INVESTIGATION OF THE HELMHOLTZ RESONATOR CONSIDERED AS A NON-ACOUSTIC OSCILLATING SYSTEM

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Abstract

In this paper the validity limits for a pulsating combustion unit have been determined, which has been designed for preheating diesel engines. An interesting phenomenon has been observed, namely the same pulsating frequency could be measured at two different resonator tube lengths. The paper presents an investigation of the basic causes of the frequency deviation of a pulsating combustion system built as a Helmholtz resonator and presents a frequency calculation for real systems. Furthermore, based on detailed investigation, it tries to explain the observed phenomenon.

Introduction

Processes in practical installations are often accompanied by oscillation phenomena. Generally these phenomena are undesirable in conventional combustion equipment, like pulsation in the combustion chambers of gas turbines as well as in the piping system of reciprocating compressors. Especially fast pressure variation may seriously increase the mechanical stresses on several structural components and may have an undesirable effect on the operation. But, at the same time there are equipment developed for using the pulsation of gas flow as a desired process to increase the heat transfer coefficient. Otherwise the pulsating flow may increase the volumetric efficiency [4, 9, 13].

At the Department of Heat Engines (TU. Budapest) a small size preheater unit has been developed for diesel engines which operates on the principle of pulsating combustion. It consists of a combustion chamber, operated by a mechanical valve and a tube for flue gas both being cooled by water-flow. In this way it operates as a heat exchanger. An interesting phenomenon has been observed, namely, the same pulsating frequency could be measured at two different tube lengths. There is a tube length range at which pulsating combustion could not be sustained [5, 6]. The paper presents an investigation of the basic causes of the frequency deviation of a pulsating combustion system built as a Helmholtz resonator.

The frequency of gas oscillation developing in pipes and small closed volumes mentioned in literature, were generally derived from the laws of acoustics. The results of measurements corresponded well with theory, up to the values of $\Delta p/p_0 \approx 0.2$ [8]. Gas oscillations developing in systems comprising a small closed volume connected with a tube can be treated as a Helmholtz resonator. The Helmholtz resonator is well known in acoustic as applied to sound analysis and noise reduction. The conditions for application are a relatively small amplitude of pressure oscillation and the characteristical geometric dimension should be smaller than 1/4th of the wave length. In pulsating combustors, however, the amplitude of pressure fluctuations may reach a value of $\Delta p/p_0 = 0.6$. The existence of gas oscillation is provided by heat input and release, respectively, occurring in phase with the pressure oscillation (the Rayleigh criteria) [7]. Consequently, the gas stream is inhomogeneous within the equipment, its temperature and density depend on location and time. Therefore an oscillating system like this cannot be regarded as an acoustic one any more.

In spite of this, in the relevant literature, the frequency of gas oscillation — developing in all Helmholtz resonator type pulsating combustors — is determined from the Helmholtz resonator frequency formula, known from acoustics, without consideration of the restriction factors of validity. The 10 to 30% deviation from the measured values is simply taken into account by a multiplication factor [4]. Because of the problem described above it was necessary to examine the relationship between gas oscillations and dimensions of the equipment, the conditions of the existence of gas oscillation, as well as the limits for application of the Helmholtz resonator analogy.

The mechanism of oscillations developing in a pulsating combustor

All kinds of oscillating systems consist of an exciter and an excited system. Undamped oscillation can only develop if the frequency of the exciter is approximately equal to the frequency of the excited system (resonator) and, on the other hand if the rate of energy — supplied to the system at adequate phases — exactly covers its losses.

The block diagram of an oscillating system of such design is shown in *Fig. 1.*

The resonator can be excited in several ways, namely:

- — by pressure fluctuations, due to eddies separating from the air stream [2];
- — by pressure fluctuations occurring in the air and fuel supply-system;
- — by oscillations developing in the flames [7].

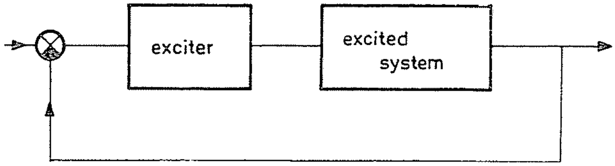

Fig. 1. Block diagram of a pulsating combustor

Pressure fluctuation $p(t)$, caused by excitation results in the fluctuation of the volume flow $V(t)$ on the fuel/air mixture. This causes certain delayed pressure fluctuation in the flue gas, and so signal intensification occurs in the exciter.

