SYNCHRONOUS MACHINE DYNAMICS WITH SATURATION

I. NAGY

Department of Electrotechnics,
Technical University, H-1521 Budapest

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Summary

The block diagram of the synchronous machine set up on the principle of its operation is presented. It reflects the casual relations besides the quantitative ones among the basic variables. It is a significant tool for acquiring a deeper understanding of the machine dynamics. The saturation is approximately taken into consideration. The phenomenon of induced voltage in cross direction as a result of saturation and its calculation is considered.

Introduction

Dynamics of symmetrically constructed synchronous machine will be discussed with the help of block diagram technique. An approximate treatment of saturation is included. Only the saturation effect on the fundamental space harmonic is considered. The hysteresis and eddy current losses are neglected. It is assumed that the unsaturated mutual reactance X_m is the same among all windings in axis d and in axis q. All leakage inductances are supposed to be constant, that is, the saturation has no effect on them. Once again it has to be stressed that the saturation is considered only with a number of approximations.

The block diagram offers significantly more than the simple differential equations of the synchronous machines. It provides not only quantitative but as well as causal relations among principal physical variables and gives a deeper insight into the dynamics of machine [7, 8, 9].

The block diagram represents a machine with round rotor connected to a large system through transmission line. The magnetic properties of the machine are assumed being identical in all directions. The block diagram was developed for large perturbations and it includes the effects of amortisseurs and variable speed.

Two rotor windings are assumed both in the direct axis and in the quadrature axis. Field winding and an amortisseur winding taking into account the eddy currents in iron as well can be found in direct axis. Beside the amortisseur winding a second, separate one takes into account the eddy current effect in iron in quadrature axis.

The equations are written in rotor coordinate system. All rotor variables are referred to the stator winding. Park vectors are used in connection of three phase time variables [1, 2, 3].

Per unit system is used. The base quantities are the peak values of the rated phase variables. The torque base in the rated voltamperes divided by the rated angular speed. The time base is the reciprocal of the rated angular frequency. The numerical value of the reactance and inductance as well as that of the linkage flux and voltage are the same at rated frequency in per unit system, respectively: $X = L$, $\Psi = u$.

Basic equations

The system discussed is shown in Fig. 1. Z_e is the external impedance between the machine terminals and the infinite bus.

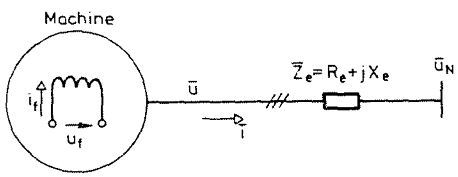

Fig. 1

By using generating sign convention, the fundamental equations are:

$$\bar{u} = -\bar{i}R + \frac{d\bar{\Psi}}{dt} + j\omega\bar{\Psi} \tag{1}$$

$$\bar{u} - \bar{u}_N = \bar{i}R_e + X_e \frac{d\bar{i}}{dt} + j\omega X_e \bar{i} \tag{2}$$

$$u_f = i_f R_f + \frac{d\Psi_f}{dt} \tag{3}$$

$$0 = i_D R_D + \frac{d\Psi_D}{dt} \tag{4}$$

$$0 = i_{Q1} R_{Q1} + \frac{d\Psi_{Q1}}{dt} \tag{5}$$

$$0 = i_{Q2} R_{Q2} \frac{d\Psi_{Q2}}{dt} \tag{6}$$

$$\Psi_d = kX_m(-i_d + i_f + i_D) - X_l i_d = k\Psi_{md} - X_l i_d \tag{7}$$
$$\Psi_q = kX_m(-i_q + i_{Q1} + i_{Q2}) - X_l i_q = k\Psi_{mq} - X_l i_q \tag{8}$$
$$\Psi_f = kX_m(-i_d + i_f + i_D) + X_{fl} i_f = k\Psi_{md} + X_{fl} i_f \tag{9}$$
$$\Psi_D = kX_m(-i_d + i_f + i_D) + X_{Dl} i_D = k\Psi_{md} + X_{Dl} i_D \tag{10}$$
$$\Psi_{Q1} = kX_m(-i_q + i_{Q1} + i_{Q2}) + X_{Q1l} i_{Q1} = k\Psi_{mq} + X_{Q1l} i_{Q1} \tag{11}$$
$$\Psi_{Q2} = kX_m(-i_q + i_{Q1} + i_{Q2}) + X_{Q2l} i_{Q2} = k\Psi_{mq} + X_{Q2l} i_{Q2} \tag{12}$$

$$t_m = M\frac{\mathrm{d}\omega}{\mathrm{d}t} + D(\omega - 1) + t_e \tag{13}$$

$$\omega = \frac{\mathrm{d}\delta}{\mathrm{d}t} + 1 \tag{14}$$

where the electric torque

$$t_e = \Psi_d i_q - \Psi_q i_d \tag{13a}$$

The nomenclature is included in Appendix 1. Equation (1) is the stator voltage equation. Its real and imaginary parts are

$$u_d = -i_d R + \frac{\mathrm{d}\Psi_d}{\mathrm{d}t} - \omega\Psi_q \tag{15}$$

$$u_q = -i_q R + \frac{\mathrm{d}\Psi_q}{\mathrm{d}t} + \omega\Psi_d \tag{16}$$

Equation (2) is the transmission line voltage equation. Its real and imaginary parts are

$$u_d - u_{Nd} = i_d R_e + X_e\frac{\mathrm{d}i_d}{\mathrm{d}t} - \omega X_e i_q \tag{17}$$

$$u_q - u_{Nq} = i_q R_e + X_e\frac{\mathrm{d}i_q}{\mathrm{d}t} + \omega X_e i_d \tag{18}$$

Eqs (3)...(6) are the voltage equations of rotor windings. Eqs (7)...(12) are the flux-current relations where

$$k = \frac{\Psi_{ms}}{\Psi_m} = \frac{u_{gs}}{u_g} \tag{19}$$

The main linkage flux versus magnetizing current $\Psi_{ms}(i_m)$ is shown in Fig. 2.a. Here the stator current Park vector $\bar{i} = i_d + ji_q$ while the rotor current Park vector $\bar{i}_r = i_f + i_D + j(i_{Q1} + i_{Q2})$. Ψ_m and Ψ_{ms} is the main flux without and with saturation, respectively. X_m is the unsaturated mutual reactance (inductance). $kX_m = X_{m\,\mathrm{stat}}$ is the saturated static mutual reactance (inductance):

$$X_{m\,\mathrm{stat}} = \frac{\Psi_{ms}}{i_m} = \frac{\Psi_{ms}}{X_m i_m} X_m = kX_m \tag{19a}$$

