sculptured surfaces—otherwise known as free-shaped, non-analytic surfaces—are characterized, among others, by the following:

— they cannot be handled analytically, or only with much difficulty;
— it is generally not possible to describe them completely;
— the surface elements and their limiter curves can generally be handled with methods of approximation [2].

When touching the sculptured surfaces, the actual tangent point cannot directly be determined, but it is enough to know the so-called equidistant surface generated from the centres of the probe sphere. It is easy to see that the normal vectors of the real and equidistant surface (Fig. 2) coincide.

Let us consider a surface element having 9 points. The 9 points determine 3 curves, each both in plane X, Z and in plane Y, Z; these are the curves of the surface to be examined. The tangent of the central point of the curves is regarded as parallel to the chord connecting the marginal points. With this approximation 6 tangent vectors can be obtained (Fig. 3).

Their direction vectors in plane X, Z are the following:

$$(X_3 - X_1, Z_{3,j} - Z_{1,j}) \qquad j = 1, 2, 3 .$$

In plane Y, Z:

$$(Y_3 - Y_1, Z_{i,3} - Z_{i,1}) \qquad i = 1, 2, 3 .$$

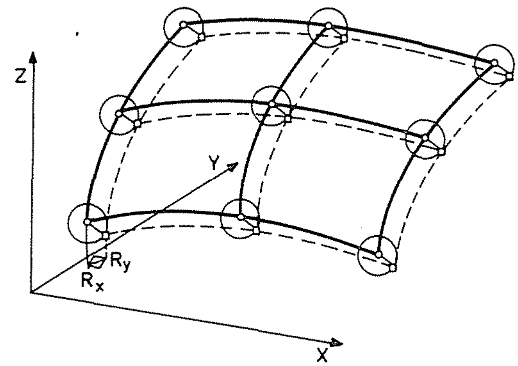

Fig. 2. Generating an equidistant surface element

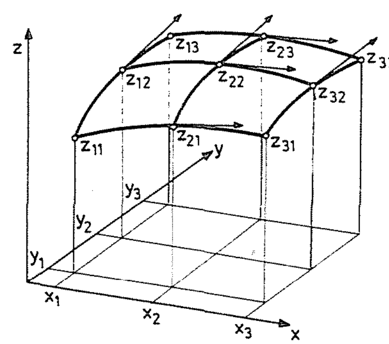

Fig. 3. The limiter curves of a surface element and the components of a normal vector

Let us generate the vectorial series of the tangent vectors starting from point Z_{22} and the vectorial series of two tangents each lying nearest to and farthest from the origin. In this way three normal vectors can be obtained. Let us regard the weighted average of the three vectors as the normal vector of the surface. As the normal vector of point Z_{22} was to be determined, the normal vector belonging to this point had to be weighted. When calculating the weighted average, there are two more vectors, therefore the normal vector of point Z_{22} was considered twice.

The normal vector can be calculated from the above tangents as follows:

$$n_x=(Y_3-Y_1)\cdot(Z_{1,j}-Z_{3,j})$$

$$n_y=(X_3-X_1)\cdot(Z_{i,1}-Z_{i,3})$$

$$n_z=(X_3-X_1)\cdot(Y_3-Y_1)$$

During calculation all tangent and normal vectors have to be normed to unit length.

Determining the distance between the measured points and the coordinates of the real tangent point

When the normal vector is known, the coordinates of the real tangent point can be determined. The calculated normal vector of the surface is (n_x, n_y, n_z) in point $(\tilde{X}, \tilde{Y}, \tilde{Z})$, while the measured values of the centre of the probe sphere are (X, Y, Z).

The coordinates of the real tangent point, with radius R of the probe sphere, are the following:

$$\tilde{X} = X - R \cdot n_x$$
$$\tilde{Y} = Y - R \cdot n_y$$
$$\tilde{Z} = Z - R \cdot n_z \,.$$

The normal vector always points outwards from the surface, consequently the correction is also performed in the suitable direction.

