

TRANSMISSION LINE OPERATING AT MEDIUM FREQUENCY

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Summary

The expanding industrial employment of medium frequency electric energy demands the energy supply of medium frequency consumers to be ensured by developing medium frequency distribution networks instead of feeding these consumers individually. The aim of this investigation was to work out theoretically variants of energy supply, ensuring on the one hand independent operation of the consumers connected to the medium frequency network, and on the other hand facilitating the transportation of medium frequency energy to large distances while consumers' requirements are fulfilled.

It has been verified that the approximately constant voltage or current along the transmission line necessary for the independence of consumers and for energy transport to great distances can be maintained with the aid of compensating elements to be inserted in the line. In the course of the investigation of the various compensating methods, the values of the compensating reactances have been determined, and the effects of the variation of certain system parameters (load, frequency) have been checked.

It has been concluded that the most favourable solution to the problem examined is obtained by the insertion of π or T arrangements of appropriate compensating reactances into the transmission line.

Introduction

It is a well-known fact that by increasing the frequency of the alternating current, the mass, the dimensions and frequently the price of certain energy converters can be reduced. The increasing role medium frequency (2–20 kHz) electrical energy plays in electrical energy consumption is primarily due to the development of semiconductor elements.

One of the main fields of application of medium frequency electrical energy is electrical thermotechniques and especially induction heating. Several advantages would result from supplying these consumers from medium frequency distribution networks, similar to the 50 Hz consumers' network, instead of the now prevalent individual supply [1]. In view of the fact that consumers could connect to any point of the distribution network, a near constant voltage should be ensured at all points.

Nowadays, the rapid increase in oil prices, the ever stricter environmental regulations urge designers to seek novel ways of supplying energy to public vehicles, one possibility being the transmission of energy through induction

[2]. Experiments in this field are also being carried out in high-speed railway transport, primarily to dispense with running and sliding contacts. In mines, the danger of explosions necessitates the employment of energy transmission without contacts. In the Soviet Union, experimental mine railways operated as early as in the nineteen-fifties with the energy supply of the engine ensured through induction, with the aid of the magnetic field of the medium frequency A. C. current carried by the system of conductors arranged along the track [3]. Inductive transmission of power to vehicles at 50 Hz would result in inadmissibly large "energy transmitting transformers" [4], a fact which well justifies the increase of frequency. In several mines of the Soviet Union, engines of 10, 14 tons tractive power are also presently operated from a 5 kHz supply system in steady preliminary operation. A 2-3 times higher productivity, 2-2.5 times lower energy-consumption and 1.5-3 times lower operating costs are obtained as compared to battery operated engines of the same power rating [5]. Along the transmission line generating the magnetic field, energy transmission services constant in space and time should be ensured, therefore a constant current should be maintained along the line.

A characteristic example of medium frequency transmission line is provided by centralized network control (ripple control). A ripple control system takes care of switching on and off usually low-voltage consumers fed by the 50 Hz energy distribution network, with the aid of a coded, acoustic frequency pulse sequence set on the network and of an evaluating receiver. In order to obtain a reliable system, it is necessary to ensure that an acoustic frequency signal of sufficient amplitude reaches all receivers mounted at any point of the area radiated, independent of disturbing factors.

One of the several difficulties encountered when designing medium frequency transmission lines results from the fact that the length of the line and the wavelength of its voltages and currents are of the same order of magnitude, thus standing waves may appear along the line. At the same time, each of the above three examples of medium frequency transmission lines necessitated the maintenance of near constant voltage or current along a line long as compared to the wavelength in order to ensure uniform conditions of energy transmission or independent operation of consumers. However, the transmission line is loaded by impedances varying in space and time, and its matching cannot be ensured in the traditional way, as at 50 Hz distribution networks or in telecommunication, due to the variation of load impedance.

The voltage and current distribution of transmission lines of lengths in the order of magnitude of the wavelength can be advantageously influenced by series or parallel connected reactances. In the following, this particular question of medium frequency transmission lines will be dealt with. In order to simplify the discussion, the examinations will only be carried out for steady state, harmonic time variation, and the medium frequency transmission line

will be taken as ideal with no losses and constant L, C parameters. It should be noted that in case of a practical design procedure, certain factors neglected in the present discussion may also have to be taken into account.

Possibilities of compensating a medium frequency transmission line loaded at its terminal

Distributed parameter (theoretical) compensation

Let us assume a constant ohmic load of R_t at the terminal of the transmission line.

If the loading resistance equals the characteristic resistance, then a harmonic current and voltage of constant amplitude is obtained along the so called matched transmission line.