In control theory, according to Nyquist, the condition for oscillation of constant amplitude — characterized by the above-mentioned block diagram — can be written as follows:

$$Y_1(i\omega) \cdot Y_2(i\omega) + 1 = 0 \tag{1}$$

The circuit is able to oscillate if the condition of Eq. (1) is satisfied and the sum of phase-delays is 180° in all units.

Modelling of oscillating systems

The modelling of a simple, closed oscillating system, free from energy dissipation, can be carried out by the mathematical as well as by the analogue method. In case of small oscillations the applied forces are proportional to amplitudes, hence the motion of the oscillating system can be described by the following equation:

$$m \frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + kx = 0 \tag{2}$$

For parts of an analogous model of the oscillating system, elements should be applied in which potential and kinetic energy are well separated from one another. Such a system is represented by e.g. a mass suspended to a spring, the oscillating circuit consisting of a capacitor and an inductor, as well as the Helmholtz resonator.

In the latter one, the frequency of oscillation is:

$$f = \frac{a}{2\pi} \sqrt{\frac{A}{VL}} \tag{3}$$

If the damping force is taken into account which is always present in oscillating systems, Eq. (2) is modified as follows:

$$m\frac{d^2x}{dt^2}+r\frac{dx}{dt}+kx=0 \tag{4}$$

Consequently, in an oscillating system, built as a Helmholtz-type resonator, the differential equation describing the pressure fluctuations is represented by:

$$T_2^2\frac{d^2}{dt^2}\Delta p+T_1\frac{d}{dt}\Delta p+\Delta p=0 \tag{5}$$

Completing Eq. (5) by periodical heat input — governed by means of pressure fluctuations — it results:

$$T_2^2\frac{d^2}{dt^2}\Delta p+T_1\frac{d}{dt}\Delta p+\Delta p=q(t) \tag{6}$$

The analogue computer model of this equation is shown in Fig. 2, [1]. Model of a pulsating combustor obtained by an analogue computer.

The exciter is governed by the excited system. According to the Rayleigh criteria the heat release has to be in phase with the pressure oscillation or mathematically

when $p>p_0$ and $dp/dt>0$ then $\Delta q>0$

and

when $p<p_0$ and $dp/dt<0$ then $\Delta q=0$ (7)

conditions must be fulfilled.

The output of the excited system $p(t)$ governs a comparator which compares the values of the momentary pressure and the mean value of pressure oscillation p_0 and according to Eq. (7), it switches the value of Δq or 0. The

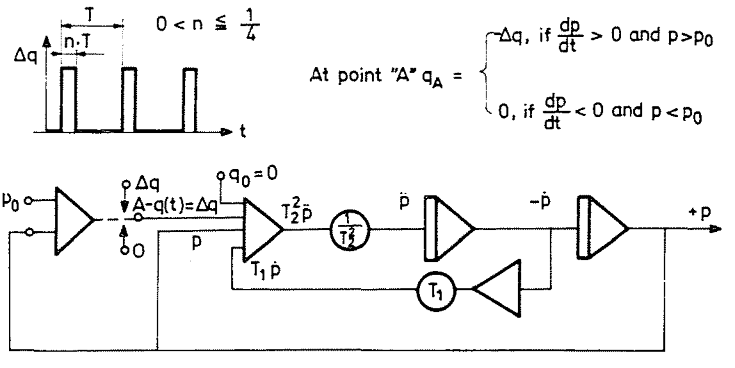

Fig. 2. The analogue computer model

duration and quantity of heat release are set by the height and width of square impulses. The output sign of the comparator is transfered to the summing element. At the input of the summing element the sum of the input values are equal with the highest derivative as it can be derived from Eq. (6).

$$T_2^2\ddot{p}=q(t)-T_1\dot{p}-p=-(-q(t)+T_1\dot{p}+p)$$

All summing elements and integrators change the sign of input values as seen in Fig. 2.

The description of the phenomenon by linear differential equation, is approximative. The magnitude of errors caused by neglections should be examined by experimental measurements in each case. The wave equation of acoustics can be written on the basis of Euler's equation, the continuity and the energy theorem by application of which the friction of the medium as well as convective acceleration have been neglected and the pressure and density fluctuations have been assumed to be "small". The so-called d'Alembert-solution of the differential equation describes the pressure and velocity fluctuations, respectively, while Bernoulli's solution gives the frequency of oscillation [9].

If pressure and density variations are considerably high, a partial differential equation system results [3] where the solution might be obtained graphically by a characteristic method and numerically by the aid of a computer. Accuracy and applicability of the solutions depend upon the ways how heat release in the oscillating process is taken into consideration. This fact also supports the significance of experimental work. This is the reason why gas oscillations developing in pulsating combustors have been studied mainly by experiments and the scientists working in this special field were satisfied mainly with approximations obtained by linear differential equations. Nevertheless, the validity range for pulsating combustors of Helmholtz resonator design has not been determined yet.