During the procedure a secant plane of the surface was substituted for the surface on a small surface element. In this way the order of magnitude of the approximation is subordinately small [3]. For us this means that with an 0.01 mm permissible fault, the distance of the base points (P_{ij}) cannot be bigger than 0.1 mm.

Checking coordinate Z of the surface belonging to a place of (X, Y) coordinates

Let us set the task of measuring a point of (X, Y) coordinates of a given surface. The slides of the three-dimensional coordinate measuring machine cannot be positioned so that the probe sphere could touch the surface at the required point (Fig. 1). This position cannot be determined, directly therefore it is advisable to apply the method of successive approximation.

Let the starting position of the measuring machine be the given point (X, Y). After touching the surface, the real tangent point is calculated with the abovementioned method. The table of the measuring machine is shifted from the obtained tangent point (X_1, Y_1) with the direction vector pointing to the required point (X, Y).

The direction vector: $(X - X_1, Y - Y_1)$.

During shifting it was supposed that the normal vectors of position (X, Y) and (X_1, Y_1) coincided. This condition does generally not come true, so the real tangent point does not get into position (X, Y) after being shifted either. If the procedure is continued with the direction vector of the resulting deviation—$(X - X_{\bar{1}}, Y - Y_i)$—the tangent point converges on position (X, Y).

Analyzing the approximation method

The proof of the convergence is based on the following: the deviation from position (X, Y) decreases step by step. This means that the series of deviations represents a monotonic decreasing sequence limited from below. A sequence like this is always convergent and converges on the lower limit. The limit in our case is the 0 deviation from (X, Y). To meet the requirements of convergence, the surface has to fulfil at least one of the following three conditions:

— concave surface, its radius of curvature has to be longer than the diameter of the probe sphere;

— convex surface;

— if the surface has a point of inflexion, it should fulfil either of the above-mentioned conditions.

The proposition can be proved in two steps. First the decrease of deviations in case of curves is proved, then it is generalized for a three-dimensional case.

The two-dimensional proof is illustrated in Figs 4 and 5. The convergence is the most evident in case of a *convex* curve. According to the symbols of Fig. 4 n_i is the normal vector of the first step; E_0 is the required point: n_0 is the normal vector belonging to the required point.

Let us shift n_i to point E_0! The curve is convex, therefore the vector obtained lies between n_i and n_0. The base point (E_{i+1}) of the perpendicular drawn from the end point of n_i'' to the curve, lies between E_0 and E_i, owing to convexity. This means that the deviation of point E_{i+1} from E_0 is smaller than previously that of E_i. So the deviations form a monotonic decreasing sequence.

In case of a *concave* curve the approximation method is applied. Let us substitute its osculating circle for the curve—since a very small environment of a point is examined. The proof is illustrated in Fig. 5, whose symbols follow those of Fig. 4. It is obvious that the end point of n_i'' obtained from n_i and shifted to E_0 after the first step is here outside points E_i' and E_0'. In spite of this fact, the base point (E_{i+1}) of the perpendicular drawn from this point to the osculating circle is nearer than point E_i, as the deviation of E_{i+1}' from position X_0 is identical with that of E_i, because normal vector n_i has been shifted in a parallel way. The deviation of E_{i+1} from position X_0 is smaller than the distance of point E_{i+1}'. This results from the fact that the radius of the osculating circle is longer than the diameter of the probe sphere. For this reason the distance of 0 from position X_0 is bigger than that of E_{i+1}. Points 0, E_{i+1}' and E_{i+1} are on the same straight line, consequently this proposition follows from their relative positions. All this means that the distance of E_{i+1} from position X_0 is smaller than that of E_i. In the first step the deviation has decreased.

