ON THE THEORY OF CROSS-MAGNETIZATION EFFECTS IN CYLINDRICAL ROTOR A. C. MACHINES, INCLUDING MAIN FLUX SATURATION

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Summary

In recently published papers concerned with variable saturation in cylindrical rotor a.c. machines, a cross-saturation effect was described, caused by variable saturation. Using the same approximations as applied in the cited papers we will show that Faraday's law applied to cylindrical rotor a.c. machines including variable saturation excludes any transformer effect that links the mutually perpendicular axes of a general reference frame. In reality the induced component voltages in the directions of coordinate axes are well defined by transformer and rotational effects in agreement with Faraday's law, showing that the coordinate axes of the applied general coordinate system, are magnetically independent.

Introduction

In the last decade or so, several papers have dealt with the theory of cylindrical rotor a.c. machines, including the effect of variable saturation [1–3]. In this paper we are applying the same simplifying assumptions as used in the works cited. These are: a) The main flux distribution around the air-gap periphery is sinusoidal at any rate of saturation. This assumption is equivalent to the supposition that the permeance of the iron is permanently infinite and the variable saturation is to be modeled by a fictitious variation of the air-gap clearance. b) The m.m.f.'s (currents) and voltages are sinusoidally distributed around the periphery. c) The time function of the magnetizing current remains sinusoidal at any rate of saturation. (The saturation curve is measured by a.c. instruments indicating r.m.s. values.) These assumptions, in particular the first one, are very useful for theoretical investigations, as we can apply the same calculating methods as usual with constant saturation. Anyhow, the real saturation effect caused by the iron core is to be replaced by the supposition of a variable air-gap clearance. In reality, we could close our investigation at this point because the cross-saturation effect (the magnetic linkage of the mutually perpendicular reference frame axes), as described in [1–3] is a priori excluded due to the first assumption. It is a well known fact that real cross-saturation can only occur if the main flux is distorted by cross magnetization as in case of saturated d.c. or salient-pole synchronous machines. Nevertheless, the papers cited claim that based on mathematical results it must be concluded that even by using the assumption of sinusoidal field distribution, a real cross-saturation does exist, magnetically linking the perpendicular coordinate axes.

We intend to show why this mathematically correct line of thought is leading to incorrect conclusions. For the sake of correct proof we will present a thorough physical analysis of machine transients using customary methods, but including variable saturation in accordance with our first assumption. Since, due to the assumption about field distribution, the use of the spatial vector method is permitted, we are applying this easily treatable method. The equations are given in p.u. system and in dimensionless form. The instantaneous speed of the general reference frame is denoted by ω_a, the mechanical speed of the rotor by ω. The time is measured in radians ($\tau = \omega_1 t$; ω_1 the synchronous speed, t the time in seconds). Assuming that the leakage reactances (X_{s1} and X_{r1}) are constants independent of the current, we can write the following voltage equations (see Ref. [4]):

$$\bar{u}_s = \bar{i}_s R_s + \frac{d\bar{\Psi}_m}{d\tau} + X_{s1}\frac{d\bar{i}_s}{d\tau} + j\omega_a(\bar{\Psi}_m + \bar{i}_s X_{s1}) \tag{1}$$

$$\bar{u}_r = \bar{i}_r R_r + \frac{d\bar{\Psi}_m}{d\tau} + X_{r1}\frac{d\bar{i}_r}{d\tau} + j(\omega_a - \omega)(\bar{\Psi}_m + \bar{i}_r X_{r1}) \tag{2}$$

Equations (1) and (2) are of general validity, i.e., the value of the main flux $\bar{\Psi}_m$ is not restricted to constant saturation. All physical quantities in Eqs (1) and (2) (voltage-, current-, and flux-vectors) are instantaneous values. As only the transient behaviour of the main flux and of the induced voltages are affected by variable saturation, we are focusing our investigation on these quantities.