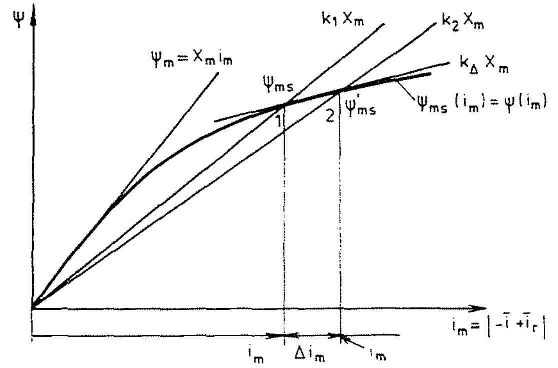

Fig. 2.a

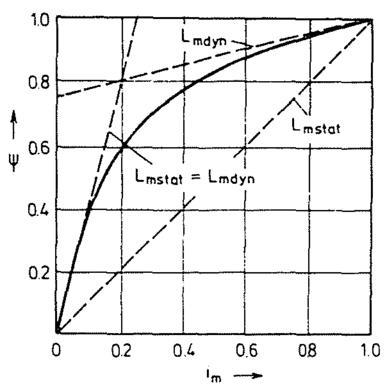

Fig. 2.b

The relation for $\Psi_{ms}(i_m)$ may be written in algebraic form for electric machines and for reactors with air gap as follows [4]

$$i_m = E\Psi_{ms} + F\Psi^n_{ms}$$

The static inductance is

$$L_{m\,\mathrm{stat}} = \frac{\Psi_{ms}}{i_m} = \frac{1}{E + F\Psi^{n-1}_{ms}}$$

$L_{m\,\mathrm{stat}}$ depends on Ψ_{ms}. The dynamic inductance

$$L_{m\,\mathrm{dyn}} = \frac{\mathrm{d}\Psi_{ms}}{\mathrm{d}i_m} = \frac{1}{E + nF\Psi^{n-1}_{ms}}$$

In a particular case in per unit system $E = 0.25$, $F = 0.75$, $n = 5$ [4] (Fig. 2.b).

Saturation

Resultant MMF Park vector $\bar{i}_m = -\bar{i} + \bar{i}_r$ developes the resultant main flux Park vector $\bar{\Psi}_{ms} = \bar{\Psi}$ (Fig. 3). Point 1 of magnetizing curve in Fig. 2.a belongs to i_m and $\Psi_{ms} = \Psi = k_1 X_m i_m$. Assuming a change Δi_m in magnetizing current the new magnetizing current Park vector is $\bar{i}'_m = \bar{i}_m + \Delta \bar{i}_m$.

Point 2 of magnetizing curve in Fig. 2.a belongs to $\bar{i}'_m$ and $\Psi'_{ms} = \Psi' = k_2 X_m i'_m$, where Ψ' is the new main flux. The vector relations are: $\bar{\Psi} = k_1 X_m \bar{i}_m$ and $\bar{\Psi}' = k_2 X_m \bar{i}'_m$.

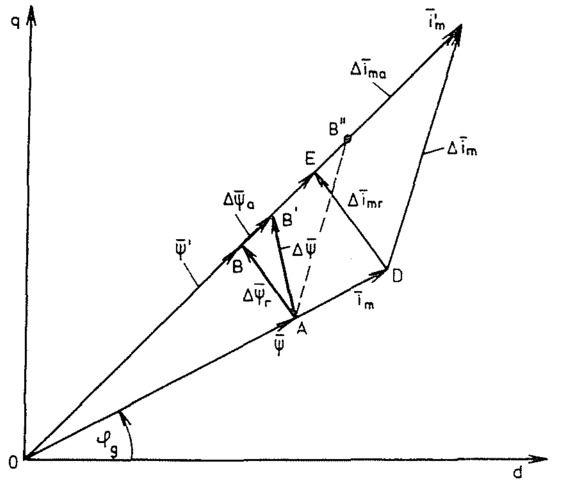

Fig. 3

$\Delta \bar{i}_m$ and $\Delta \bar{\Psi}$ can be composed by two components: $\Delta \bar{i}_m = \Delta \bar{i}_{mr} + \Delta \bar{i}_{ma}$ and $\Delta \bar{\Psi} = \Delta \bar{\Psi}_r + \Delta \bar{\Psi}_a$ where $\Delta \bar{i}_{mr}$ and $\Delta \bar{\Psi}_r$ is the rotating component, $\Delta \bar{i}_{ma}$ and $\Delta \bar{\Psi}_a$ is the amplitude changing component, respectively. It means, that $\overline{OD} = \overline{OE}$ and $\overline{OA} = \overline{OB}$, respectively.

The relation between the perturbation of rotating components is

$$\Delta \bar{\Psi}_r = k_1 X_m \Delta \bar{i}_{mr} \tag{20}$$

that is

$$\bar{\Psi} + \Delta \bar{\Psi}_r = k_1 X_m (\bar{i}_m + \Delta \bar{i}_{mr})$$

since $|\bar{i}_m| = |\bar{i}_m + \Delta \bar{i}_{mr}|$ and $|\bar{\Psi}| = |\bar{\Psi} + \Delta \bar{\Psi}_r|$.

The relation between the perturbation of amplitude changing components is

$$\Delta \bar{\Psi}_a = k_\Delta X_m \Delta \bar{i}_{ma} \tag{21}$$

For infinitesimal changes

$$X_{m\,\mathrm{dyn}} = k_\Delta X_m = \frac{\mathrm{d}\Psi}{\mathrm{d}i_m}$$

$X_{m\,\mathrm{dyn}}$ is the dynamic inductance, that is $k_\Delta X_m$ is the tangent of function $\Psi(i_m)$.

As a result of saturation the direction of $\Delta\bar{\Psi}$ is different of that of $\Delta\bar{i}_m$:

$$\Delta\bar{\Psi}=\left(k_1\frac{\Delta\bar{i}_{mr}}{\Delta\bar{i}_m}+k_\Delta\frac{\Delta\bar{i}_{ma}}{\Delta\bar{i}_m}\right)X_m\Delta\bar{i}_m=\bar{k}X_m\Delta\bar{i}_m \tag{22}$$

Now $\Delta\bar{\Psi}$ is leading $\Delta\bar{i}_m$ and $|k|<1$.
In linear case $k_1=k_\Delta=k$ and

$$\Delta\bar{\Psi}=kX_m\Delta\bar{i}_m$$

as well as the end point of $\Delta\bar{\Psi}$ would be point B''.

Summarizing the effect of saturation, the following can be stated:

— The relation for large signals between $\bar{\Psi}$ and i_m is

$$\bar{\Psi}=k_iX_m\bar{i}_m \tag{23}$$

where k_iX_m is the saturated static mutual reactance (inductance) or large signal reactance.