In the second step let us shift normal vector n_{i+1} to E_0; vector n_{i+1}'' is obtained in this way. The end point of n_{i+1}'' is between E_0' and E_i'. The base

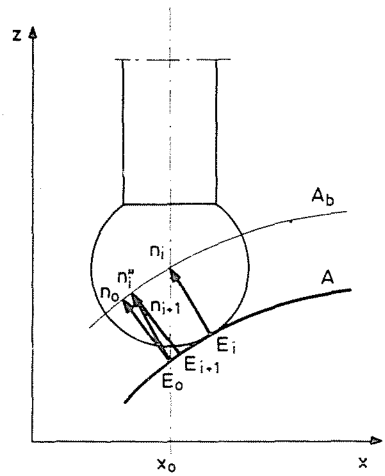

Fig. 4. Approximating the normal vector of a convex curve

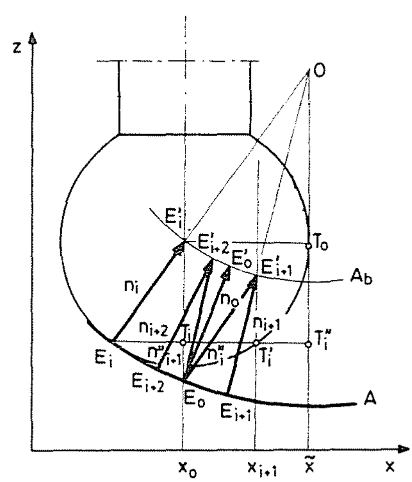

Fig. 5. Approximating the normal vector of a concave curve

point (E_{i+2}) of the perpendicular drawn from this point to the osculating circle is between E_i and E_0. If this condition should not be fulfilled, the condition that the radius of curvature is longer than the diameter of the probe sphere would be contradicted. If the base point does not lie between E_0 and E_i, vector n_{i+2} would have to intersect either n_i or n_0. This would mean that the radius of curvature is shorter than the diameter of the probe sphere; and that would contradict our initial assumption.

From this it is evident that the deviation of E_{i+2} from X_0 is smaller than that of E_i. If we go on with the successive approximation, it becomes clear that, the deviation of E_{i+3} from position X_0 is smaller than that of E_{i+2}. Therefore sequence E_{i+2k} and sequence E_{i+2k-1} are monotonic decreasing distance sequences. Their lower limit is identical, the limit value is E_0. In case of a curve with a point of inflexion, it depends on the position of the measured point which of the two abovementioned cases realizes [3].

The generalization for three dimensions will not be set forth in detail either. The prevailing idea is identical with the one outlined for the curves. Here, considering cone surfaces instead of triangles formed by normal vectors, discs instead of the intervals of deviations, the above ideas can be followed precisely. It must also be remarked that when examining a concave surface, the osculating sphere of the concavity is used during proof.

The number of the steps of iteration is determined by the required accuracy of measurement. When the deviation from position (X, Y) reaches the accuracy of measurement, the rteration can be stopped.

Measuring a turbine blade on a three-dimensional coordinate measuring machine

When using the three-dimensional coordinate measuring machine—considered a milestone in measurement engineering—the demand has arisen to make it suitable for more complex measurements than before, e.g., to measure die-casting machines and stamping dies for processing workpieces of intricate shape as well as turbine blades [4].

To try the touching method outlined above in practice, a steam-turbine blade was chosen the profile points of whics were given in coordinates (X, Y). The turbine blade with a cross section constant along its length, is manufactured in the LÁNG Machine Factory.

The blade was fastened to the table of the measuring machine in a vertical position. The whole measuring equipment—seen in Fig. 6—consists of the following sets:

— DKM 1–300 DP measuring machine;
— EMG-666 desk computer;
— interface to connect the measuring machine to the computer;
— $X-Y$ plotter;
— interface to connect the computer to the plotter.

The authors used a probe with its spherical end fixed in the three-dimensional probe head in directions $-X$ and $+X$. The surface of the turbine blade was uniformly smooth which made it possible to use the smallest possible probe sphere—in this case $R=0.75$ mm.