To give a better insight, in Fig. 1 we have indicated the spatial orientation of the stationary and of the arbitrarily chosen general coordinates (instantaneous speed ω_a). We have also plotted the instantaneous direction of the main flux. As we are neglecting the iron losses, the direction of the magnetizing current ($\bar{i}_m$) is permanently coaxial to the direction of $\bar{\Psi}_m$. In any case,

$$\bar{\Psi}_m = \bar{i}_m X_m \tag{3}$$

X_m stands for the main field reactance. In Fig. 1, the axes of the rectangular stationary frame are denoted by α (real direction) and β (imaginary direction); the two axes of the general coordinate system are denoted by a real direction and b imaginary direction. The instantaneous value of the angle formed by the directions a and $\bar{\Psi}_m$ is denoted by μ.

Based on Fig. 1, we can write:

$$\bar{\Psi}_m = |\bar{\Psi}_m| e^{j\mu} = \Psi_m e^{j\mu} \tag{4}$$

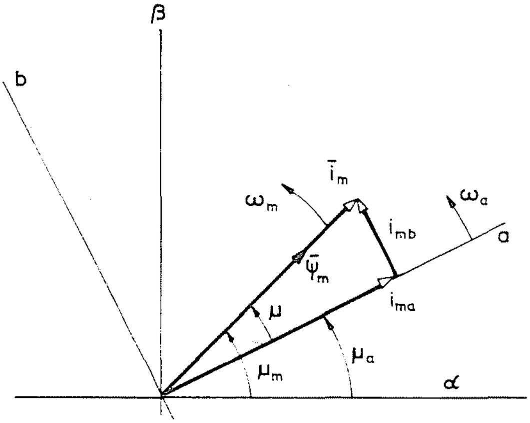

Fig. 1. Spatial orientation of stationary coordinates (α, β) of rotating coordinates (a, b; speed ω_a); of the main flux and magnetizing current ($\bar{\Psi}_m$, $\bar{i}_m$; speed ω_m). ω_a and ω_m are related to the stationary frame

By differentiation we obtain the induced voltage:

$$\bar{u}_i = \frac{d\bar{\Psi}_m}{d\tau} = e^{j\mu}\left(\frac{d\Psi_m}{d\tau} + j\Psi_m \frac{d\mu}{d\tau}\right) \tag{5}$$

where $d\mu/d\tau = \omega_\mu$ is the speed of the main flux ($\bar{\Psi}_m$) relative to the general reference frame. The total induced voltage given in Eq. (5) consists of a transformer voltage ($d\Psi_m/d\tau$), pointing in the direction of the main flux, and of a rotational voltage ($\omega_\mu \Psi_m$), pointing perpendicular to the direction of the flux. This mode of dividing the total induced voltage into transformer and rotational voltages is a well known physical consequence of Faraday's law applied to rotating a.c. machines.

Under general transient conditions, the peak of the spatial distribution of flux changes with time, while the spatial distribution remains sinusoidal. The induced voltage in this direction is proportional to the rate of change of the flux. At the same time, the flux is moving relative to the reference frame. The voltage induced by motion is proportional to the value of the flux and to its relative speed. The total induced voltage is, in any case, composed of a transformer and of a rotational voltage. There are only two exceptions: under steady-state conditions Ψ_m = const. and, as $d\Psi_m/d\tau = 0$, the total induced voltage is generated by rotation only. The rotational voltage can permanently be zero only if μ = const. ($d\mu/d\tau = 0$), i.e., the speed of the reference frame is permanently synchronous to the speed of the main flux. This can be effected only if the reference frame is assumed to be fixed to the direction on the flux.

The effect of variable saturation

With variable saturation the saturation curve is non-linear. The saturation curve on the graph gives an unambiguous correlation between the main flux and the magnetizing current. In Fig. 2 we have plotted the function $\Psi_m = \Psi_m(i_m)$.

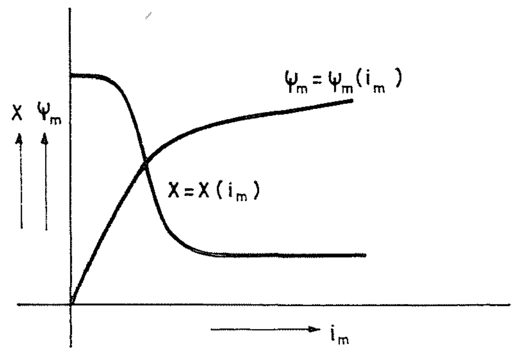

Fig. 2. Saturation curve $[\Psi_m = \Psi_m(i_m)]$ and the dynamic reactance $[X = X(i_m) = \mathrm{d}\Psi_m/\mathrm{d}i_m]$, measured by a.c. instruments, indicating r.m.s. values