If e.g. the loading resistance is greater than the characteristic resistance:

$$R_t > Z_o = \sqrt{\frac{L}{C}}$$

where

L is the inductance per unit length of the transmission line,
 C is the capacitance per unit length of the transmission line,
 Z_o is the characteristic resistance,

the condition of matching can be fulfilled by increasing the characteristic resistance of the transmission line to $Z'_o = R_t$, i.e. by increasing the inductive and/or capacitive reactance per unit length of the line. In theory, this can be attained by inserting series or parallel inductances into the line distributed uniformly along the transmission line. E.g. in case of series compensation:

$$R_t = Z'_o = \sqrt{\frac{L'}{C}} = \sqrt{\frac{L + L_S}{C}}$$

where

L' is the inductance per unit length of the compensated transmission line,
 L_S is the per unit length value of the series inductance to be inserted: $L_S = C(R_t^2 - Z_o^2)$.

The wavelength on an uncompensated transmission line is

$$\lambda = \frac{1}{f\sqrt{LC}}$$

where f is the frequency. Due to the inserted series inductance it changes to

$$\lambda_s = \frac{1}{f\sqrt{L'C}} = \frac{Z_0}{R_t} \lambda.$$

Similarly, the values of parallel inductances, and in case of $R_t < Z_0$, those of the series or parallel capacitances as well as the characteristics of the compensated transmission line can be obtained.

The per unit length values of the compensating elements, the characteristic resistance and the wavelength of the compensated transmission line are given in Table 1. In case $R_t > Z_0$ the compensation should be inductive while in case $R_t < Z_0$ it is to be capacitive.

	$R_t > Z_0$		$R_t < Z_0$	
Compensating element	$L_s = C(R_t^2 - Z_0^2)$	$L_p = \frac{1}{\omega^2 C \left(1 - \frac{Z_0^2}{R_t^2}\right)}$	$C_s = \frac{1}{\omega^2 C (Z_0^2 - R_t^2)}$	$C_p = C \left(\frac{Z_0^2}{R_t^2} - 1\right)$
Characteristic resistance Z_0 of compensated line	$\sqrt{\frac{L + L_s}{C}}$	$\sqrt{\frac{L}{C - \frac{1}{\omega^2 L_p}}}$	$\sqrt{\frac{L - \frac{1}{\omega^2 C_s}}{C}}$	$\sqrt{\frac{L}{C + C_p}}$
Wavelength of compensated line	$\frac{Z_0}{R_t} \lambda$	$\frac{R_t}{Z_0} \lambda$	$\frac{Z_0}{R_t} \lambda$	$\frac{R_t}{Z_0} \lambda$

Denotes the wavelength, and the subscripts S and P refer to series and parallel compensation, respectively.

It can be stated that assuming a variable terminal load with $R_t > Z_0$, a parallel inductive (L_p) compensation, while with $R_t < Z_0$, a series capacitive (C_s) compensation offers greater advantages. Namely, in these cases the wavelength on the transmission line increases which may result in a negligible variation of voltage and current along the line. In an extreme case, by compensating the whole of the shunt reactances the entire capacitive reactive power of the transmission line ($C - 1/\omega^2 L_p = 0$), a constant current can be ensured along the line. Compensation of the whole of the series reactances the entire inductive reactive power of the transmission line, results in a constant voltage at any value of the load impedance.

Compensation by series or parallel lumped reactances

The equivalent circuit of the compensated transmission line can be obtained by describing the system as a network composed of distributed parameter transmission line sections and of inserted compensating elements. The effect of inserting various compensating elements (series reactances in Fig. 1) will be studied on the models of chain connected transmission line sections of lengths l_c compensated at their middle point, as shown in Fig. 1. The values of

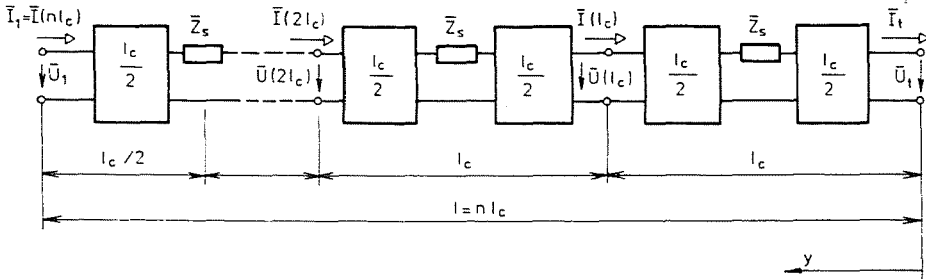


Fig. 1. Equivalent circuit of transmission line compensated with lumped reactances

the compensating reactances and relationships concerning the voltage and current distribution will be obtained as functions of the transmission line characteristic parameters, the load resistance and the length l_c .