The 9 points giving the surface element were taken as the stationary points of a 0.1 mm square net. The order of measuring the points can be seen in

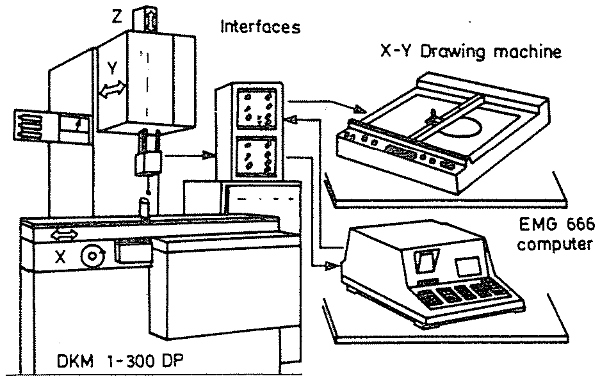

Fig. 6. A measuring system for the measurement of non-analytic surfaces

Fig. 7a. Let us try to find the normal vector of the surface in point 5. The coordinates of the probe sphere during feeling are stored in the computer. All data that may become necessary in the future (the radius of the probe sphere, the direction of touching the accuracy of iteration) can be asked by the program of approximation in an interactive way.

If the distance between the required point and the point obtained as a result of approximation is bigger than the accuracy of iteration, the new base points are printed out. The measurement has to be repeated with the new base points. If the distance between the required point and the obtained one is smaller than the accuracy of iteration, the end of the iteration is indicated by the program and the most accurate value of the approximated point is printed out. It is in this case that the theoretical coordinates are best approximated by the real tangent point.

The previous data can be used to measure the following point, in this way only three more points have to be measured. This is illustrated in Fig. 7b where the normal vector of point 0 is searched for.

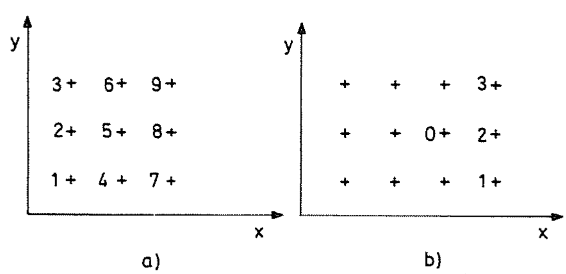

Fig. 7. Order of touching for the computational processing of approximation

Graphic analysis of the measured data

The measurements have been carried out on a turbine blade with constant cross section, to be seen in Fig. 8. The coordinates of the 25 points of the profile and the data of circular arcs connecting the pairs of points are known. The constant cross section would have made it possible to solve the problem as a two-dimensional case. Because of the possible errors of clamping and those of the surface, however, the three-dimensional approximation was