Expressing the fluxes by currents and reactances and using Eq. (5) we obtain the spatial vector of the induced voltage:

$$\bar{u}_i = \frac{\mathrm{d}\bar{\Psi}_m}{\mathrm{d}\tau} = \mathrm{e}^{j\mu}\left(\frac{\mathrm{d}(i_m X_m)}{\mathrm{d}\tau} + j\omega_\mu i_m X_m\right) \tag{6}$$

In case of variable saturation i_m and X_m are real time variables, determined by the saturation curve. Therefore we can write:

$$\frac{\mathrm{d}\Psi_m}{\mathrm{d}\tau} = \frac{\mathrm{d}(i_m X_m)}{\mathrm{d}\tau} = \frac{\mathrm{d}(i_m X_m)}{\mathrm{d}i_m}\cdot\frac{\mathrm{d}i_m}{\mathrm{d}\tau} = \frac{\mathrm{d}\Psi_m}{\mathrm{d}i_m}\cdot\frac{\mathrm{d}i_m}{\mathrm{d}\tau} \tag{7}$$

The multiplying term in Eq. (7), $\mathrm{d}\Psi_m/\mathrm{d}i_m$, can be obtained from the saturation curve, using step-by-step, or graphical differentiation. We denote $\mathrm{d}\Psi_m/\mathrm{d}i_m = X$ (dynamic reactance) and indicate the function X versus i_m in Fig. 2. By using the dynamic reactance we can rewrite Eq. (6). That is

$$\bar{u}_i = \frac{\mathrm{d}\bar{\Psi}_m}{\mathrm{d}\tau} = \mathrm{e}^{j\mu}\left(X\frac{\mathrm{d}i_m}{\mathrm{d}\tau} + j\omega_\mu X_m i_m\right) \tag{8}$$

The only difference for the above, compared to that for constant saturation ($X_m = \text{const.}$) is that in Eq. (8) the differential quotient $\mathrm{d}i_m/\mathrm{d}\tau$ is multiplied by the variable dynamic reactance (X) instead of the constant X_m. In Eq. (8) the spatial vector of the transformer voltage is: $\mathrm{e}^{j\mu}X\,\mathrm{d}i_m/\mathrm{d}\tau = \bar{u}_t$; the rotational voltage is:

$je^{j\mu}i_m X_m \cdot \omega_\mu = je^{j\mu}i_m X_m \, d\mu/d\tau = \bar{u}_{rot}$ and the vector of the total induced voltage is: $\bar{u}_i = \bar{u}_t + \bar{u}_{rot}$.

In Fig.3 we indicated the spatial orientation of the vector quantities given by Eq. (8) that are derived by Faraday's law applied to cylindrical rotor a.c. machines including variable saturation. The component induced voltages u_{ia} and u_{ib} are also plotted in Fig. 3.

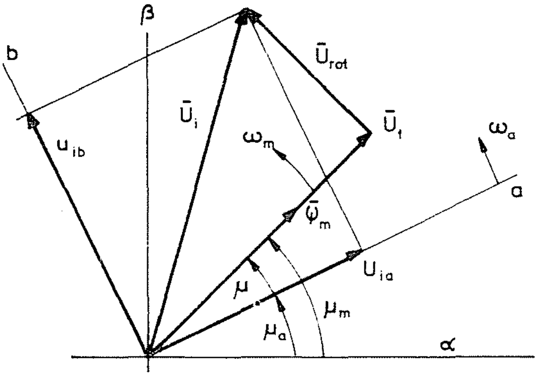

Fig. 3. Spatial orientation of the transformer voltage: $\bar{u}_t$; of the rotational voltage: $\bar{u}_{rot}$ and of the total induced voltage: $\bar{u}_i = \bar{u}_t + \bar{u}_{rot}$. We indicated the component voltages in directions a and b (u_{ia} and u_{ib})