— The relation for small signals between the rotating components $\Delta\bar{\Psi}_r$ and $\Delta\bar{i}_{mr}$ is

$$\Delta\bar{\Psi}_r=k_iX_m\Delta\bar{i}_{mr} \tag{24}$$

where k_iX_m is determined by Ψ and i_m.

— The relation for small signals between the amplitude changing components $\Delta\bar{\Psi}_a$ and $\Delta\bar{i}_{ma}$ is

$$\Delta\bar{\Psi}_a=k_\Delta X_m\Delta\bar{i}_{ma} \tag{25}$$

where $k_\Delta X_m$ is the saturated dynamic mutual reactance (inductance) or small signal reactance belonging to point $\Psi-i_m$.

Block diagrams

Instead of writing the basic equations a block diagram can be set up on the principle of machine operation. The block diagram is shown in Fig. 4. Here all variables are time functions. p is the differential operator: $p=\mathrm{d}/\mathrm{d}t$.

Each rotor winding has three induced voltage components. For instance, the induced voltages in the direct axis amortisseur winding are (Fig. 4.a)

$$u_{iDD}=p(kX_m+X_{dl})i_D \tag{26}$$

$$u_{iD}=-pkX_mi_d \tag{27}$$

$$u_{iDf}=pkX_mi_f \tag{28}$$

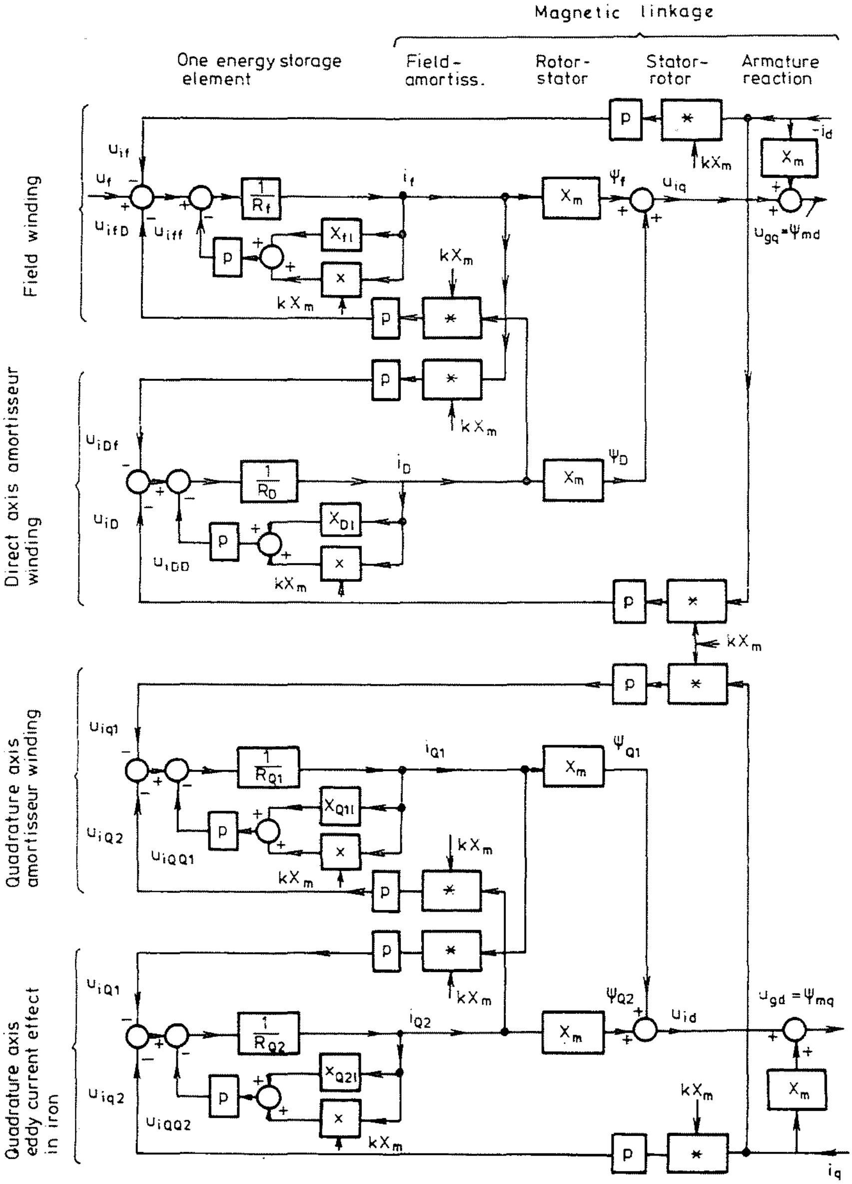
Magnetic linkage
One energy storage element
Field-amortiss.
Rotor-stator
Stator-rotor
Armature reaction
Field winding
Direct axis amortisseur winding
Quadrature axis amortisseur winding
Quadrature axis eddy current effect in iron
u_{if}
u_f
u_{ifD}
u_{iff}
$\frac{1}{R_f}$
i_f
X_m
Ψ_f
u_{iq}
$-i_d$
kX_m
X_{fl}
$u_{gq}=\Psi_{md}$
u_{iDf}
u_{iD}
u_{iDD}
$\frac{1}{R_D}$
i_D
Ψ_D
X_{Dl}
u_{iq1}
u_{iQ2}
u_{iQQ1}
$\frac{1}{R_{Q1}}$
i_{Q1}
Ψ_{Q1}
X_{Q1l}
u_{iQ1}
u_{iq2}
u_{iQQ2}
$\frac{1}{R_{Q2}}$
i_{Q2}
Ψ_{Q2}
X_{Q2l}
u_{id}
$u_{gd}=\Psi_{mq}$
i_q

Fig. 4.a

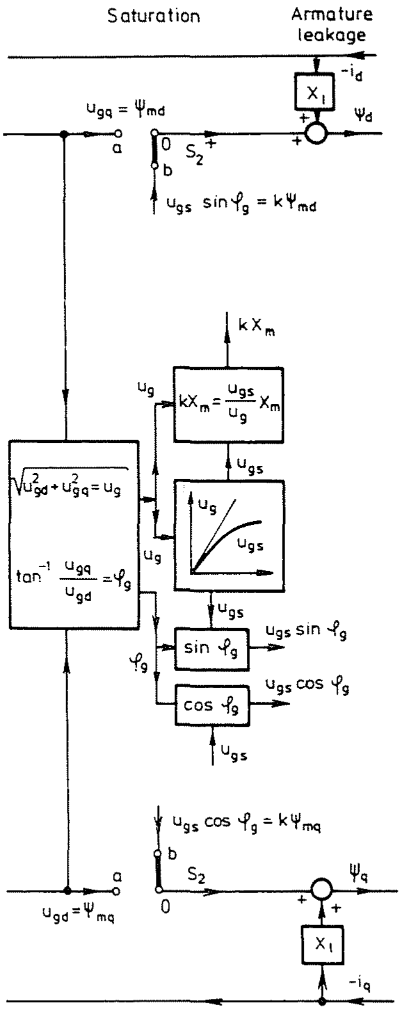

Fig. 4.b

Assuming that no saturation takes place, the two components of the air gap voltage in steady-state and at $\omega = 1$ are