The relationship between the currents and voltages at the terminal point $y=0$ and at the point $y=l_c$ is given with the aid of the chain matrix $\mathbf{T}(l_c)$ of the transmission line section:

$$\begin{bmatrix} \bar{U}(l_c) \\ \bar{I}(l_c) \end{bmatrix} = \mathbf{T}(l_c/2) \mathbf{K} \mathbf{T}(l_c/2) \begin{bmatrix} \bar{U}_t \\ \bar{I}_t \end{bmatrix} = \mathbf{T}(l_c) \begin{bmatrix} \bar{U}_t \\ \bar{I}_t \end{bmatrix} = \begin{bmatrix} \bar{A}(l_c) & \bar{B}(l_c) \\ \bar{C}(l_c) & \bar{D}(l_c) \end{bmatrix} \begin{bmatrix} \bar{U}_t \\ \bar{I}_t \end{bmatrix}$$

where

$\mathbf{T}(l_c/2)$ is the chain matrix of a transmission line section of length l_c [6].

\mathbf{K} is the chain matrix of the compensating four-pole (in this case an impedance \bar{Z}_s or \bar{Z}_p),

\bar{U}_t, \bar{I}_t are the complex r.m.s. values of the terminal ($y=0$) voltage and current,

$\bar{U}(l_c), \bar{I}(l_c)$ are the complex r.m.s. values of the voltage and current at $y=l_c$,

$\bar{A}(l_c), \bar{B}(l_c), \bar{C}(l_c), \bar{D}(l_c)$ are general chain parameters.

Aiming at a uniform voltage distribution, the compensating reactance four-pole should be chosen so that the voltage $\bar{U}(l_c)$ should be of equal amplitude as the load voltage \bar{U}_i :

$$|\bar{U}(l_c)| = |\bar{U}_i|$$

i.e. the equation

$$\left| \bar{A}(l_c) + \frac{\bar{B}(l_c)}{R_t} \right| = 1 \quad (1)$$

should hold.

Similarly, if the value of the compensating reactance is chosen so that

$$|\bar{C}(l_c)R_t + \bar{D}(l_c)| = 1 \quad (2)$$

then

$$|\bar{I}(l_c)| = |\bar{I}_i|.$$

The compensating reactances

$$\frac{1}{\omega C_{SR}} = Z_0 \sin \beta l_c \frac{Z_0^2 - R_t^2}{Z_0^2 \cos^2 \beta \frac{l_c}{2} + R_t^2 \sin^2 \beta \frac{l_c}{2}} \quad \text{if } R_t < Z_0$$

$$\omega L_{SR} = Z_0 \sin \beta l_c \frac{R_t^2 - Z_0^2}{Z_0^2 \cos^2 \beta \frac{l_c}{2} + R_t^2 \sin^2 \beta \frac{l_c}{2}} \quad \text{if } R_t > Z_0$$

$$\frac{1}{\omega C_{PR}} = \frac{Z_0}{\sin \beta l_c} \frac{Z_0^2 \sin^2 \beta \frac{l_c}{2} + R_t^2 \cos^2 \beta \frac{l_c}{2}}{Z_0^2 - R_t^2} \quad \text{if } R_t < Z_0$$

$$\omega L_{PR} = \frac{Z_0}{\sin \beta l_c} \frac{Z_0^2 \sin^2 \beta \frac{l_c}{2} + R_t^2 \cos^2 \beta \frac{l_c}{2}}{R_t^2 - Z_0^2} \quad \text{if } R_t > Z_0$$

where $\beta = \omega \sqrt{LC}$ is the wavenumber satisfy both (1) and (2) while the reactances

$$\frac{1}{\omega C_{SO}} = 2Z_0 \operatorname{tg} \beta \frac{l_c}{2}$$

$$\frac{1}{\omega C_{PO}} = \frac{1}{2} Z_0 \operatorname{tg} \beta \frac{l_c}{2}$$

satisfy (1) only, and the reactances

$$\omega L_{SO} = 2Z_0 \frac{1}{\operatorname{tg} \beta \frac{l_c}{2}}$$

$$\omega L_{PO} = \frac{1}{2} Z_0 \frac{1}{\operatorname{tg} \beta \frac{l_c}{2}}$$

satisfy (2) only.

A detailed study of the effect of inserting these reactances reveals that at one group of compensating elements (C_{SR} , C_{PR} , L_{SR} , L_{PR} where the subscript R refers to the fact that the selected values of the compensating elements depend on the terminal load resistance) the voltages and currents at the points $y = kl_c$ as well as the input resistance equal the terminal values, and further that the voltage and current distribution along the sections of lengths l_c vary in phase only. The reactive power of the series or parallel reactances to be inserted in this fashion is of a value ensuring a balance of reactive power in the line either along its whole length or a compensating length of l_c . This means that the difference of the inductive reactive power stored in the distributed series inductances of the transmission line terminated by a resistance $R_t \neq Z_0$ and that of the capacitive reactive power stored in the distributed parallel capacitances of the transmission line is balanced with the aid of compensating elements. This equilibrium is only attained at one fixed value of the terminal load resistance.

