A REMARK CONCERNING SURFACE TYPOLOGY

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Abstract

In this paper a formula to a probability density function of metal surfaces is given.

Introduction

There are many real parameters defined concerning metal surfaces to give some quantitative information on the smoothness and other properties of the surface.

Whitehouse [1] has introduced the idea to compute the cuttings of a line segment into the surface, which line segment is parallel to the main plane of the surface. He conjectured that the number of cutting in depth h is proportional to the value of an applicable beta probability density function.

In this paper the number of such cuttings—using only the depth distribution function of the surface—is computed with mathematical methods.

The result

Drawing a set of parallel lines to the mean line of the surface, (Fig. 1) the crossings of each line with the surface can be calculated. This occurs so, that an interval of the corresponding line lies "in the air", the subsequent one in the metal, and so on.

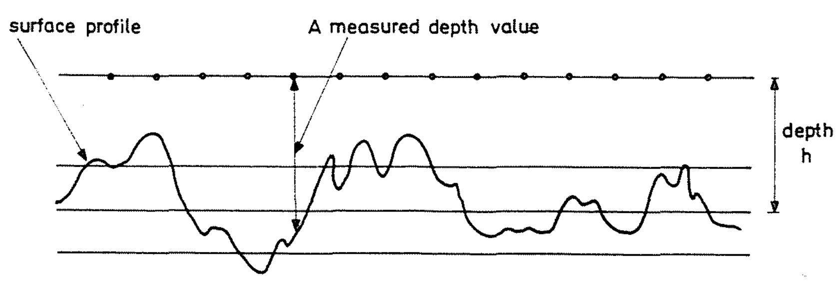

Fig. 1

On the figure, dots are indicating depth measuring places on the surface. The line in depth h has four complete crossings with the surface, i.e. has four subintervals lying in the air, and four in the metal. We can assume that the profile depth has the same probability distribution at each measured point, and the depths are totally independent.

Now, any interval's length can be measured with the number of measurements "above" the corresponding interval. By the independence of the depth in the measured points, a well-known idea of probability theory implies that the now-defined length of an interval has Poisson distribution. This means that

$$P(\text{interval length} = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

for some fixed parameter $\lambda > 0$, where $P(\cdot)$ denotes probability.

Clearly, this λ is a function of the depth h (in larger depth, one waits a larger λ in the metal intervals) Denoting the depth distribution function by F, we can compute this λ parameter for metal and air, too.

In the case of air,

$$P(\text{interval length } h = 0) = P(\text{depth at this point } > h) = 1 - F(h)$$

and by the definition of the Poisson distribution

$$P(\text{interval length} = 0) = e^{-\lambda a} = 1 - F(h),$$

$$\lambda_a = -\ln(1 - F(h)) \qquad (1)$$

In case of metal,

$$P(\text{interval length} = 0) = P(\text{depth at this point } < h) = F(h),$$

so,

$$\lambda_m = -\ln(F(h)). \qquad (2)$$

Clearly, the lengths of the subsequent intervals are independent.

Using the well-known fact that the convolution of two Poisson distributions is also Poisson with the sum of the two parameters, the length distribution of the union of two subsequent intervals is Poisson with parameter $\lambda_a + \lambda_m$.

Using this fact, the mean value of such pairs of cuttings can be computed. The reneval theorem [2] says that

$$P\left(\lim_{d \to \infty} \frac{d}{C_d} = \lambda_a + \lambda_m\right) = 1,$$

where C_d denotes the number of crossings of an interval with length d.

This implies that for relatively large d,

$$C_d \approx \frac{d}{\lambda_a + \lambda_m}$$

So, using (1) and (2), the number of crossings in depth h is

$$\frac{d}{-\ln(F(h)) - \ln(1 - F(h))}.$$

Remark

In a further paper concerning this subject—using experimental data—further considerations will be discussed, in the hope, that these investigations shall have practical significance.

Acknowledgement

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References

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