$$u_{gq} = \Psi_{md} = X_m(i_f + i_D - i_d) \tag{29}$$

$$u_{gd} = \Psi_{mq} = X_m(i_{Q1} + i_{Q2} - i_q) \tag{30}$$

and its absolute value and phase angle is (Fig. 4.b)

$$u_g = \sqrt{u_{gd}^2 + u_{gq}^2} \tag{31}$$

$$\varphi_g = \tan^{-1} \frac{u_{gq}}{u_{gd}} \tag{32}$$

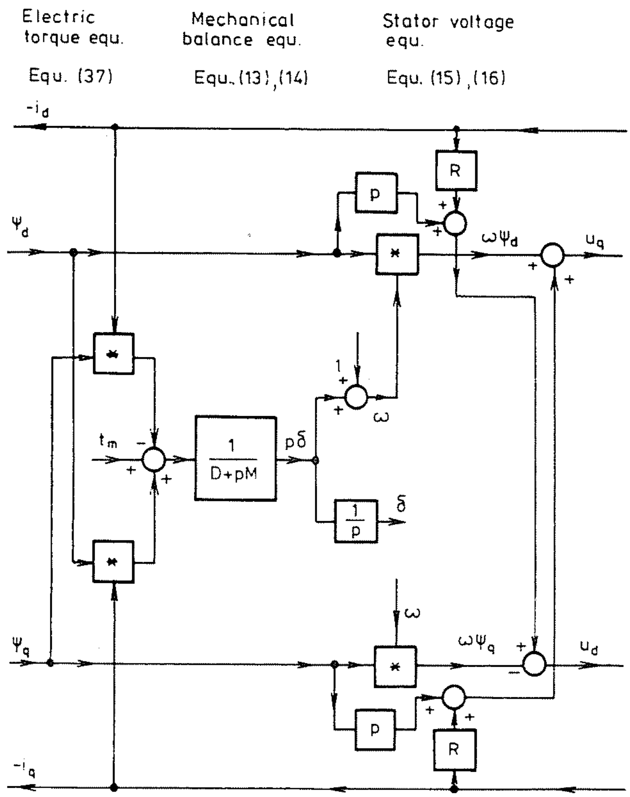

Fig. 4.c

By knowing the no load characteristic of the machine, the saturated air gap voltage u_{gs} can be easily found. The instantaneous value of the saturation constant $k = u_{gs}/u_g$. The two components of the saturated air gap voltage or those of the main flux are

$$u_{gsd} = u_{gs} \cos \varphi_g = k\Psi_{mq} \tag{33}$$

$$u_{gsq} = u_{gs} \sin \varphi_g = k\Psi_{md} \tag{34}$$

Relations (29)...(34) hold for flux linkages in transient-state and $\omega \neq 1$ as well. u_g and u_{gs} are fictitious voltages in transient case since $\omega \neq 1$.

Saturation is taken into consideration in total stator flux linkages when switch S_2 is in position o—b and then (Fig. 4.b)

$$\Psi_d = k\Psi_{md} - X_l i_d \tag{35}$$

$$\Psi_q = k\Psi_{mq} - X_l i_q \tag{36}$$

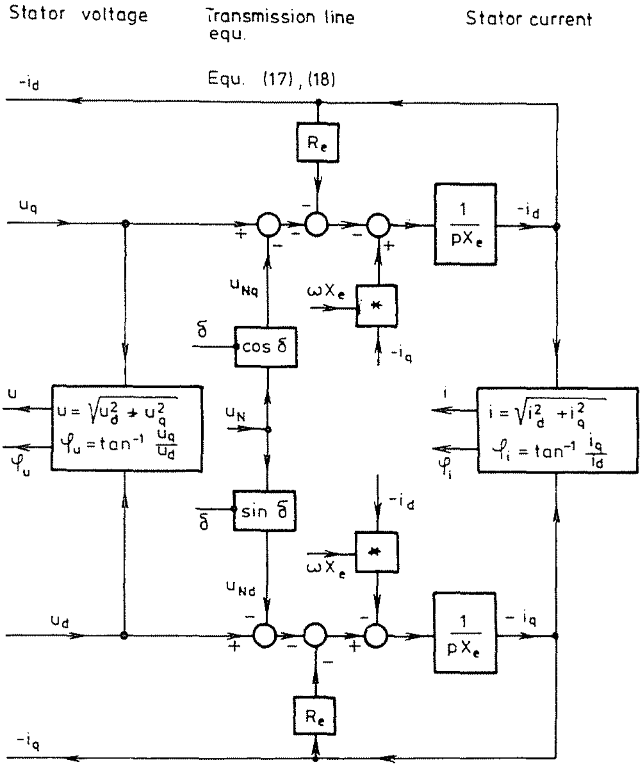

Fig. 4.d

On the other hand, when switch S_2 is in position o—a the saturation effect is neglected ($k=1$).

The rest of the block diagram can be followed by consulting the equations written on the head of figure 4.c and d. Variables u, φ_u and i, φ_i are the amplitude and phase angle of the stator terminal voltage Park vector and those of the stator current Park vector in rotor coordinate system, respectively.

Because of saturation the superposition theorem must not be applied for calculating the flux changes one by one excited by the single current changes and adding them up. Although the three induced voltages of each rotor winding appear separately in Fig. 4.a they can be united into a resultant induced voltage in each winding (Fig. 5). For instance, in direction d the change of main flux linkage $k\Psi_{md}$ and the change of the respective leakage flux generate the resultant induced voltage. Furthermore the resultant induced voltage in