Another group of compensating elements (C_{SO} , C_{PO} , L_{SO} , L_{PO}) can be selected independent of the terminal load resistance. E.g. in the case of C_{SO} , the voltages of the points $y = kl_c$ (their currents in case of L_{PO}) equal the terminal value not only in amplitude but also in phase. (Eq. (1) is satisfied not only for absolute values.) At the same time, in the case of C_{SO} , the current (in case of L_{PO} , the voltage) differs from the terminal value also in amplitude. The reactive power inserted equals the entire inductive (capacitive) reactive power of the original transmission line. Thus, e.g. the capacitive reactive power of the series compensating capacitors C_{SO} balances the whole of the inductive power stored in the distributed series inductances of the transmission line. This equilibrium is maintained independent of the load resistance of the current flowing through the series elements while the entire reactive power of the transmission line is balanced at one particular load only, e.g. in case of C_{SO} compensation at $R_t = 0$.

The qualitative phasor diagrams in Fig. 2 clearly show the differences in the various ways of compensation. Figure 2.a shows the simplified model of the terminal section of length l_c of a compensated transmission line terminated by a resistance R_t , while the voltage-current phasor diagram of the case without compensation is drawn in Fig. 2.b. The effects of the compensating elements

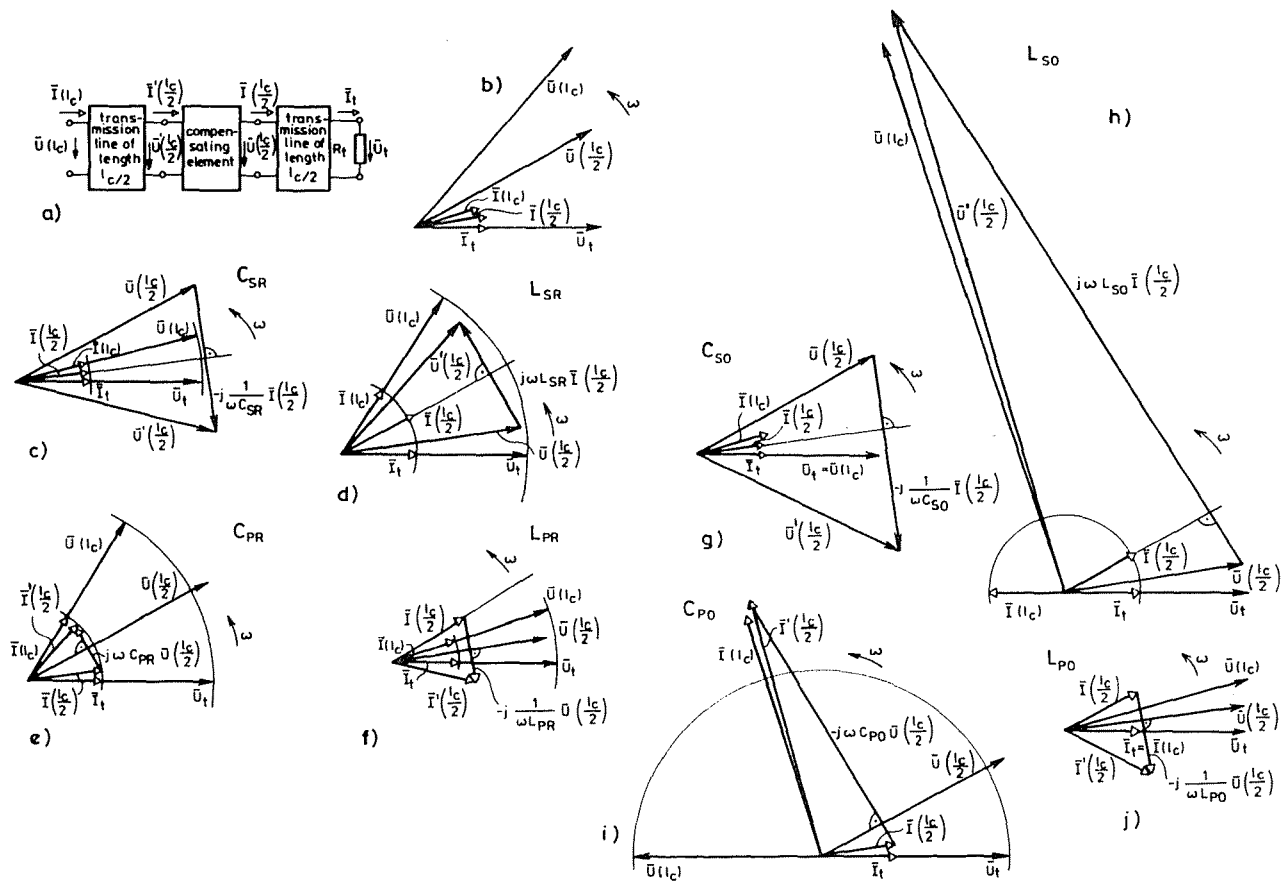


Fig. 2. a) Equivalent circuit
 b) Phasor diagram without compensation
 c) . . . j) Phasor diagrams at various compensations
 of a length l_c section of a compensated transmission line