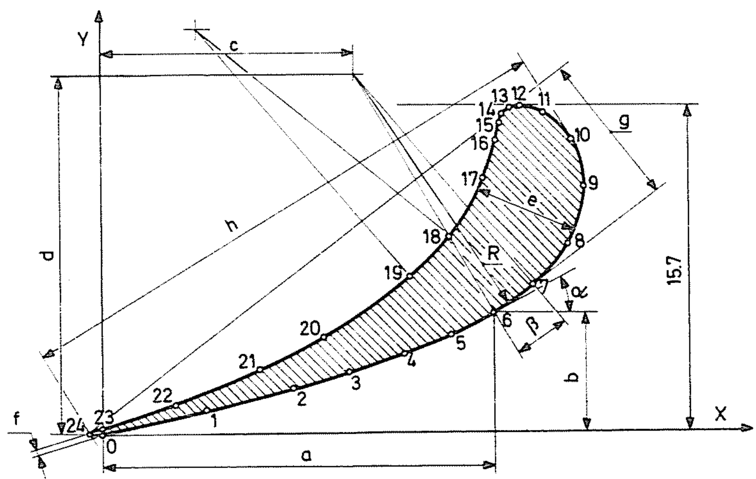

Fig. 8. The cross section of the checked turbine blade

chosen for the first time. But the first measurements unambiguously showed that components Z of the normal vectors are negligible. The subsequent measurements were performed considering this fact. The points indicated in Fig. 8 were regarded as the first step of iteration. Point 24 cannot be checked with this method. At the beginning of iteration the blade surface was not touched by the probe sphere at the required point at all. Using the tangent points corrected with calculations, three or four steps are necessary to achieve the appropriate approximation. In this case the authors managed to approximate 21 points with an accuracy of 2 μm, two points with an accuracy of 3 μm, one point with an accuracy of 4 μm. When the profile was being touched with the probe head moving in direction X, coordinate Y of the tangent point corrected with calculations was considered the nominal position. Obviously, when touching in direction Y, value X was used.

The measured data were also displayed on an $X-Y$ plotter joined to the desk computer. Figure 9 shows the geometrical tolerance range of the given

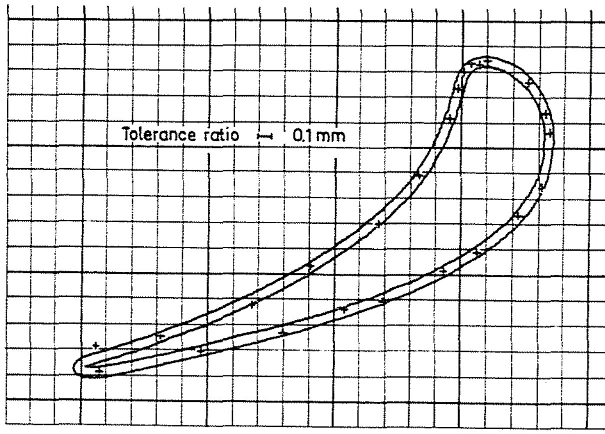

Fig. 9. The tolerance range of the turbine blade and the measured data

profile according to the specifications of standard MSZ KGST 301-76. The curves of the tolerance range limits were plotted with the maximum (250) number of points allowed in this case by the storage capacity of the computer.

Although the deviations of the measured points can be seen well in the figure, the numerical qualification of the product is not very likely to succeed in this way. To this it seems necessary to elaborate quality degrees for curves and surfaces that cannot be described analytically and to establish the hierarchy of accuracy.

Summary

The results proved that the three-dimensional coordinate measuring machines without special measuring program can also be used for checking intricate surfaces. The outlined method has proved to be most valuable because it directly checks the surface to be measured. No approximation concerning the description of the surface was used. The authors tried to check the points of the surface to be measured that had been given by the designer as characteristic coordinates (and coordinates to be measured).

In spite of the multi-step iteration, the approximation of the required tangent points can be performed quickly. Because of the hand-controlled slide, most of the measuring time is spent on touching the workpiece anyway.

The measuring system consisted of mass-production equipment. It was only the interface to connect the three-dimensional coordinate measuring machine to the desk computer that had to be prepared by the authors.

The computer iteration program used during the measurements is an all-purpose program, the subroutines for the description of the curve and those for the drawing are special and always have to be prepared in accordance with the aim of the measurement.

References

1. PETERS, J.: Annals of the CIRP *26/2*, 415–421 (1977)
2. ENDRŐDY, T.: Szoborfelület-tervezési és megmunkálási rendszerek összehasonlító elemzése (Comparative Analysis of Systems to Design and Manufacture Sculptured Surfaces). Dr. techn. dissertation 85–86 (1981)
3. SZALAY, T.: Nem analitikus (szobor-) felületek mérése koordináta-méréstechnikai módszerekkel (Measurement of Sculptured Surfaces with Coordinate Measuring Methods) Diploma work, Technical University of Budapest 1983.
4. WECKENMANN, A.:–GOCH, G.: VDI-Z. 120 *21*, 992–996 (1978)

Károly SULYOK
Tibor SZALAY
H-1521 Budapest