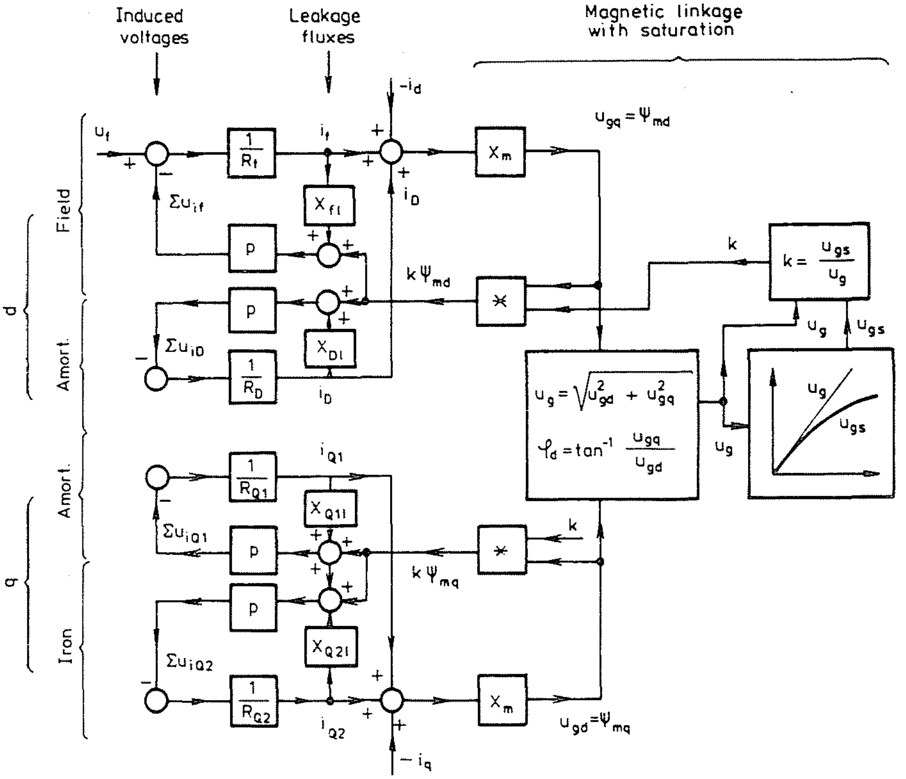

Fig. 5

each winding can be calculated from the respective component of the change in the total main flux linkage. It means, that

$$\frac{d\bar{\Psi}_{ms}}{dt} = \frac{dk\bar{\Psi}_m}{dt} = \frac{dk\Psi_{md}}{dt} + j\frac{dk\Psi_{mq}}{dt} = u_{imd} + ju_{imq} \tag{37}$$

where u_{imd} and u_{imq} is the induced voltage generated by the change in the main flux linkage $dk\Psi_{md}/dt$ and $dk\Psi_{mq}/dt$, respectively.

Calculation

The state equations are best suited for numerical calculation. Their block diagram representation is given in Fig. 6. The first column has four integrators belonging to the four rotor energy storage elements. The corresponding state

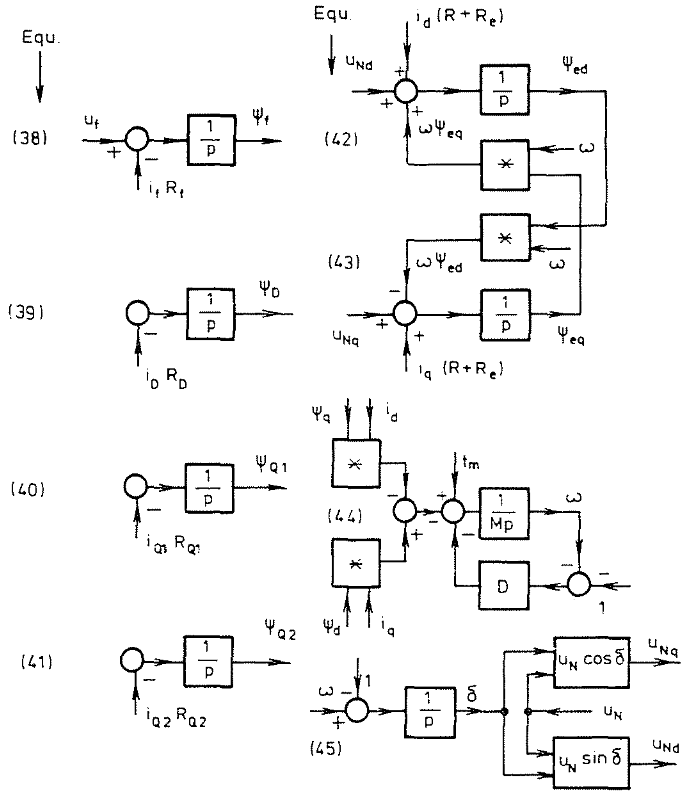

Fig. 6.a

equations are

$$\frac{\mathrm{d}\Psi_f}{\mathrm{d}t}=u_f-i_fR_f \tag{38}$$

$$\frac{\mathrm{d}\Psi_D}{\mathrm{d}t}=-i_DR_D \tag{39}$$

$$\frac{\mathrm{d}\Psi_{Q1}}{\mathrm{d}t}=-i_{Q1}R_{Q1} \tag{40}$$

$$\frac{\mathrm{d}\Psi_{Q2}}{\mathrm{d}t}=-i_{Q2}R_{Q2} \tag{41}$$

The second column has again four integrators. Two of them corresponds to the stator and transmission line voltage equations:

$$\frac{\mathrm{d}\Psi_{ed}}{\mathrm{d}t}=u_{Nd}+i_d(R+R_e)+\omega\Psi_{eq} \tag{42}$$