selected depending on the load resistance (C_{SR}, \dots, L_{PR}) can be seen in Figs. 2.c, . . . 2.f. Comparing the effects of C_{SR} and C_{PR} (Figs 2.c and 2.e), it is worth noting that the parallel capacitive compensation causes a considerably greater "rotation" of the voltage and current phasors. Thus, the statement that a series capacitive compensation tends to increase the wavelength and a parallel one to decrease it, may be deemed valid for lumped compensation as well. The phasor diagrams of L_{SR} and L_{PR} (Figs 2.d and 2.f) indicate a greater rotational effect of the series inductive compensation.

Comparison of Figs 2.g and 2.i suggests that the employment of compensating capacitors C_{PO} is not recommendable due to the considerable increase in current. The employment of L_{SO} (Fig. 2.h) is disadvantageous due to the inadmissible increase in voltage.

*Compensation at arbitrary terminal load
(π or T compensation)*

At the points $y=kl_c$, voltages and currents with both their amplitudes and phases identical to the terminal voltage and current independent of the value of the load impedance \bar{Z}_l can be obtained if the resultant chain matrix of the compensated transmission line section of length l_c is a unit matrix:

$$\mathbf{T}(l_c) = \mathbf{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Let the compensating device be located at a distance l_k from the terminal of the transmission line section of length l_c . The solution of the matrix equation

$$\mathbf{E} = \mathbf{T}(l_c - l_k) \mathbf{K} \mathbf{T}(l_k)$$

is

$$\mathbf{K} = \begin{bmatrix} \cos \beta l_c & -j Z_0 \sin \beta l_c \\ -j \frac{1}{Z_0} \sin \beta l_c & \cos \beta l_c \end{bmatrix}. \quad (3)$$

The series and parallel impedances of the compensating π element (Fig. 3) are:

$$\bar{Z}_{S\pi} = -j Z_0 \sin \beta l_c, \quad \bar{Z}_{P\pi} = j \frac{Z_0}{\tan \beta l_c / 2}$$

which is equivalent to the insertion of the reactances

$$\frac{1}{\omega C_S} = Z_0 \sin \beta l_c \quad \text{and} \quad \omega L_P = \frac{Z_0}{\tan \beta l_c / 2}.$$

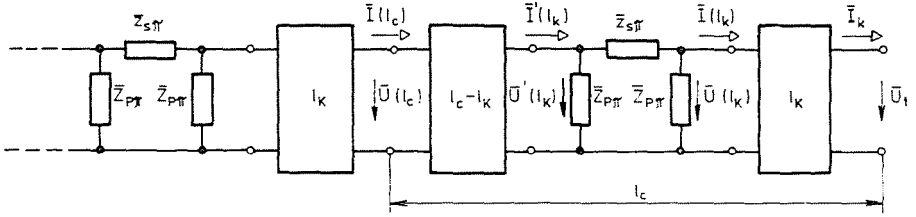


Fig. 3. Compensation of transmission lines with π elements

In the case of the compensating T element (Fig. 4):

$$\bar{Z}_{ST} = -jZ_0 \tan \beta \frac{l_c}{2} \quad \bar{Z}_{PT} = j \frac{Z_0}{\sin \beta l_c}$$

$$\frac{1}{\omega C_S} = Z_0 \tan \beta \frac{l_c}{2} \quad \omega L_P = \frac{Z_0}{\sin \beta l_c}$$

It is noted that with real (lossy) transmission lines taken into account.

$$\bar{Z}_{S\pi} = -\bar{Z}_0 \operatorname{sh} \bar{\gamma} l_c \quad \bar{Z}_{ST} = -\bar{Z}_0 \operatorname{th} \bar{\gamma} \frac{l_c}{2}$$

$$\bar{Z}_{P\pi} = -\frac{\bar{Z}_0}{\operatorname{th} \bar{\gamma} \frac{l_c}{2}} \quad \text{and} \quad \bar{Z}_{PT} = -\frac{\bar{Z}_0}{\operatorname{sh} \bar{\gamma} l_c}$$

is obtained.