Physical analysis

As can be seen from Figs 1 and 3, the angular velocities ω_m and ω_a were related to the stationary reference frame (stator frame). This natural reference frame is fixed to the stator of the machine and the real physical behaviour of the individual electromagnetical quantities inside the machine are related to this frame, and could be, in principle, measured by a stationary observer. The use of any other moving coordinate system does not alter the course of transients; only the mathematical formulation of the phenomena is to be transformed. The instantaneous physical state related to the stationary frame is plotted in Fig. 4.a. We indicated the direction and value of the main flux $\bar{\Psi}_m$ and the transformer voltage $\bar{u}_t^\alpha$ in the same direction. The total induced voltage ($\bar{u}_i^\alpha$) is the sum of transformer and rotational ($\bar{u}_{rot}^\alpha$) voltages; the latter has the value in the given frame: $i_m X_m \omega_m = u_{rot}^\alpha$ and is perpendicular to the transformer voltage.

It is interesting to mention that the direct observation of the main physical quantities (main flux or induced voltages) is practically very difficult. The transient flux and transient induced voltages are moving inside the machine with very irregular speeds while the absolute values are changing rapidly. However, it is practically possible to determine the total induced voltage ($\bar{u}_i^{\alpha}$) by the measurement of induced voltage components in the stationary directions α and β, which can be done by means of detecting coils, located in these directions (voltages: $u_{i\alpha}$ and $u_{i\beta}$ in Fig. 4.a). It seems important to note that the transient behaviour of the main flux and of the total induced voltage is strictly dependent on fundamental physical rules, determined by electromagnetical and electro-mechanical properties beyond our control. On the other hand, to obtain a possibly simple, or in some cases a general

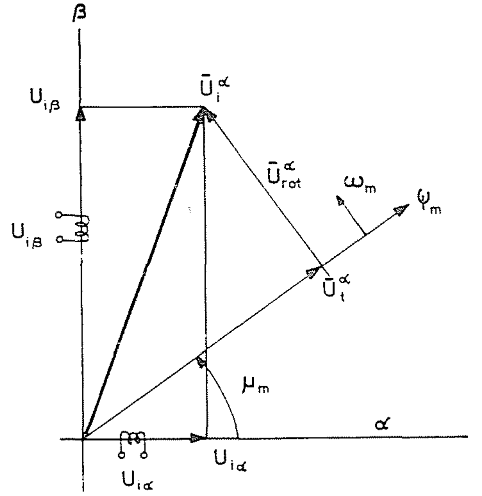

Fig. 4.a. Physical quantities (flux and induced voltages) inside the machine, under general transient conditions, related to a stationary observer

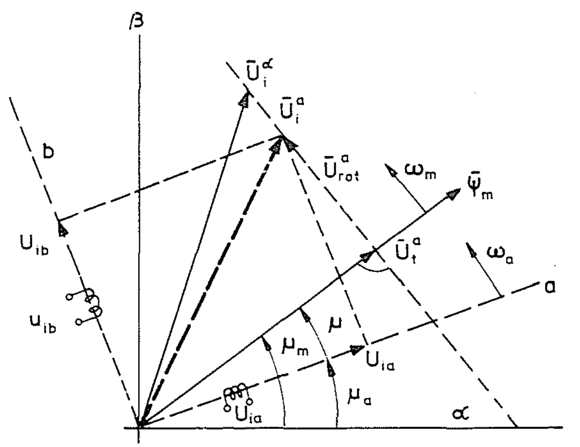

Fig. 4.b. Physical quantities as in Fig. 4.a related to a moving observer (speed ω_a; dotted lines)

mathematical model, we can arbitrarily choose the most convenient reference frame. The new frame must be taken as a seat of an observer moving together with the reference coordinate system.