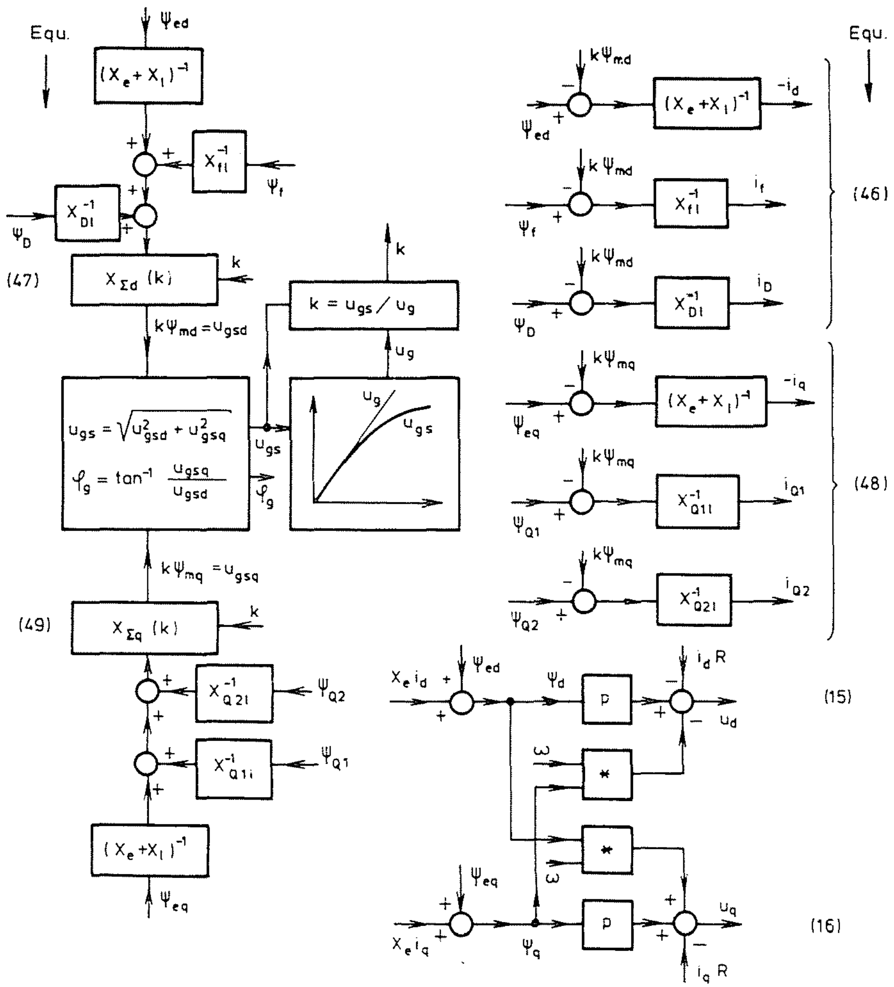

Fig. 6.b

$$\frac{\mathrm{d}\Psi_{eq}}{\mathrm{d}t} = u_{Nq} + i_q(R + R_e) - \omega\Psi_{ed} \tag{43}$$

The other two integrators belong to the mechanical energy storage elements:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{1}{M}(t_m - t_e) - \frac{D}{M}(\omega - 1) \tag{44}$$

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \omega - 1 \tag{45}$$

By knowing Ψ_{ed}, Ψ_f, Ψ_D, the flux linkage $(k\Psi_{md})$ can be calculated. On the basis

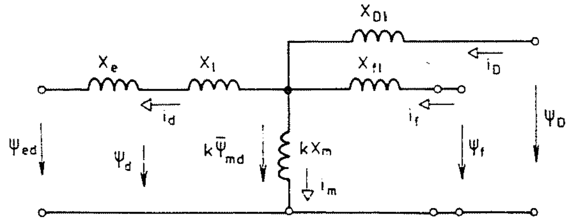

Fig. 7

of flux equivalent circuit (Fig. 7)

$$\left.\begin{aligned} \Psi_{ed} &= k\Psi_{md} - (X_e + X_l)i_d \\ \Psi_f &= k\Psi_{md} + X_{fl}i_f \\ \Psi_D &= k\Psi_{md} + X_{Dl}i_D \end{aligned}\right\} \tag{46}$$

Taking into account

$$k\Psi_{md} = kX_m(-i_d + i_f + i_D)$$

the unknown variable

$$k\Psi_{md} = X_{\Sigma d}(k)\left(\frac{\Psi_{ed}}{X_e + X_l} + \frac{\Psi_f}{X_{fl}} + \frac{\Psi'_D}{X_{Dl}}\right) \tag{47}$$

where

$$X_{\Sigma d}^{-1}(k) = \frac{1}{X_e + X_l} + \frac{1}{X_{fl}} + \frac{1}{X_{Dl}} + \frac{1}{kX_m} \tag{47a}$$

Similarly

$$\begin{aligned} \Psi_{ed} &= k\Psi_{mq} - (X_e + X_l)i_q \\ \Psi_{Q1} &= k\Psi_{mq} + X_{Q1l}i_{Q1} \\ \Psi_{Q2} &= k\Psi_{mq} + X_{Q2l}i_{Q2} \end{aligned} \tag{48}$$

Taking into account

$$k\Psi_{mq} = kX_m(-i_q + i_{Q1} + i_{Q2})$$

the other unknown flux linkage

$$k\Psi_{mq} = X_{\Sigma q}(k)\left(\frac{\Psi_{eq}}{X_e + X_l} + \frac{\Psi_{Q1}}{X_{Q1l}} + \frac{\Psi_{Q2}}{X_{Q2l}}\right) \tag{49}$$

$$X_{\Sigma q}^{-1}(k) = \frac{1}{X_e + X_l} + \frac{1}{X_{Q1l}} + \frac{1}{X_{Q2l}} + \frac{1}{kX_m} \tag{49a}$$

The saturated air gap voltage u_{gs} is obtained from $k\Psi_{md}$ and $k\Psi_{mq}$. Now the unsaturated air gap voltage u_g is taken from the no load characteristic. The currents are calculated from eqs (46) and (48).

Induced voltage and saturation [5, 6]

The time change of mutual linkage flux $\bar{\Psi}_{ms}$ induces voltages in both the stator and rotor windings. The induced voltages $\mathrm{d}(k\Psi_{md})/\mathrm{d}t$ and $\mathrm{d}(k\Psi_{mq})/\mathrm{d}t$ in stator windings are usually negligible in comparison to $\omega\Psi_d$ and $\omega\Psi_q$. On the other hand, they have decisive effect in the rotor windings in transient-state. As a result of saturation voltage is induced in cross direction. The change of exciting current $\bar{i}_m=(-\bar{i}+\bar{i}_r)$ in one direction, for instance in direction d, modifies the value k and it induces voltages not only in direction d but in direction q as well (Fig. 4 and Fig. 5). Without saturation $k=1$ and a change in $\bar{i}_m$ in direction d would induce voltages only in direction d.

The Park vector of induced voltage

$$\bar{u}_i=\frac{\mathrm{d}\bar{\Psi}_{ms}}{\mathrm{d}t}=\frac{\mathrm{d}\Psi_{ms}e^{j\varphi_g}}{\mathrm{d}t}=\left(\frac{\mathrm{d}\Psi_{ms}}{\mathrm{d}t}+j\frac{\mathrm{d}\varphi_g}{\mathrm{d}t}\Psi_{ms}\right)e^{j\varphi_g} \tag{50}$$

where

$$\frac{\mathrm{d}\Psi_{ms}}{\mathrm{d}t}=\frac{d\Psi_{ms}}{di_m}\frac{\mathrm{d}i_m}{\mathrm{d}t}=L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}t} \tag{51}$$

$$\frac{\mathrm{d}\varphi_g}{\mathrm{d}t}=\omega_g \tag{52}$$

$$\bar{\Psi}_{ms}=k\bar{\Psi}_m=L_{m\,\mathrm{stat}}\bar{i}_m \tag{53}$$

Introducing the last three equations into eq. (50)

$$\bar{u}_i=\frac{\mathrm{d}\bar{\Psi}_{ms}}{\mathrm{d}t}=L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}t}e^{j\varphi_g}+j\omega_g L_{m\,\mathrm{stat}}\bar{i}_m \tag{54}$$

The first term on the right side is the induced voltage u_{ia} generated by the change in the magnitude of the magnetizing current i_m. Its direction is the same as that of $\Psi_{ms}(t)$ (Fig. 8). The second term on the right side is the induced voltage u_{ir} generated by the rotation of $\bar{i}_m$. It is in right angle to $\bar{\Psi}_{ms}(t)$ (Fig. 8).