It is worth observing that if the compensating four-pole is selected according to (3), the compensating elements are independent of the terminal load resistance and of the location l_k of compensation. It hence follows that for any compensated section of length l_c of the transmission line, the chain matrix is a unit matrix, the voltage and current are periodic with an l_c period.

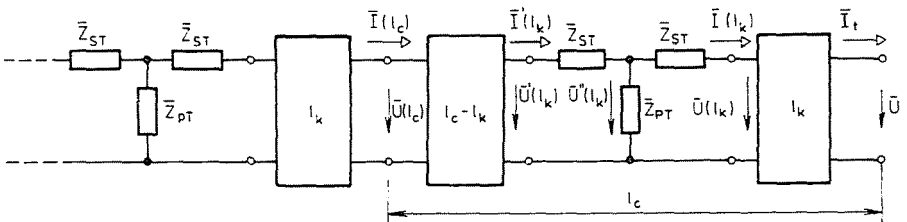


Fig. 4. Compensation of transmission lines with T elements

The selection of the compensating four-pole according to (3) ensures the balancing of the entire inductive and capacitive reactance and reactive power of the transmission line. Regarding the special points $y = kl_c$, the compensated, ideal transmission line behaves as an ideal (short) conductor pair.

Compensation of medium frequency lines loaded at an arbitrary point

When discussing transmission lines loaded at an intermediate point, a distinction should be made according to whether the consumers require a voltage or current generator supply.

The case of parallel loads will only be considered here, i.e. the possibility of maintaining constant voltage along the line with the transmission line compensated by π or T elements.

1. If the load impedance \bar{Z}_{t1} connects to the special points $y = k_1 l_c$, the resultant chain matrix giving the relationship between the voltages and currents of the points

$$y = kl_c \quad k > k_1 \quad k, k_1 = 1, 2, 3, \dots$$

and the terminal voltages and currents is:

$$\begin{bmatrix} 1 & 0 \\ 1/\bar{Z}_{t1} & 1 \end{bmatrix}$$

In case of distinct impedances \bar{Z}_{ti} connecting at several points k_i ($i = 1, 2, \dots, j$), the resultant chain matrix at the points

$$y = kl_c \quad k > k_j$$

is:

$$\begin{bmatrix} 1 & 0 \\ \sum_{i=1}^j 1/\bar{Z}_{ti} & 1 \end{bmatrix}$$

Thus, load connected to the lossfree line at these special points has no effect upon the voltages at the points $y = kl_c$, the voltages equal the terminal value.

2. If the load impedance is connected at an arbitrary intermediate point $y \neq kl_c$, the transmission line section compensated at its midpoint can be created e.g. as the resultant of the chain connected four-poles shown in Fig. 5.

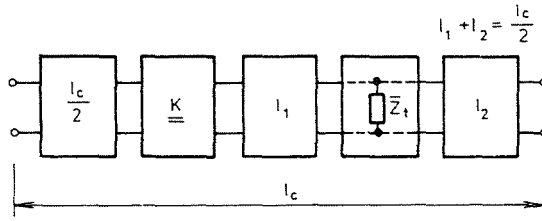


Fig. 5. Transmission line loaded at intermediate ($y \neq kl_c$) point

The resultant chain matrix of the arrangement is:

$$\begin{bmatrix} 1 - j \frac{Z_0}{\bar{Z}_t} \sin \beta l_2 \cos \beta l_2 & \frac{Z_0^2}{\bar{Z}_t} \sin^2 \beta l_2 \\ \frac{1}{\bar{Z}_t} \cos^2 \beta l_2 & 1 + j \frac{Z_0}{\bar{Z}_t} \sin \beta l_2 \cos \beta l_2 \end{bmatrix}$$

The voltage difference between the two terminal points of the transmission line section is seen to depend on the characteristics of the transmission line (Z_0, β), on the value and character of the load (\bar{Z}_t) and on its location (l_2).

However, considering the transmission line section between the load impedance \bar{Z}_t and the feed point, where no further loads are assumed, it must still hold that the resultant chain matrix of any compensated transmission line section is a unit matrix and the voltage and current of the transmission line recur after a distance of l_c .

Hence, the effect of any load impedance \bar{Z}_t modifying the voltage and current distribution is present within a distance of l_c towards the feed point, however, beyond that, the thus formed voltage and current distribution will recur. Only the effect of the load impedance within a distance l_c will prevail.

It is clearly futile to seek a compensation for impedances \bar{Z}_t at arbitrary locations and of any value and phase (since then the case of e.g. $\bar{Z}_t = 0$ had to be also included). However, knowing the limits of the load impedance variation, with the aid of π or T compensation, primarily by appropriately selecting the compensating distance l_c , the variation of voltage (or current) can be kept within range, independent of the transmission line length.