Now let us go over to another coordinate system moving with the instantaneous angular velocity ω_a (Fig. 4.b dotted lines). It may be seen that the absolute value of the transformer voltage and its position relative to the stationary frame remained unchanged. The rotational voltage is plotted perpendicular to the transformer voltage through the endpoint of this voltage vector, maintaining its direction relative to the stator frame. But from the viewpoint of the co-moving observer the value of rotational voltage is becoming less by the difference $\omega_m - \omega_a = \omega_\mu$ and the relative angle of the new axis will be: $\mu = \mu_m - \mu_a$. This means that the value of the rotational voltage in "ω_a" frame will be: $u^a_{\text{rot}} = i_m X_m(\omega_m - \omega_a) = i_m X_m \omega_\mu = i_m X_m \mathrm{d}\mu/\mathrm{d}\tau$. The locus of the endpoint of the total induced voltage is at any instant given by a straight line perpendicular to the transformer voltage through the endpoint of this voltage. The total induced voltage in "ω_a" coordinates ($\bar{u}^a_i$) is indicated by a dotted line in Fig. 4.b. The induced component voltages are the orthogonal projections of $\bar{u}^a_i$ into the axes a and b (u_{ia} and u_{ib} in Fig. 4.b.). By means of detecting coils in directions a and b the induced component voltages are, in principle, measurable. It may be seen that Fig. 4.b, is a combination of Fig. 3 and Fig. 4.a. From Fig. 4.b we can see that the value of the total induced voltage, related to the new frame, is—at a given instant—determined solely by the angular velocity of the axis a. The coordinate axis b of the rectangular frame has no role in determining the total induced voltage. This means that the induced voltages detectable in freely chosen directions, moving synchronously to axis a, are depending only on the angle formed by the total induced voltage and the axis in question. The individual induced voltages in different directions are mutually independent quantities. They have a common source: the change of the main flux (equivalent to the total induced voltage). The coordinate axes are not sources of flux changes but passive perceiving directions, only. The component induced voltages are determined by the orthogonal projections of the total induced voltage onto the chosen coordinate directions. This means, at the same time, that the coordinate axes are magnetically independent. A magnetic linkage across the reference frame axes has no meaning; such linkage does not exist.

This final conclusion was to be expected as a natural consequence of the simplifying assumption of sinusoidal flux distribution.

We will show, at last, how Eq. (8) could be changed into another form which leads to the idea of a cross-saturation effect, described in the works cited [1–3].[1]

[1] Paper [3] describes the cross-saturation effect in its Eqs (8) through (8.e), inclusive.

The apparent cross-saturation effect[2]

Eq. (8) may be rewritten in the following form:

$$u_i = e^{j\mu}\left(X \frac{\mathrm{d}i_m}{\mathrm{d}\tau} + j i_m X_m \frac{\mathrm{d}\mu}{\mathrm{d}\tau} \right) \tag{8a}$$

The induced component voltages are obtainable from Eq. (8a) by separating it into real and imaginary parts (orthogonal projections of $\bar{u}_i^a$ onto the axes a and b; see Fig. 4.b). We obtain:

$$u_{ia} = X \frac{\mathrm{d}i_m}{\mathrm{d}\tau} \cos\mu - i_m X_m \frac{\mathrm{d}\mu}{\mathrm{d}\tau} \sin\mu \tag{9a}$$

and

$$u_{ib} = X \frac{\mathrm{d}i_m}{\mathrm{d}\tau} \sin\mu + i_m X_m \frac{\mathrm{d}\mu}{\mathrm{d}\tau} \cos\mu \tag{9b}$$

From Fig. 1, assuming sinusoidal distribution, we can find the component currents (the real and imaginary parts of $\bar{i}_m$). That is

$$i_{ma} = i_a \cos\mu \tag{10a}$$

and

$$i_{mb} = i_m \sin\mu \tag{10b}$$

By differentiation we obtain

$$\frac{\mathrm{d}i_{ma}}{\mathrm{d}\tau} = \cos\mu \frac{\mathrm{d}i_m}{\mathrm{d}\tau} - i_m \sin\mu \frac{\mathrm{d}\mu}{\mathrm{d}\tau} \tag{11a}$$

and

$$\frac{\mathrm{d}i_{mb}}{\mathrm{d}\tau} = \sin\mu \frac{\mathrm{d}i_m}{\mathrm{d}\tau} + i_m \cos\mu \frac{\mathrm{d}\mu}{\mathrm{d}\tau} \tag{11b}$$