Experimental equipment and results

The layout and instrumentation of the experimental equipment is illustrated in *Fig. 3*. For measuring the pressure oscillation, two piezoquartz pressure-sensors, type AVL 8 QP 500, have been built into the system (eigenfrequency: 100 kHz). The first sensor was built directly in the vicinity of the combustion chamber, to the starting end of the resonator tube, while the second one was placed near to its open end. The running-down of the pressure wave and the mean velocity of sound, prevailing at different performance stages have been determined by the sensors'. The measurements were carried out at $\lambda=0.95=\text{const}$, in order to keep the value of the velocity of combustion constant, too.

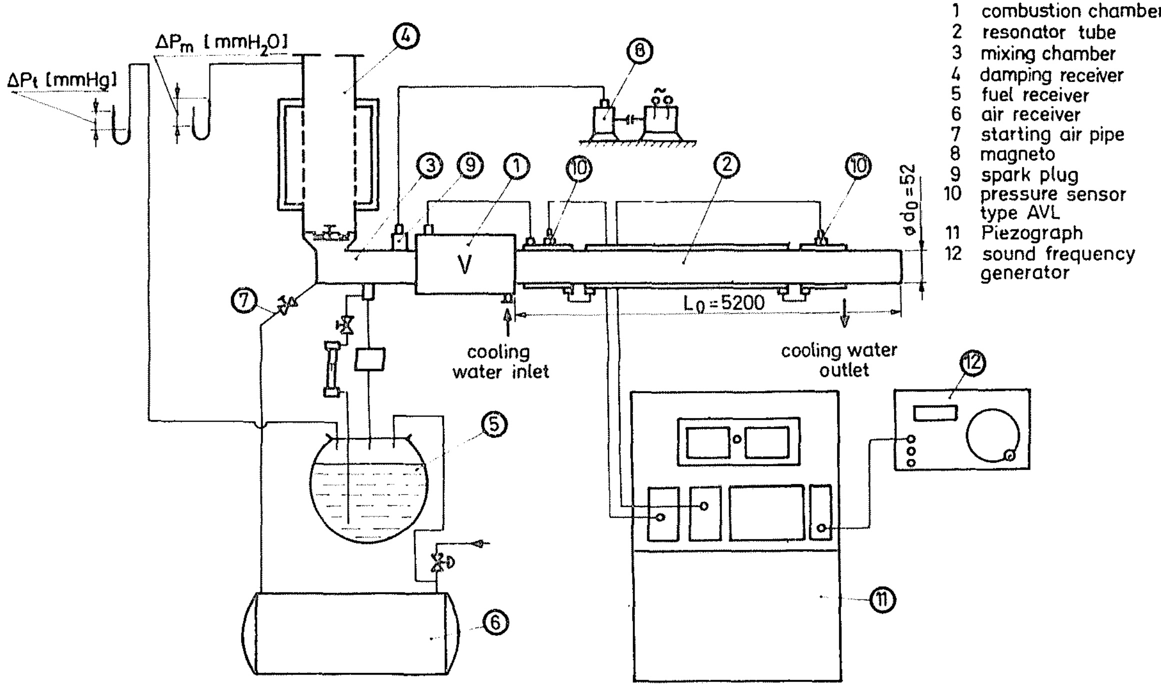

Fig. 3. Scheme of experimental equipment — layout and instrumentation

The influence of length variation of the resonator tube on the operation of a pulsating combustor and on the frequency of the oscillating system has been examined for three different volumes of the combustion chamber: $V_1 = 644\ \text{cm}^3$, $V_2 = 832\ \text{cm}^3$, $V_3 = 1008\ \text{cm}^3$. The result is shown in *Fig. 4.*

Besides frequency curve *1*, obtained by measurements, the curve of frequency *2* of the Helmholtz resonator, calculated by the value of mean sound velocity, measured in the resonator of given tube length and *3* of a tube closed