Test computations

The transmission line examined is formed by two parallel copper rails facing each other (see Fig. 6), surrounded by isolating material.

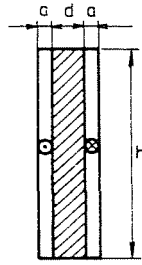


Fig. 6. Arrangement of conductor rail

The dimensions are:

$$\begin{aligned} a &= 1 \text{ mm} \\ h &= 100 \text{ mm} \\ d &= 6 \text{ mm} \end{aligned}$$

Electrical characteristics at 10 kHz:

$$\begin{aligned} R &\cong 0.6 \Omega/\text{km} \\ L &\cong 40 \cdot 10^{-6} \text{ H/km} \\ C &\cong 0.7 \cdot 10^{-6} \text{ F/km.} \end{aligned}$$

In order to simplify the calculation, the losses of the transmission line are neglected, hence:

$$\begin{aligned} \beta &= 0.3325 \text{ km}^{-1} \\ Z_0 &= 7.56 \\ \lambda &= 18.9 \text{ km.} \end{aligned}$$

Further assumptions:

- the length of the transmission line is $l = 5 \text{ km}$, greater than a quarter of the transmission line wavelength without compensation;
- the load is at the terminal of the transmission line;
- compensation is carried out at a frequency $f_a = 10 \text{ kHz}$.

The load condition of the transmission line is characterized by the ratio of the characteristic resistance and the load resistance. The value $Z_0/R_t = 1$ is not admissible since then the voltage drop on the resistance $R = 3 \Omega$ of the transmission line neglected during further calculations would be close to 30% of the feed voltage. Thus the strongly loaded condition of the transmission line has been computed at $Z_0/R_t = 10^{-1}$ compared to which $Z_0/R_t = 10^{-3}$ is approximately an open circuit condition.

At a given load condition, the effects of the variation of load and frequency were investigated employing different methods of compensation.

Some curves characterizing the voltage distribution along the line are shown in Figs 7, 8 and 9 for a transmission line loaded at its terminal.

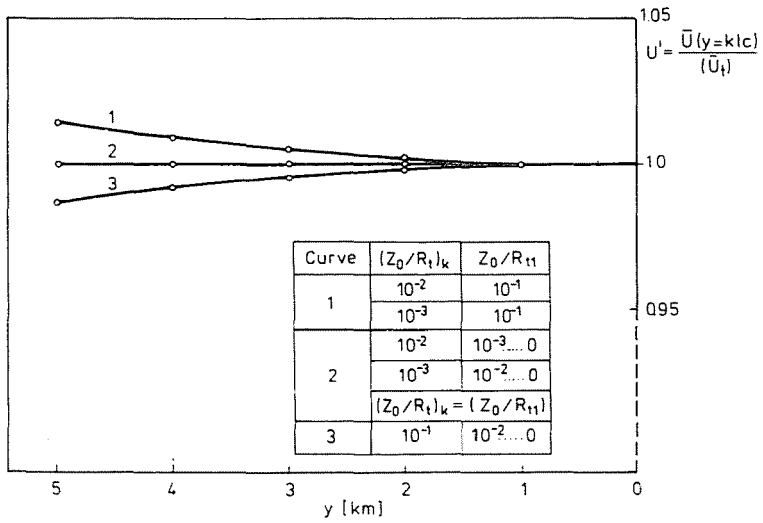


Fig. 7. Effect of load variation at distributed L_p shunt compensation (U')
 $Z_0 \cong 7.56 \Omega$, $\lambda \cong 18.9 \text{ km}$
 selection of compensating L_p : for R_t actual load resistance, R_{t1}

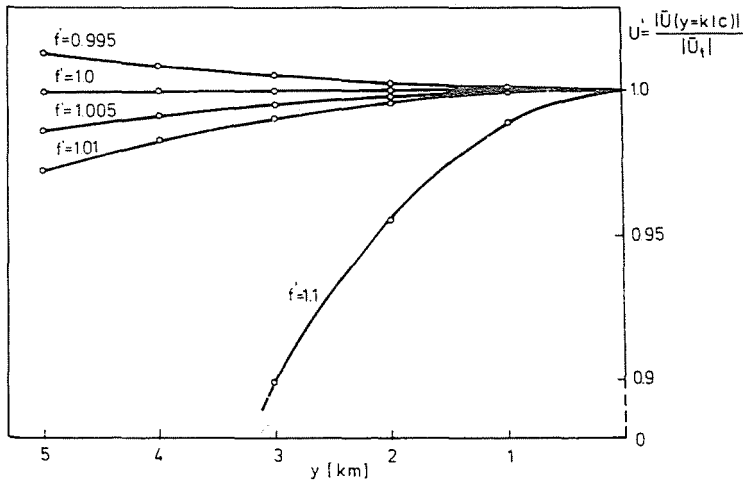


Fig. 8. Effect of frequency variation at distributed L_p shunt compensation (U')
 $Z_0 \cong 7.56 \Omega$, $\lambda = 18.9 \text{ km}$