From Eqs (11a) and (11b) the differencial quotients $\mathrm{d}i_m/\mathrm{d}\tau$ and $\mathrm{d}\mu/\mathrm{d}\tau$ can be obtained in fuctions of component current changes: $\mathrm{d}i_{ma}/\mathrm{d}\tau$ and $\mathrm{d}i_{mb}/\mathrm{d}\tau$. Substituting back into Eqs (9a) and (9b) we obtain

$$u_{ia} = (X\cos^2\mu + X_m \sin^2\mu) \frac{\mathrm{d}i_{ma}}{\mathrm{d}\tau} + (X - X_m) \frac{\sin 2\mu}{2} \frac{\mathrm{d}i_{mb}}{\mathrm{d}\tau} \tag{12a}$$

and

$$u_{ib} = (X - X_m) \frac{\sin 2\mu}{2} \frac{\mathrm{d}i_{ma}}{\mathrm{d}\tau} + (X\sin^2\mu + X_m \cos^2\mu) \frac{\mathrm{d}i_{mb}}{\mathrm{d}\tau} \tag{12b}$$

[2] We are using the word *apparent* in a usual physical sense; e.g., the motion of the Sun around the Earth is an apparent property of the Sun and has no physical significance.

By setting

$$X\cos^2\mu+X_m\sin^2\mu=X_A$$
$$X\sin^2\mu+X_m\cos^2\mu=X_B$$
$$(X-X_m)\frac{\sin 2\mu}{2}=X_{BA}=X_{AB}$$

and by using Eqs (12a) and (12b) we can write

$$u_{ia}=X_A\frac{\mathrm{d}i_{ma}}{\mathrm{d}\tau}+X_{BA}\frac{\mathrm{d}i_{mb}}{\mathrm{d}\tau} \tag{13a}$$

and

$$u_{ib}=X_{AB}\frac{\mathrm{d}i_{ma}}{\mathrm{d}\tau}+X_B\frac{\mathrm{d}i_{mb}}{\mathrm{d}\tau} \tag{13b}$$

The papers cited [1–3] concluded from equations of the form of (13a) and (13b) that the two perpendicular coordinate axes are magnetically linked by the mutual reactance (inductivity in natural units)

$$X_{BA}=X_{AB}=(X-X_m)\sin 2\mu/2\,.$$

Indeed, from the form of Eqs (13a, b) the impression may be gained that a magnetic connection exists between the perpendicular coordinate axes due to variable saturation. However, we have to consider that Eqs (13a, b) were obtained by simple mathematical operations from Eq. (8) which described the physically intelligible Faraday's law, applied to cylindrical rotor a.c. machines including variable saturation. It is a fundamental rule that the physical content of a given phenomenon can not be altered due to mathematical transformations. The physical content and physical consequences of the phenomenon described by Eq. (8) were analyzed in detail in the previous section. The unambiguous result of our analysis is that the induced voltages in arbitrarily chosen directions are mutually independent; they have a single common origin: the change of the main flux, that is, the total induced voltage. Therefore the seeming cross-magnetization across the perpendicular coordinate axes, as concluded from Eqs (13a, b), is an apparent property of the machine and has no physical significance.

For digital calculations the "cross-saturation" equations have a decisive advantage since the component voltages are expressed by component current changes.

Conclusions

In recently published papers about saturation effects in cylindrical rotor a.c. machines a so-called cross-saturation effect is defined, i.e., a direct transformer linkage seems to act across the mutually perpendicular axes of the orthogonal reference frame. Using Fraday's law, applied to cylindrical rotor a.c. machines, we have shown that the concept of cross-saturation has no physical significance; it is an erroneous conclusion only, obtained from an otherwise correct mathematical formulation.

We could state that the common source of the induced voltages, detectable in directions of the reference frame axes, is the change of the main flux, or in other words, the instantaneous value of the total induced voltage.

It may be stated that if the real effect of the iron saturation will be taken into account, the use of the vector method or the usual matrix calculus is no more permitted. This means that the correct modelling must be based on the use of the effect of all higher harmonic field and current components.

Summing up: the theories, given in papers [1] through [3] are not covering the effects of any cross-magnetization due to iron saturation. The theory, given in the mentioned papers is a very useful tool to describe and to calculate the overall effect of variable saturation on the behaviour of the machine under transient conditions. The claime that this theory is an explanation of any cross-magnetization effect, as is observable in saturated d.c. or salient-pole synchronous machines, was proved to be incorrect.

Acknowledgement

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