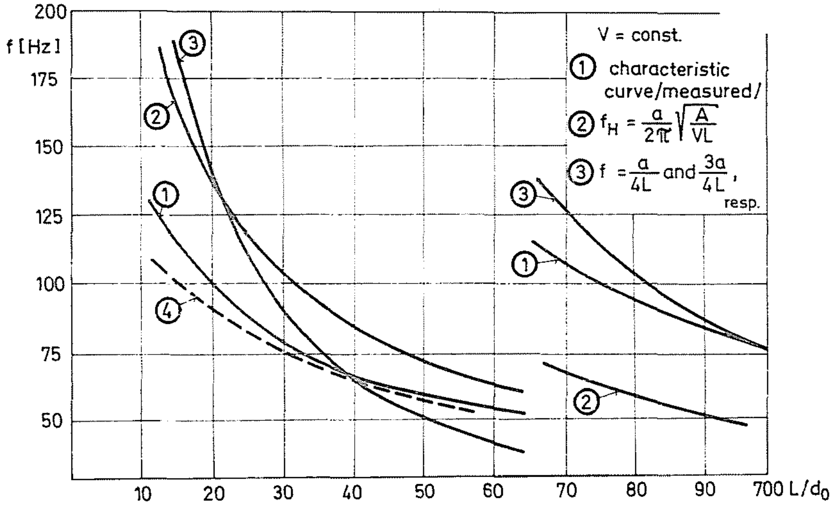

Fig. 4. Influence of resonator tube length on frequency of the oscillating system

at one end, has been plotted, too. It can be seen that the measured curve is not continuous, within the range of $L/d_0 > 70$ it smoothly approaches frequency curve *3* of the tube closed at one end, but keeps above the frequency curve of the Helmholtz resonator all along. Within the range $L/d_0 < 70$, however, it approaches the frequency curve of the Helmholtz resonator and with the increase of the tube length the frequency decreases, while the convergence of the two curves increases. With the value of $V/A = \text{const}$, according to our experiments the frequency formation in different equipment will be about the same. See curve *4* in Fig. 4.

By graphical evaluation of the measured and theoretical curve, introducing the ratio:

$$\Omega = f_{\text{measured}} / f_{\text{theoretical}}$$

it can be determined that the analogy of Helmholtz's resonator applies to $AL/V < 5.0$ as seen from Fig. 5. In the range of $L/d_0 < 70$ the measured deviation of

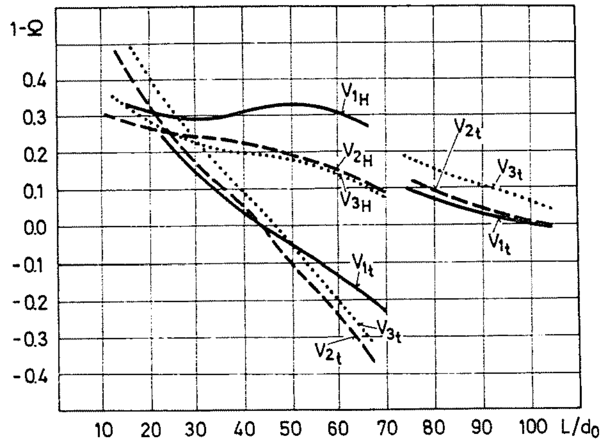

Fig. 5. Deviation of measured and calculated frequency curves

the theoretically determined frequency with three different combustion chamber volumes considered as a Helmholtz resonator the oscillating system is

$$0.08<(1-\Omega)<0.3$$

but considering the oscillating system as a tube closed at one end, the deviation from the measured frequency to the theoretically determined frequency is:

$$-0.4<(1-\Omega)<0.4$$

which is more then three times higher than in the previous case.

It can be seen from Fig. 5 that in the case of $AL/V>5.0$ the dimensions of the combustion chamber volume are so small compared to the resonator tube that the eigenoscillation of the oscillating system turns into eigenoscillation of the tube closed at one end. If $L/d_0 \to 100$, then the value of $(1-\Omega)\to 0$ relations valid if the volume ot the combustion chamber is V_1 and V_2, and in the case of V_3 the deviation of the above relation is less than 0.2. In the range of $L/d_0>70$ the equipment can run only by an increased mass flow of the fuel/air mixture, similar to the phenomenon observed by wind-instruments, where at a given geometry the pitch of the tone depends on the intensity of blowing.

Proving the presence of standing waves, it was confirmed that with a combustion chamber of given volume V, the increase of the resonator tube length leads to an alteration of the type of oscillation and, on the other hand, the extent of the losses in the oscillating system could not be very high [8].

According to our investigations on the experimental pulsating combustor, we have stated that the value of the damping factor depends on the volume of the combustion chamber and the length of the resonator tube. After detailed analysis of these parameters the following equation was obtained for the frequency of the oscillating system examined:

$$f=\frac{1}{2\pi}\sqrt{a^2\cdot\left(\frac{A}{LV}\right)-\left(\frac{R_0}{L^{p+1}V^c}\right)^2\cdot\left(\frac{1}{2\rho A}\right)^2} \tag{8}$$

The measurement values for the equipment were:

$$R_0=253, \qquad b=1.4-1.7, \qquad c=1.0$$

which are valid for pulsating combustion systems with a geometry of $\frac{AL}{V}<5.0$ and with a pressure ratio $\Delta p/p_0<0.4$.