Valid both for $\frac{Z_0}{R_t} = 10^{-1}$ and $\frac{Z_0}{R_t} 10^{-2}$

$$f_a = 10 \text{ kHz} \quad f' = \frac{f}{f_a}$$

The computations carried out for lumped element compensation by inductances L_{PR} , L_{PO} yielded almost identical results to the case with distributed L_p compensation shown in Figs 7 and 8. At lumped element compensation, the curves are only valid at the points $y = kl_c$, but with l_c chosen appropriately, the voltage distribution of intermediate points is well approximated by the curves plotted.

Compensation by π and T elements yields considerably more favourable results than L_p shunt compensation.

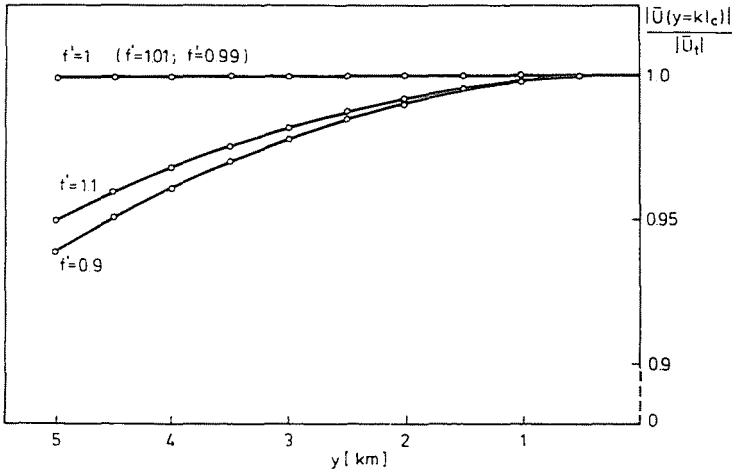


Fig. 9. Effect of frequency variation at π or T compensation. Curves only valid at points $y = 0.5, 1, \dots, 5$ km. Valid both for $\frac{Z_0}{R_t} = 10^{-1}$ and $\frac{Z_0}{R_t} = 10^{-2}$

$$f_a = 10 \text{ kHz}, f' = \frac{f}{f_a}$$

Conclusions

Three, principally distinct possibilities exist for maintaining approximately uniform voltage and current distribution along long transmission lines, with case $R_t = Z_0$ excluded.

One of them is applicable when the terminal load resistance differs from the characteristic resistance, but is of constant value. The characteristic resistance can be changed to $Z'_0 = R_t$ with the aid of series or parallel, capacitive or inductive reactances. The reactive power balance of the transmission line is

ensured by the insertion of capacitive or inductive reactive power. In theory, the reactive power balance would have to be maintained at each point of the transmission line, in practice this can be attained in a preselected neighbourhood of lumped compensating elements (C_{SR} , C_{PR} , L_{SR} , L_{PR}) located at a distance l_c from each other. Since the insufficiency or excess of reactive power in the transmission line depends upon the ratio of the load resistance and characteristic resistance, for a given transmission line characteristic resistance and fixed compensating reactance, the most favourable voltage and current distribution (voltage and/or current of identical amplitude at points located at kl_c from the terminal of the transmission line) can only be attained at one particular load.

A second way of compensation is successful even in case of variable terminal load by increasing the wavelength to infinite. To this end, the entire series or shunt impedance (inductive or capacitive reactive power) of the transmission line should be balanced, in theory at each point of the line, in practice within a distance l_c . While in the previous case the series and parallel compensation ensured the uniformity of both current and voltage, now balancing the inductive reactive power (C_{SO}) yields constant voltage but variable current and balancing the capacitive reactive power (L_{PO}) results in constant current but variable voltage along the line. In regard of the special points $y = kl_c$ the same result is obtained by equating the half wavelength of the transmission line to l_c by decreasing the wavelength (L_{SO} , C_{PO}). However, due to the great variation of the voltage or current of intermediate points ($y \neq kl_c$) this latter way of compensation is not recommendable.

The most favourable result, a uniform distribution of voltage or current regardless of the value and location of load impedance, is obtained by balancing both the series inductive and parallel capacitive reactance (inductive and capacitive reactive power) of the transmission line with the aid of series capacitors and shunt inductors. If this is carried out at each point of the transmission line, the lossfree transmission line behaves as an ideal line without voltage drop. In practice this condition can be well approximated with the aid of a π or T arrangement of series capacitors and shunt inductors.

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