From Equation (51)

$$\frac{\mathrm{d}(\Psi_{msd}+j\Psi_{msq})}{\mathrm{d}t}=L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}t}\cos\varphi_g-\omega_g L_{m\,\mathrm{stat}}i_{mq}+$$

$$+j\left(L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}t}\sin\varphi_g+\omega_g L_{m\,\mathrm{stat}}i_{md}\right) \tag{55}$$

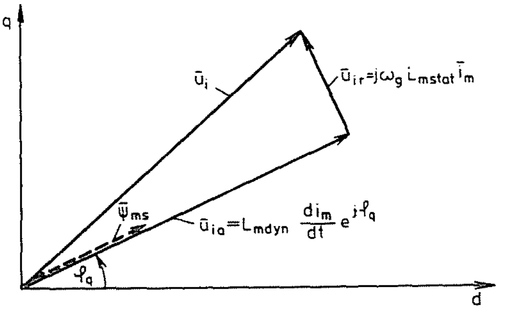

Fig. 8

Let us assume, that $\Delta i_{md}(t)\neq 0$ and $\Delta i_{mq}=0$. In general, both i_m and φ_g is changing. The induced voltage in cross direction

$$u_{imq}=\frac{d\Psi_{msq}}{dt}=L_{m\,\mathrm{dyn}}\frac{di_m}{dt}\sin\varphi_g+\omega_g L_{m\,\mathrm{stat}}i_{md} \tag{56}$$

Since $i_{mq}=i_m\sin\varphi_g$, its derivate

$$\frac{di_{mq}}{dt}=\frac{di_m}{dt}\sin\varphi_g+\omega_g i_{md}=0 \tag{57}$$

where $i_{md}=i_m\cos\varphi_g$.
Without saturation $L_m=L_{m\,\mathrm{dyn}}=L_{m\,\mathrm{stat}}$ and $u_{imq}=0$. There is no induced voltage in cross direction.

On the other hand at saturation $L_{m\,\mathrm{stat}}>L_{m\,\mathrm{dyn}}$ and the voltage component $\omega_g L_{m\,\mathrm{stat}}i_{md}$ generated by the rotation is higher than the other one $L_{m\,\mathrm{dyn}}(di_m/dt)\sin\varphi_g$ generated by the amplitude change. There is an induced voltage u_{imq} in cross direction as a result of saturation except when $i_m=i_{md}$ since then $\varphi_g=0$, $\omega_g=0$.

Assuming $\Delta i_{md}=0$ and $\Delta i_{mq}(t)\neq 0$, it can be shown in the same way as above that there is no induced voltage u_{imd} in cross direction without saturation and there is an induced voltage in cross direction with saturation except when $\bar{i}_m=ji_{mq}$.

Assuming a change $\Delta\bar{i}_{mr}=\bar{i}_m(e^{j\Delta\varphi_g(t)}-1)$, the amplitude changing component in the induced voltage is zero in eq. (54) ($di_m/dt=0$). There are induced voltages in both directions proportional to L_m without saturation and to $L_{m\,\mathrm{stat}}$ with saturation.

Finally, assuming a change $\Delta\bar{i}_{ma}=\Delta i_{ma}e^{j\varphi_g}$, there will be no rotational component in the induced voltage in eq. (54) ($\omega_g=0$). There will be induced voltages again in general in both directions proportional to L_m without saturation and to $L_{m\,\mathrm{dyn}}$ with saturation.

Naturally a current change $\Delta i_d(t) \neq 0$ and $\Delta i_q = 0$ can be composed by $\Delta \bar{i}_{mr}$ and $\Delta \bar{i}_{ma}$, that is

$$\Delta i_d(t) = \Delta \bar{i}_{mr}(t) + \Delta \bar{i}_{ma}(t) \tag{58}$$

(see Fig. 9), $\bar{i}_m(0)$ is the current at $t=0$, $\bar{i}_m(t) = \bar{i}_m(0) + \Delta i_d(t)$. The induced voltage components u_{ia} and u_{ir} can be seen at a particular time t in Fig. 9 ($\omega_g < 0$). When $L_{m\,\mathrm{stat}} = L_{m\,\mathrm{dyn}}$ the induced voltage u_{imq} in cross direction is zero, otherwise it can be determined from eq. (56),

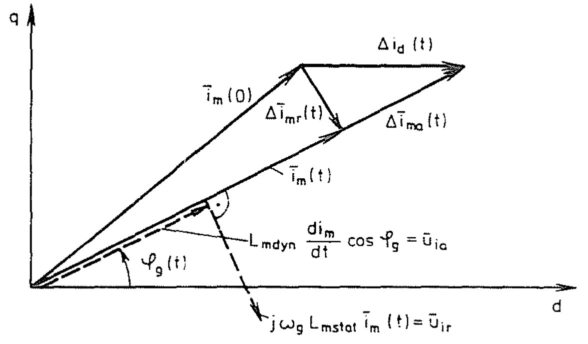

Fig. 9

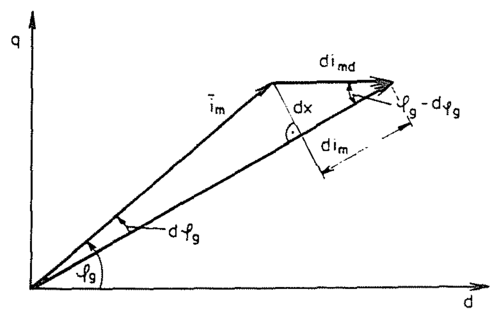

Fig. 10

The induced voltages in cross direction can be written in the following form as well

$$u_{id} = M(\varphi_g, i_m)\,\frac{di_{mq}}{dt} \tag{59}$$

$$u_{iq} = M(\varphi_g, i_m)\,\frac{di_{md}}{dt} \tag{60}$$

In order to determine $M(\varphi_g, i_m)$ the variation of i_{md} and $i_{mq} = \text{const.}$ will be supposed (Fig. 10). From eq. (55,

$$u_{iq}=L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}t}\sin\varphi_g+L_{m\,\mathrm{stat}}\frac{\mathrm{d}\varphi_g}{\mathrm{d}t}i_{md}$$

or

$$u_{iq}=L_{m\,\mathrm{dyn}}\frac{\mathrm{d}i_m}{\mathrm{d}i_{md}}\frac{\mathrm{d}i_{md}}{\mathrm{d}t}\sin\varphi_g+$$
$$+L_{m\,\mathrm{stat}}\frac{\mathrm{d}\varphi_g}{\mathrm{d}i_{md}}\frac{\mathrm{d}i_{md}}{\mathrm{d}t}i_{md} \tag{61}$$