According to measurements, the laws of acoustic can be applied up to the value of pressure amplitude $\Delta p/p_0<0.4$ (error: $<8\%$) satisfactory for determining the frequency of gas oscillations developing in pulsating combustors. With a growing frequency, damping increases in the oscillating system. The energy—necessarily introduced to keep pressure oscillation at constant amplitude—will increase, too.

The frequency of the oscillating system, however, decreases:

$$f=\sqrt{f_0^2-(K\cdot r)^2} \tag{9}$$

where f_0 = the eigenfrequency of the undamped oscillating system (damping factor $r=0$), and f = the eigenfrequency of the real damped ($r>0$) oscillating system if $\Delta p/p_0<0.4$. As already mentioned, with frequency growth of the damping factor of the oscillating system increases, too; after reaching a certain point where the energy required to cover the losses cannot be compensated any longer within one cycle — either another type of oscillation with smaller damping factor will develop within the equipment or the equipment itself will turn completely unserviceable. Equation (3), referring to the Helmholtz resonator, will present satisfactory results only if the damping factor is: $r<0.1$

It has been proved by an analogue computer aided model (see Fig. 2) that the frequency of oscillations — for amplitudes maintained at constant values by means of periodical heat input — is always smaller than that of undamped oscillations, furthermore, that the frequency of the oscillating system depends not only on the place of heat input to the oscillation process but on its duration, too. Namely the shorter this duration is, the higher the extent of frequency decrease of the oscillating system at given damping.

Conclusions

On the basis of investigations carried out by means of the experimental equipment and the model obtained by analogue computer, the results can be summarized, as follows:

— for the study of pressure fluctuations developing in a pulsating combustor, the laws of acoustics will apply satisfactorily, up to the values of pressure amplitudes $\Delta p/p_0 < 0.4$;
— the Helmholtz resonator analogy to pulsating combustors can be applied if: $AL/V < 5.0$ is observed;
— the operating frequency of a non-acoustic system is always lower than the frequency of an acoustic system having the same geometry;
— revealing the dependency of damping from the geometry of the equipment, we have succeeded to give a formula for frequency calculation, by the application of which the frequency of pulsating combustors can be determined.

Increasing the resonator tube length exceeding the above determined geometrical limit the oscillation will be extinguished. Detailed investigation has shown that the cause of this phenomenon was the change of the oscillation pattern, namely the Helmholtz resonator type oscillation turned into the oscillation pattern of a tube closed at one end. The cause of the discontinuity in the frequency function is the fact that the changed oscillation pattern results a higher frequency of oscillation. But this higher frequency can be maintained only by significantly increased energy surplus.

There are many papers dealing with pulsating combustors, verifying the topical interest of the problems in question. Because of the increase of the heat transfer coefficient in pulsating gas streams, the heating surfaces of combustors can be considerably reduced, enabling herewith the development of small-size engine pre-heaters, steam generators and hot water producing units, respectively [10]. Their design and dimensioning, however, can be carried out only after due theoretical and experimental studies.

List of Symbols

A	m^2	cross-section of resonator tube
a	$\frac{m}{s}$	mean velocity of sound
b	—	constant
c	—	constant
d_0	m	diameter of resonator tube
f	Hz	frequency of oscillating system
f_0	Hz	frequency of the undamped oscillating system
k	$\frac{1}{m}$	proportionality factor

$K=$	$\frac{1}{\mathrm{kg}}$	system parameter $= \frac{1}{4\pi \mathrm{m}}$
L	m	length of resonator tube
L_0	m	maximum length of resonator tube (5200 mm)
m	kg	mass
p	$\frac{\mathrm{N}}{\mathrm{m}^2}$	amplitude of pressure fluctuation
p_0	$\frac{\mathrm{N}}{\mathrm{m}^2}$	mean pressure, prevailing in the oscillating system
q	(t)	quantity of heat addition to the oscillating system
R_0	—	constant
r	$\frac{\mathrm{N}}{\mathrm{m/s}}$	damping factor
T_1	s	time constant
T_2	s	time constant
t	(s)	time
V	m^3	volume of combustion chamber
x	m	deflection
Y_1		frequency function of exciter
Y_2		frequency function of excited system

Greek letters

λ	air excess factor
ρ	mean density of medium
ω	angular velocity

Subscripts

H	Helmholtz
t	tube

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