From Fig. 10 ($\mathrm{d}\varphi_g<0$)

$$\mathrm{d}x=-i_m\mathrm{d}\varphi_g=\sin\varphi_g\mathrm{d}i_{md}$$

or

$$\frac{\mathrm{d}\varphi_g}{\mathrm{d}i_{md}}=-\frac{\sin\varphi_g}{i_m} \tag{62}$$

furthermore

$$\frac{\mathrm{d}i_m}{\mathrm{d}i_{md}}=\cos\varphi_g \tag{63}$$

and

$$\frac{i_{md}}{i_m}=\cos\varphi_g \tag{64}$$

Substituting the last three equations into eq. (61).

$$u_{iq}=L_{m\,\mathrm{dyn}}\frac{1}{2}\sin 2\varphi_g\left(1-\frac{L_{m\,\mathrm{stat}}}{L_{m\,\mathrm{dyn}}}\right)\frac{\mathrm{d}i_{md}}{\mathrm{d}t} \tag{65}$$

or

$$M(\varphi_g, i_m)=L_{m\,\mathrm{dyn}}\frac{1}{2}\sin 2\varphi_g\left(1-\frac{L_{m\,\mathrm{stat}}}{L_{m\,\mathrm{dyn}}}\right) \tag{66}$$

It can be shown likewise that $M(\varphi_g, i_m)$ in eq. (59) has the same expression as the one given in eq. (66). Equation (66) indicates that the induced voltage in cross direction is zero at angles $\varphi_g=0$, and $\pi/2$ and it is maximum at angle $\varphi_g=\pi/2$. When there is no saturation: $L_{m\,\mathrm{stat}}=L_{m\,\mathrm{dyn}}$ and $M(\varphi_g, i_m)=0$.

(φ_g, i_m) can be considered being a nonlinear mutual inductance. Its value is changing with φ_g and i_m since both $L_{m\,\mathrm{stat}}$ and $L_{m\,\mathrm{dyn}}$ depend on the value i_m.

The induced voltage in cross direction can be shown by the following test. The rotor of a synchronous machine is in rest and its field winding is excited by d.c. current i_f. The magnetic axis of the field winding and that of phase winding R coincide (Fig. 11.a). A.C. network voltage u is connected across terminal S

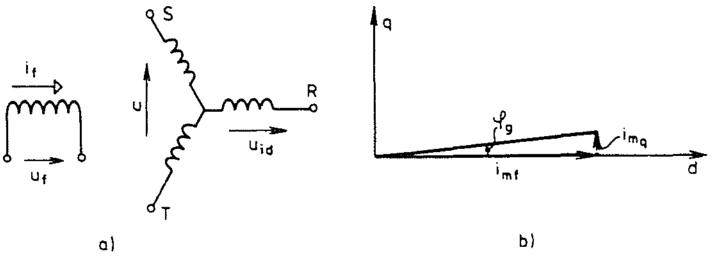

Fig. 11

and T. When $u=0$ the magnetizing current i_{mf} is generated by i_f alone. When $u \neq 0$, a sinusoidal magnetizing component

$$i_{mq} = i_{mqm} \sin \Omega t \tag{67}$$

is added to i_{mf} (Fig. 11.b) ($i_{mf} \gg i_{mqm}$). The induced voltage in cross direction [eq. (55)]:

$$u_{id} = L_{m\,\mathrm{dyn}} \frac{\mathrm{d}i_m}{\mathrm{d}t} \cos \varphi_g - \omega_g L_{m\,\mathrm{stat}} i_{mq} \tag{68}$$

Assuming high saturation, $L_{m\,\mathrm{stat}} \gg L_{m\,\mathrm{dyn}}$ and

$$u_{id} \cong -\omega_g L_{m\,\mathrm{stat}} i_{mq} \tag{69}$$

From Fig. 10.b.

$$\varphi_g = \cong \mathrm{tg}\, \varphi_g = \frac{i_{mq}}{i_{mf}} = \frac{i_{mqm}}{i_{mf}} \sin \Omega t$$

$$\omega_g = \frac{\mathrm{d}\varphi_g}{\mathrm{d}t} = \varphi_{mg} \Omega \cos \Omega t \tag{70}$$

where $\varphi_{gm} = \dfrac{i_{mqm}}{i_{mf}}$.

From equations (67), (69) and (70)

$$u_{id} \cong -\frac{1}{2} L_{m\,\mathrm{stat}} (\varphi_{gm} \Omega) i_{mqm} \sin 2\Omega t \tag{71}$$

It can easily be shown by a laboratory test that the frequency of induced voltage u_{id} is twice as high as the frequency of the network.

Appendix 1. Nomenclature

Complex vectors with superposed bar "—" are Park vectors expressed in d—q rotor coordinate system. Δ quantities denote small excursions around initial operating point. Lower case letter denotes time function. R tor quantities are referred to stator and normalized in the same way as stator quantities.

D	combined prime mover and generator damping factor
u	machine terminal voltage
i	armature line current
$j=\sqrt{-1}$	imaginary unit
$k=\dfrac{\Psi_{ms}}{\Psi_m}$	saturation constant
$kX_m=X_{m\,\mathrm{stat}}$	saturated static mutual reactance (inductance)
$k_\Delta X_m=X_{m\,\mathrm{dyn}}=\dfrac{\mathrm{d}\Psi_{ms}}{\mathrm{d}i_m}$	saturated dynamic mutual reactance (inductance)
$M=4\pi f H$	where H is per unit inertia coefficient
p	time derivate operator $\dfrac{\mathrm{d}}{\mathrm{d}t}$
R, R_e	armature and external resistances, respectively
t	time in radians
t_m	prime mover torque
$t_e=\Psi_d i_q-\Psi_q i_d$	electrical torque
$X=L$	external inductive reactance (inductance)

Greek letters

δ	load angle between the q axis and the infinite bus voltage vector
φ	angle between vector $\bar{\Psi}_{ms}$ or $\bar{i}_m$ and d axis
Ψ	flux linkage
$\omega=1+p\delta$	rotor angular speed

The symbols above denote per unit quantities

Subscripts

a	amplitude changing component
d, q	direct and quadrature axis, respectively
D, $Q1$, $Q2$	amortisseur windings in direct and in quadrature axis, respectively
e	external quantities
f	field winding
g	air gap
i	induced voltage
l	leakage
m	magnetizing or mutual or maximum
N	infinite bus
o	initial state
r	rotor or rotating component
s	saturated value

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Prof. Dr. István NAGY H-1521 Budapest